Computer algebra independent integration tests

4-Trig-functions/4.2-Cosine/4.2.10-c+d-x-^m-a+b-cos-^n

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May 18, 2020 Compiled on May 18, 2020 at 10:40am

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4 Listing of Grading functions **683**

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [189]. This is test number [83].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (189)	% 0. (0)
Mathematica	% 98.94 (187)	% 1.06 (2)
Maple	% 70.9 (134)	% 29.1 (55)
Maxima	% 70.9 (134)	% 29.1 (55)
Fricas	% 70.37 (133)	% 29.63 (56)
Sympy	% 26.46 (50)	% 73.54 (139)
Giac	% 42.33 (80)	% 57.67 (109)

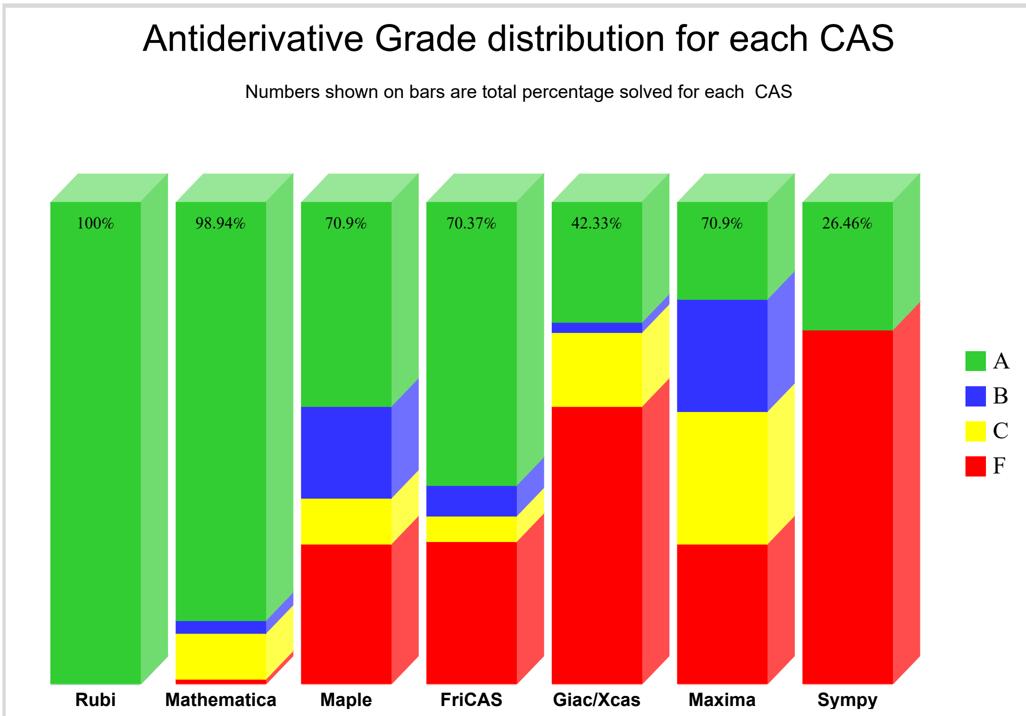
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

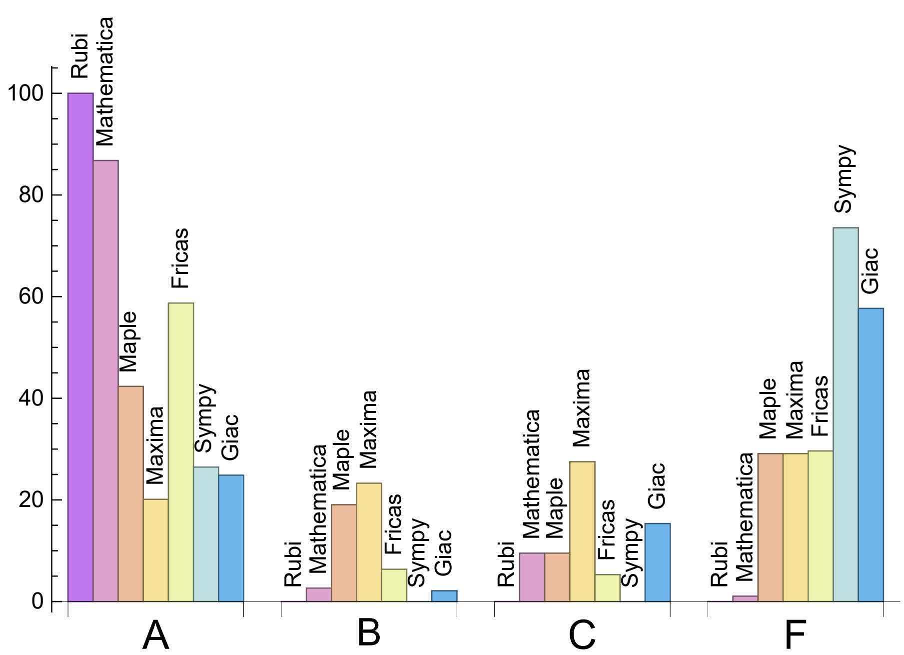
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	86.77	2.65	9.52	1.06
Maple	42.33	19.05	9.52	29.1
Maxima	20.11	23.28	27.51	29.1
Fricas	58.73	6.35	5.29	29.63
Sympy	26.46	0.	0.	73.54
Giac	24.87	2.12	15.34	57.67

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.16	108.02	0.86	89.	1.
Mathematica	1.03	113.16	0.86	77.	0.87
Maple	0.22	210.84	1.81	143.	1.35
Maxima	1.6	506.76	3.43	274.5	2.96
Fricas	1.47	505.14	3.49	290.	2.65
Sympy	1.94	139.98	1.29	61.	1.51
Giac	0.86	1090.08	10.76	147.5	1.74

1.4 list of integrals that has no closed form antiderivative

{32, 36, 40, 75, 77, 78, 80, 82, 83, 85, 86, 88, 98, 102, 103, 131, 132, 136, 137, 141, 142, 174, 179, 183, 184, 188}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {31, 139, 189}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

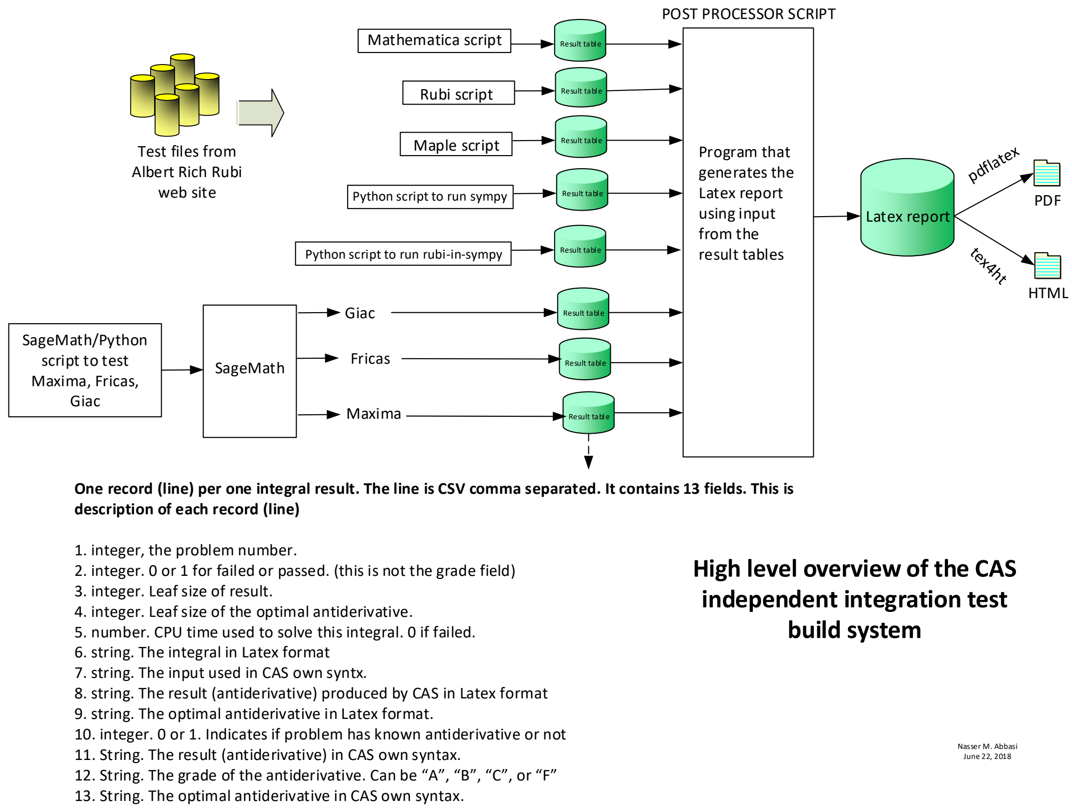
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.


POST PROCESSOR SCRIPT
Mathematica script
Rubi script
Maple script
Python script to run sympy
Python script to run rubi-in-sympy
Result table
Test files from Albert Rich Rubi web site
Program that generates the Latex report using input from the result tables
Latex report
pdflatex
tex4ht
PDF
HTML
SageMath/Python script to test Maxima, Fricas, Giac
SageMath
Giac
Fricas
Maxima
One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)
1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.
High level overview of the CAS independent integration test build system
Nasser M. Abbasi
June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 48, 49, 52, 54, 56, 57, 60, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 188 }

B grade: { 39, 62, 139, 187, 189 }

C grade: { 41, 42, 43, 44, 45, 46, 47, 50, 51, 53, 55, 58, 59, 61, 63, 64, 65, 66 }

F grade: { 75, 86}

2.1.3 Maple

A grade: { 4, 5, 6, 7, 8, 13, 14, 15, 19, 20, 21, 22, 25, 26, 27, 28, 32, 35, 36, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 75, 77, 78, 80, 82, 83, 85, 86, 87, 98, 102, 103, 121, 122, 126, 127, 130, 131, 132, 135, 136, 137, 140, 141, 142, 146, 153, 160, 174, 178, 179, 183, 184, 188 }

B grade: { 1, 2, 3, 9, 10, 11, 12, 16, 17, 18, 23, 24, 29, 30, 31, 33, 34, 37, 38, 39, 76, 79, 118, 119, 120, 123, 124, 125, 128, 129, 133, 134, 138, 139, 187, 189 }

C grade: { 84, 104, 105, 106, 107, 108, 109, 110, 143, 144, 145, 150, 151, 152, 157, 158, 159, 173 }

F grade: { 67, 68, 69, 70, 71, 72, 73, 74, 81, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 111, 112, 113, 114, 115, 116, 117, 147, 148, 149, 154, 155, 156, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 175, 176, 177, 180, 181, 182, 185, 186 }

2.1.4 Maxima

A grade: { 4, 12, 19, 23, 24, 25, 32, 75, 77, 78, 80, 82, 83, 85, 86, 88, 98, 102, 103, 125, 136, 137, 144, 145, 146, 150, 151, 152, 153, 160, 164, 165, 166, 174, 179, 183, 184, 188

B grade: { 1, 2, 3, 9, 10, 11, 16, 17, 18, 29, 30, 33, 34, 35, 37, 38, 67, 68, 69, 70, 71, 72, 73, 74, 118, 119, 120, 123, 124, 128, 129, 130, 133, 134, 135, 138, 139, 140, 143, 157, 158, 159, 173, 178 }

C grade: { 5, 6, 7, 8, 13, 14, 15, 20, 21, 22, 26, 27, 28, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 121, 122, 126, 127, 147, 148, 149, 154, 155, 156, 167, 168, 169 }

F grade: { 31, 36, 39, 40, 76, 79, 81, 84, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 131, 132, 141, 142, 161, 162, 163, 170, 171, 172, 175, 176, 177, 180, 181, 182, 185, 186, 187, 189 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 32, 35, 36, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 91, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 130, 131, 132, 135, 136, 137, 140, 141, 142, 146, 153, 160, 173, 174, 178, 179, 183, 188 }

B grade: { 7, 8, 22, 31, 34, 39, 55, 129, 134, 139, 187, 189 }

C grade: { 29, 30, 33, 37, 38, 128, 133, 138, 185, 186 }

F grade: { 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 97, 143, 144, 145, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 175, 176, 177, 180, 181, 182, 184 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 9, 10, 11, 12, 16, 17, 18, 19, 23, 24, 25, 26, 32, 36, 40, 63, 64, 65, 66, 75, 77, 83, 85, 98, 102, 103, 118, 119, 120, 123, 124, 125, 130, 131, 132, 135, 136, 137, 140, 141, 142, 174, 179, 183, 184, 188 }

B grade: { }

C grade: { }

F grade: { 5, 6, 7, 8, 13, 14, 15, 20, 21, 22, 27, 28, 29, 30, 31, 33, 34, 35, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 67, 68, 69, 70, 71, 72, 73, 74, 76, 78, 79, 80, 81, 82, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 121, 122, 126, 127, 128, 129, 133, 134, 138, 139, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 180, 181, 182, 185, 186, 187, 189 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 9, 10, 11, 12, 16, 17, 18, 19, 23, 24, 25, 32, 36, 40, 75, 77, 78, 80, 82, 83, 85, 86, 88, 98, 102, 103, 118, 119, 120, 123, 124, 125, 131, 132, 136, 137, 141, 142, 174, 179, 183, 184, 188 }

B grade: { 35, 130, 135, 140 }

C grade: { 5, 6, 7, 8, 13, 14, 15, 20, 26, 27, 28, 41, 42, 43, 44, 48, 49, 50, 51, 56, 57, 58, 59, 63, 64, 65, 121, 122, 126 }

F grade: { 21, 22, 29, 30, 31, 33, 34, 37, 38, 39, 45, 46, 47, 52, 53, 54, 55, 60, 61, 62, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 79, 81, 84, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 127, 128, 129, 133, 134, 138, 139, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 180, 181, 182, 185, 186, 187, 189 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	76	539	649	347	311	230
normalized size	1	1.	0.84	5.92	7.13	3.81	3.42	2.53
time (sec)	N/A	0.093	0.337	0.029	1.116	1.026	2.878	1.134

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	61	302	375	228	202	149
normalized size	1	1.	0.87	4.31	5.36	3.26	2.89	2.13
time (sec)	N/A	0.066	0.222	0.029	1.012	1.065	1.313	1.123

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	44	143	184	136	112	86
normalized size	1	1.	0.9	2.92	3.76	2.78	2.29	1.76
time (sec)	N/A	0.041	0.167	0.026	1.01	1.129	0.638	1.101

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	51	68	69	46	41
normalized size	1	1.	0.96	1.89	2.52	2.56	1.7	1.52
time (sec)	N/A	0.016	0.056	0.029	0.96	1.058	0.234	1.1

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	50	72	192	200	0	779
normalized size	1	1.	0.96	1.38	3.69	3.85	0.	14.98
time (sec)	N/A	0.098	0.099	0.027	1.218	1.06	0.	1.182

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	65	109	224	302	0	4298
normalized size	1	1.	0.89	1.49	3.07	4.14	0.	58.88
time (sec)	N/A	0.11	0.393	0.03	1.315	1.107	0.	1.49

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	89	143	271	471	0	7449
normalized size	1	1.	0.86	1.38	2.61	4.53	0.	71.62
time (sec)	N/A	0.138	0.609	0.033	1.601	1.16	0.	1.538

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	144	179	339	639	0	11310
normalized size	1	1.	1.13	1.41	2.67	5.03	0.	89.06
time (sec)	N/A	0.159	0.552	0.032	1.934	1.174	0.	1.886

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	132	1027	968	593	660	300
normalized size	1	1.	0.82	6.38	6.01	3.68	4.1	1.86
time (sec)	N/A	0.102	0.658	0.047	1.189	1.164	5.988	1.101

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	106	587	578	394	456	207
normalized size	1	1.	0.79	4.38	4.31	2.94	3.4	1.54
time (sec)	N/A	0.074	0.451	0.027	1.114	1.158	3.263	1.125

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	77	289	300	247	264	127
normalized size	1	1.	0.81	3.04	3.16	2.6	2.78	1.34
time (sec)	N/A	0.053	0.312	0.026	1.069	1.094	1.424	1.122

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	50	112	122	130	126	65
normalized size	1	1.	0.91	2.04	2.22	2.36	2.29	1.18
time (sec)	N/A	0.025	0.16	0.028	1.054	1.026	0.619	1.104

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	65	105	217	236	0	824
normalized size	1	1.	0.83	1.35	2.78	3.03	0.	10.56
time (sec)	N/A	0.153	0.131	0.029	1.207	1.162	0.	1.203

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	75	156	231	319	0	3976
normalized size	1	1.	0.9	1.88	2.78	3.84	0.	47.9
time (sec)	N/A	0.134	0.64	0.03	1.314	1.157	0.	1.341

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	102	193	278	502	0	6934
normalized size	1	1.	0.91	1.72	2.48	4.48	0.	61.91
time (sec)	N/A	0.197	0.925	0.032	1.617	1.424	0.	1.972

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	385	1023	1249	759	772	474
normalized size	1	1.	1.71	4.55	5.55	3.37	3.43	2.11
time (sec)	N/A	0.254	1.03	0.071	1.223	1.546	10.42	1.151

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	121	560	722	494	495	312
normalized size	1	1.	0.69	3.2	4.13	2.82	2.83	1.78
time (sec)	N/A	0.156	0.969	0.029	1.113	1.504	5.387	1.123

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	93	265	360	297	284	185
normalized size	1	1.	0.76	2.15	2.93	2.41	2.31	1.5
time (sec)	N/A	0.097	0.603	0.029	1.062	1.47	3.029	1.201

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	52	95	139	153	126	93
normalized size	1	1.	0.69	1.27	1.85	2.04	1.68	1.24
time (sec)	N/A	0.042	0.164	0.027	0.997	1.38	1.18	1.2

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	103	166	373	406	0	8201
normalized size	1	1.	0.85	1.37	3.08	3.36	0.	67.78
time (sec)	N/A	0.244	0.262	0.03	1.347	1.328	0.	2.088

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	200	242	410	571	0	0
normalized size	1	1.	1.38	1.67	2.83	3.94	0.	0.
time (sec)	N/A	0.225	0.698	0.048	1.614	1.436	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	221	311	458	861	0	0
normalized size	1	1.	1.2	1.69	2.49	4.68	0.	0.
time (sec)	N/A	0.345	0.852	0.035	1.962	1.614	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	100	440	409	271	253	146
normalized size	1	1.	0.58	2.56	2.38	1.58	1.47	0.85
time (sec)	N/A	0.154	0.414	0.056	1.056	1.41	9.483	1.113

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	92	241	254	217	209	113
normalized size	1	1.	0.69	1.8	1.9	1.62	1.56	0.84
time (sec)	N/A	0.108	0.176	0.029	1.067	1.31	6.461	1.129

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	53	110	132	161	138	86
normalized size	1	1.	0.66	1.38	1.65	2.01	1.72	1.08
time (sec)	N/A	0.047	0.133	0.026	1.017	1.36	3.419	1.136

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	52	52	123	274	60	578
normalized size	1	1.	0.88	0.88	2.08	4.64	1.02	9.8
time (sec)	N/A	0.158	0.102	0.03	1.207	1.472	5.737	1.141

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	79	90	980	312	0	4347
normalized size	1	1.	1.2	1.36	14.85	4.73	0.	65.86
time (sec)	N/A	0.151	0.22	0.03	1.35	1.442	0.	1.259

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	119	124	1073	389	0	5292
normalized size	1	1.	1.32	1.38	11.92	4.32	0.	58.8
time (sec)	N/A	0.297	0.303	0.033	1.384	1.448	0.	1.265

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	196	685	961	2367	0	0
normalized size	1	1.	0.96	3.34	4.69	11.55	0.	0.
time (sec)	N/A	0.157	0.188	0.397	2.075	1.813	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	130	392	535	1534	0	0
normalized size	1	1.	0.95	2.86	3.91	11.2	0.	0.
time (sec)	N/A	0.092	0.109	0.366	1.851	1.707	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	87	167	0	837	0	0
normalized size	1	1.	1.16	2.23	0.	11.16	0.	0.
time (sec)	N/A	0.042	0.006	0.368	0.	1.561	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	4.468	0.243	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	109	316	1426	2010	0	0
normalized size	1	1.	0.96	2.77	12.51	17.63	0.	0.
time (sec)	N/A	0.215	0.487	0.432	2.733	1.781	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	75	170	437	1187	0	0
normalized size	1	1.	0.91	2.07	5.33	14.48	0.	0.
time (sec)	N/A	0.134	0.243	0.374	2.513	1.734	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	36	37	215	115	0	1970
normalized size	1	1.	1.29	1.32	7.68	4.11	0.	70.36
time (sec)	N/A	0.027	0.013	0.026	2.115	1.408	0.	1.595

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	5.597	0.332	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	311	1127	5168	3213	0	0
normalized size	1	1.	0.92	3.34	15.34	9.53	0.	0.
time (sec)	N/A	0.269	2.935	0.458	7.551	2.489	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	184	584	2556	2034	0	0
normalized size	1	1.	0.95	3.03	13.24	10.54	0.	0.
time (sec)	N/A	0.143	1.234	0.411	2.997	2.037	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	389	267	0	1170	0	0
normalized size	1	1.	3.32	2.28	0.	10.	0.	0.
time (sec)	N/A	0.069	3.798	0.197	0.	1.786	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	1.448	0.018	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	124	232	886	466	0	1374
normalized size	1	1.	0.64	1.2	4.57	2.4	0.	7.08
time (sec)	N/A	0.423	0.056	0.031	2.246	1.482	0.	1.264

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	122	189	855	394	0	767
normalized size	1	1.	0.72	1.12	5.06	2.33	0.	4.54
time (sec)	N/A	0.236	0.101	0.03	2.23	1.478	0.	1.209

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	124	144	776	328	0	335
normalized size	1	1.	0.87	1.01	5.46	2.31	0.	2.36
time (sec)	N/A	0.172	0.092	0.029	2.053	1.415	0.	1.166

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	124	100	714	269	0	224
normalized size	1	1.	1.05	0.85	6.05	2.28	0.	1.9
time (sec)	N/A	0.134	0.06	0.031	2.123	1.35	0.	1.145

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	147	140	632	363	0	0
normalized size	1	1.	1.06	1.01	4.55	2.61	0.	0.
time (sec)	N/A	0.186	0.33	0.03	1.568	1.428	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	190	180	632	510	0	0
normalized size	1	1.	1.13	1.07	3.76	3.04	0.	0.
time (sec)	N/A	0.262	0.303	0.03	1.511	1.476	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	228	220	632	680	0	0
normalized size	1	1.	1.18	1.14	3.27	3.52	0.	0.
time (sec)	N/A	0.296	0.388	0.03	1.56	1.587	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	194	242	937	613	0	1419
normalized size	1	1.	0.84	1.05	4.06	2.65	0.	6.14
time (sec)	N/A	0.435	1.897	0.038	2.202	1.853	0.	1.38

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	175	197	902	479	0	772
normalized size	1	1.	0.86	0.97	4.44	2.36	0.	3.8
time (sec)	N/A	0.342	1.532	0.036	2.221	1.922	0.	1.279

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	146	150	824	369	0	331
normalized size	1	1.	0.92	0.95	5.22	2.34	0.	2.09
time (sec)	N/A	0.278	0.545	0.036	2.285	1.883	0.	1.202

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	145	108	747	279	0	220
normalized size	1	1.	1.12	0.83	5.75	2.15	0.	1.69
time (sec)	N/A	0.243	0.247	0.039	2.137	1.704	0.	1.183

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	133	146	640	333	0	0
normalized size	1	1.	0.99	1.08	4.74	2.47	0.	0.
time (sec)	N/A	0.258	0.633	0.037	1.473	1.851	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	181	189	644	498	0	0
normalized size	1	1.	1.06	1.11	3.79	2.93	0.	0.
time (sec)	N/A	0.312	1.418	0.036	1.551	1.979	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	244	230	644	733	0	0
normalized size	1	1.	1.13	1.06	2.98	3.39	0.	0.
time (sec)	N/A	0.324	1.29	0.037	1.652	2.07	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	237	273	644	927	0	0
normalized size	1	1.	0.96	1.11	2.61	3.75	0.	0.
time (sec)	N/A	0.407	0.857	0.036	1.718	2.239	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	542	474	1863	921	0	2719
normalized size	1	1.	1.32	1.16	4.54	2.25	0.	6.63
time (sec)	N/A	1.139	3.232	0.043	2.627	2.131	0.	1.608

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	390	386	1793	763	0	1515
normalized size	1	1.	1.1	1.09	5.06	2.16	0.	4.28
time (sec)	N/A	0.991	1.696	0.043	2.702	2.068	0.	1.442

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	254	294	1651	645	0	662
normalized size	1	1.	0.84	0.97	5.43	2.12	0.	2.18
time (sec)	N/A	0.484	0.443	0.04	2.611	1.992	0.	1.293

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	236	212	1530	551	0	443
normalized size	1	1.	0.92	0.82	5.95	2.14	0.	1.72
time (sec)	N/A	0.417	0.409	0.043	2.577	1.711	0.	1.161

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	299	286	1260	682	0	0
normalized size	1	1.	1.1	1.06	4.65	2.52	0.	0.
time (sec)	N/A	0.564	1.548	0.042	1.892	1.984	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	268	368	1261	914	0	0
normalized size	1	1.	0.92	1.26	4.32	3.13	0.	0.
time (sec)	N/A	0.738	2.059	0.043	1.735	2.329	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	C	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	1429	450	1261	1229	0	0
normalized size	1	1.	4.01	1.26	3.54	3.45	0.	0.
time (sec)	N/A	0.831	6.333	0.042	1.706	2.531	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	55	34	100	134	83	93
normalized size	1	1.	1.12	0.69	2.04	2.73	1.69	1.9
time (sec)	N/A	0.057	0.013	0.027	1.78	1.59	16.241	1.142

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	48	27	90	105	61	72
normalized size	1	1.	1.33	0.75	2.5	2.92	1.69	2.
time (sec)	N/A	0.035	0.006	0.028	1.749	1.629	1.193	1.135

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	51	19	81	76	37	47
normalized size	1	1.	2.12	0.79	3.38	3.17	1.54	1.96
time (sec)	N/A	0.02	0.007	0.026	1.878	1.656	0.996	1.137

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	63	28	28	111	61	0
normalized size	1	1.	1.8	0.8	0.8	3.17	1.74	0.
time (sec)	N/A	0.037	0.042	0.028	2.438	1.665	4.173	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	122	0	770	331	0	0
normalized size	1	1.	0.67	0.	4.21	1.81	0.	0.
time (sec)	N/A	0.24	0.107	0.191	1.564	1.755	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	124	0	701	258	0	0
normalized size	1	1.	0.82	0.	4.61	1.7	0.	0.
time (sec)	N/A	0.149	0.118	0.178	1.597	1.727	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	124	0	701	258	0	0
normalized size	1	1.	0.82	0.	4.61	1.7	0.	0.
time (sec)	N/A	0.162	0.1	0.174	1.558	1.789	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	124	0	636	208	0	0
normalized size	1	1.	0.92	0.	4.71	1.54	0.	0.
time (sec)	N/A	0.116	0.06	0.177	1.62	1.739	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	124	0	636	208	0	0
normalized size	1	1.	0.92	0.	4.71	1.54	0.	0.
time (sec)	N/A	0.12	0.064	0.165	1.616	1.665	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	121	0	632	290	0	0
normalized size	1	1.	0.8	0.	4.19	1.92	0.	0.
time (sec)	N/A	0.146	0.054	0.181	1.473	1.749	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	121	0	632	290	0	0
normalized size	1	1.	0.79	0.	4.13	1.9	0.	0.
time (sec)	N/A	0.155	0.057	0.183	1.558	1.733	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	125	0	632	452	0	0
normalized size	1	1.	0.69	0.	3.47	2.48	0.	0.
time (sec)	N/A	0.204	0.054	0.186	1.491	1.866	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.	0.464	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	133	0	0	0	0
normalized size	1	1.	1.	8.31	0.	0.	0.	0.
time (sec)	N/A	0.009	0.028	1.411	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.704	0.141	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	60	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	2.026	0.122	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	36	179	0	0	0	0
normalized size	1	1.	0.86	4.26	0.	0.	0.	0.
time (sec)	N/A	0.02	0.031	2.089	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	16.498	0.122	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	40	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.415	0.275	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	5.124	0.085	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.299	0.125	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	18	0	0	0	0
normalized size	1	1.	1.	1.12	0.	0.	0.	0.
time (sec)	N/A	0.009	0.015	0.03	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.199	0.132	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	54	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.	0.123	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	101	0	0	0	0
normalized size	1	1.	1.	2.66	0.	0.	0.	0.
time (sec)	N/A	0.017	0.054	2.207	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	F	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	15.343	180.	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	33	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.388	0.22	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	17	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.086	0.22	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	54	0	0
normalized size	1	1.	0.71	0.	0.	2.25	0.	0.
time (sec)	N/A	0.046	0.075	0.213	0.	1.598	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	33	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.129	0.298	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	29	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	0.145	0.226	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.09	0.166	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	0.139	0.161	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	45	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.107	0.167	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	51	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.154	0.105	0.151	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.726	0.559	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	253	0	0	470	0	0
normalized size	1	1.	0.92	0.	0.	1.71	0.	0.
time (sec)	N/A	0.304	0.182	0.342	0.	1.848	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	150	0	0	340	0	0
normalized size	1	1.	0.93	0.	0.	2.1	0.	0.
time (sec)	N/A	0.213	0.198	0.287	0.	1.733	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	122	0	0	223	0	0
normalized size	1	1.	0.93	0.	0.	1.7	0.	0.
time (sec)	N/A	0.097	0.048	0.211	0.	1.767	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	5.456	0.205	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.826	0.194	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	455	0	153	0	0
normalized size	1	1.	1.	6.07	0.	2.04	0.	0.
time (sec)	N/A	0.08	0.019	0.09	0.	1.706	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	79	354	0	153	0	0
normalized size	1	1.	1.	4.48	0.	1.94	0.	0.
time (sec)	N/A	0.077	0.016	0.078	0.	1.68	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	291	0	153	0	0
normalized size	1	1.	1.	3.88	0.	2.04	0.	0.
time (sec)	N/A	0.077	0.015	0.077	0.	1.691	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	79	379	0	136	0	0
normalized size	1	1.	1.	4.8	0.	1.72	0.	0.
time (sec)	N/A	0.071	0.014	0.073	0.	1.698	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	62	427	0	142	0	0
normalized size	1	1.	0.95	6.57	0.	2.18	0.	0.
time (sec)	N/A	0.073	0.023	0.078	0.	1.701	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	530	0	153	0	0
normalized size	1	1.	1.	7.07	0.	2.04	0.	0.
time (sec)	N/A	0.074	0.015	0.082	0.	1.714	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	600	0	153	0	0
normalized size	1	1.	1.	8.	0.	2.04	0.	0.
time (sec)	N/A	0.074	0.015	0.086	0.	1.653	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	92	0	0	235	0	0
normalized size	1	1.	0.93	0.	0.	2.37	0.	0.
time (sec)	N/A	0.157	0.112	180.	0.	1.7	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	96	0	0	235	0	0
normalized size	1	1.	0.93	0.	0.	2.28	0.	0.
time (sec)	N/A	0.144	0.103	180.	0.	1.679	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	90	0	0	235	0	0
normalized size	1	1.	0.93	0.	0.	2.42	0.	0.
time (sec)	N/A	0.137	0.098	180.	0.	2.107	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	90	0	0	203	0	0
normalized size	1	1.	0.87	0.	0.	1.97	0.	0.
time (sec)	N/A	0.133	0.119	180.	0.	1.943	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	77	0	0	193	0	0
normalized size	1	1.	0.91	0.	0.	2.27	0.	0.
time (sec)	N/A	0.126	0.058	180.	0.	1.955	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	91	0	0	227	0	0
normalized size	1	1.	0.9	0.	0.	2.25	0.	0.
time (sec)	N/A	0.138	0.108	180.	0.	2.035	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	95	0	0	235	0	0
normalized size	1	1.	1.	0.	0.	2.47	0.	0.
time (sec)	N/A	0.142	0.092	180.	0.	1.835	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	122	476	616	362	264	211
normalized size	1	1.	1.37	5.35	6.92	4.07	2.97	2.37
time (sec)	N/A	0.115	0.557	0.038	1.317	1.69	1.659	1.153

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	80	236	317	228	151	127
normalized size	1	1.	1.19	3.52	4.73	3.4	2.25	1.9
time (sec)	N/A	0.085	0.334	0.04	1.213	1.631	0.786	1.175

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	52	89	123	126	68	62
normalized size	1	1.	1.18	2.02	2.8	2.86	1.55	1.41
time (sec)	N/A	0.042	0.23	0.033	1.168	1.624	0.316	1.143

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	55	95	232	234	0	934
normalized size	1	1.	0.85	1.46	3.57	3.6	0.	14.37
time (sec)	N/A	0.15	0.129	0.036	1.421	1.673	0.	1.171

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	78	143	265	332	0	4419
normalized size	1	1.	0.88	1.61	2.98	3.73	0.	49.65
time (sec)	N/A	0.163	0.32	0.036	1.525	1.707	0.	1.274

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	217	1129	1281	751	779	458
normalized size	1	1.	0.92	4.76	5.41	3.17	3.29	1.93
time (sec)	N/A	0.261	1.436	0.04	1.189	1.787	4.306	1.124

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	193	564	667	448	456	279
normalized size	1	1.	1.15	3.36	3.97	2.67	2.71	1.66
time (sec)	N/A	0.179	0.656	0.042	1.2	1.662	1.962	1.153

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	80	218	266	228	219	144
normalized size	1	1.	0.68	1.85	2.25	1.93	1.86	1.22
time (sec)	N/A	0.097	0.483	0.037	1.214	1.627	0.821	1.128

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	114	192	455	467	0	9360
normalized size	1	1.	0.79	1.32	3.14	3.22	0.	64.55
time (sec)	N/A	0.338	0.232	0.04	1.527	1.661	0.	1.617

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	206	276	500	683	0	0
normalized size	1	1.	1.3	1.74	3.14	4.3	0.	0.
time (sec)	N/A	0.317	0.498	0.046	1.83	1.838	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	151	364	1264	1019	0	0
normalized size	1	1.	1.13	2.72	9.43	7.6	0.	0.
time (sec)	N/A	0.282	0.305	0.453	1.721	1.77	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	125	197	382	564	0	0
normalized size	1	1.	1.24	1.95	3.78	5.58	0.	0.
time (sec)	N/A	0.199	0.33	0.381	1.606	1.683	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	70	60	216	149	70	316
normalized size	1	1.	1.43	1.22	4.41	3.04	1.43	6.45
time (sec)	N/A	0.064	0.076	0.054	1.201	1.669	0.75	1.145

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	2.748	0.293	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	2.729	0.335	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	250	678	4420	1796	0	0
normalized size	1	1.	0.92	2.5	16.31	6.63	0.	0.
time (sec)	N/A	0.367	1.018	0.628	4.917	1.911	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	212	358	1041	956	0	0
normalized size	1	1.	1.	1.69	4.91	4.51	0.	0.
time (sec)	N/A	0.256	1.064	0.349	2.651	1.754	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	113	123	1030	298	146	1022
normalized size	1	1.	0.92	1.	8.37	2.42	1.19	8.31
time (sec)	N/A	0.095	0.508	0.11	1.27	1.671	1.432	1.503

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	12.181	2.144	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	13.232	2.551	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	164	468	1295	1173	0	0
normalized size	1	1.	1.23	3.52	9.74	8.82	0.	0.
time (sec)	N/A	0.284	1.238	0.46	2.068	1.777	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	292	247	425	743	0	0
normalized size	1	1.	2.86	2.42	4.17	7.28	0.	0.
time (sec)	N/A	0.203	5.482	0.398	1.959	1.72	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	57	85	216	151	90	309
normalized size	1	1.	1.14	1.7	4.32	3.02	1.8	6.18
time (sec)	N/A	0.065	0.263	0.063	1.167	1.63	0.896	1.161

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	2.541	0.295	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	2.405	0.328	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	53	132	278	0	0	0
normalized size	1	1.	0.48	1.2	2.53	0.	0.	0.
time (sec)	N/A	0.134	0.21	0.29	2.869	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	44	105	165	0	0	0
normalized size	1	1.	0.5	1.19	1.88	0.	0.	0.
time (sec)	N/A	0.112	0.158	0.231	2.812	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	34	80	82	0	0	0
normalized size	1	1.	0.64	1.51	1.55	0.	0.	0.
time (sec)	N/A	0.061	0.128	0.218	2.52	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	29	43	27	84	0	0
normalized size	1	1.	1.12	1.65	1.04	3.23	0.	0.
time (sec)	N/A	0.013	0.033	0.496	2.676	1.501	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	55	0	82	0	0	0
normalized size	1	1.	0.65	0.	0.98	0.	0.	0.
time (sec)	N/A	0.121	0.091	0.268	2.584	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	75	0	267	0	0	0
normalized size	1	1.	0.68	0.	2.43	0.	0.	0.
time (sec)	N/A	0.133	0.162	0.215	2.65	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	98	0	313	0	0	0
normalized size	1	1.	0.65	0.	2.07	0.	0.	0.
time (sec)	N/A	0.162	0.253	0.223	2.519	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	33	87	65	0	0	0
normalized size	1	1.	0.49	1.28	0.96	0.	0.	0.
time (sec)	N/A	0.111	0.055	0.214	2.338	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	29	70	49	0	0	0
normalized size	1	1.	0.55	1.32	0.92	0.	0.	0.
time (sec)	N/A	0.096	0.044	0.155	2.172	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	22	55	32	0	0	0
normalized size	1	1.	0.69	1.72	1.	0.	0.	0.
time (sec)	N/A	0.05	0.021	0.154	2.3	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	18	25	16	57	0	0
normalized size	1	1.	1.2	1.67	1.07	3.8	0.	0.
time (sec)	N/A	0.011	0.008	0.373	2.381	1.561	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	0	23	0	0	0
normalized size	1	1.	1.	0.	1.	0.	0.	0.
time (sec)	N/A	0.087	0.007	0.197	2.535	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	33	0	31	0	0	0
normalized size	1	1.	0.79	0.	0.74	0.	0.	0.
time (sec)	N/A	0.091	0.058	0.149	2.186	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	44	0	26	0	0	0
normalized size	1	1.	0.66	0.	0.39	0.	0.	0.
time (sec)	N/A	0.106	0.075	0.152	2.134	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	34	86	174	0	0	0
normalized size	1	1.	0.47	1.19	2.42	0.	0.	0.
time (sec)	N/A	0.115	0.051	0.163	1.971	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	30	69	135	0	0	0
normalized size	1	1.	0.54	1.23	2.41	0.	0.	0.
time (sec)	N/A	0.098	0.042	0.096	1.987	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	23	54	97	0	0	0
normalized size	1	1.	0.68	1.59	2.85	0.	0.	0.
time (sec)	N/A	0.051	0.024	0.095	1.823	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	19	25	31	59	0	0
normalized size	1	1.	1.19	1.56	1.94	3.69	0.	0.
time (sec)	N/A	0.011	0.007	0.961	1.946	1.552	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.021	0.137	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	34	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	0.026	0.096	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	45	0	0	0	0	0
normalized size	1	1.	0.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.057	0.096	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	67	0	132	0	0	0
normalized size	1	1.	0.36	0.	0.71	0.	0.	0.
time (sec)	N/A	0.183	0.298	0.129	2.253	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	54	0	97	0	0	0
normalized size	1	1.	0.37	0.	0.67	0.	0.	0.
time (sec)	N/A	0.144	0.241	0.088	1.989	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	45	0	65	0	0	0
normalized size	1	1.	0.51	0.	0.73	0.	0.	0.
time (sec)	N/A	0.072	0.066	0.089	2.34	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	36	0	39	0	0	0
normalized size	1	1.	0.65	0.	0.71	0.	0.	0.
time (sec)	N/A	0.128	0.02	0.089	2.429	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	53	0	50	0	0	0
normalized size	1	1.	0.67	0.	0.63	0.	0.	0.
time (sec)	N/A	0.126	0.085	0.089	2.104	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	66	0	45	0	0	0
normalized size	1	1.	0.61	0.	0.41	0.	0.	0.
time (sec)	N/A	0.165	0.055	0.092	2.248	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	374	374	199	0	0	0	0	0
normalized size	1	1.	0.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.212	0.127	0.197	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	146	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.167	0.077	0.184	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	89	0	0	0	0	0
normalized size	1	1.	0.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	0.051	0.178	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	40	54	122	347	0	0
normalized size	1	1.	0.87	1.17	2.65	7.54	0.	0.
time (sec)	N/A	0.022	0.015	0.243	2.139	1.663	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.972	0.178	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	170	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.173	0.098	0.125	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	117	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.141	0.049	0.089	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	83	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	0.033	0.093	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	36	25	109	273	0	0
normalized size	1	1.	0.97	0.68	2.95	7.38	0.	0.
time (sec)	N/A	0.02	0.015	0.889	2.026	1.632	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	3.124	0.087	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	423	257	0	0	0	0	0
normalized size	1	1.	0.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.26	0.459	0.112	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	185	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.189	0.118	0.116	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	165	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.18	0.105	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	11.306	0.112	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	1.071	0.164	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	383	290	0	0	2719	0	0
normalized size	1	1.	0.76	0.	0.	7.1	0.	0.
time (sec)	N/A	0.559	0.899	0.231	0.	2.537	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	379	0	0	3218	0	0
normalized size	1	1.	1.15	0.	0.	9.78	0.	0.
time (sec)	N/A	0.663	0.701	0.44	0.	2.513	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	756	414	0	2338	0	0
normalized size	1	1.	3.53	1.93	0.	10.93	0.	0.
time (sec)	N/A	0.4	0.811	0.418	0.	2.375	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.819	0.191	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	296	296	933	674	0	3575	0	0
normalized size	1	1.	3.15	2.28	0.	12.08	0.	0.
time (sec)	N/A	0.523	9.653	0.795	0.	2.884	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [180] had the largest ratio of [0.6429]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	2	1.	14	0.143
2	A	4	2	1.	14	0.143
3	A	3	2	1.	14	0.143
4	A	2	2	1.	12	0.167
5	A	3	3	1.	14	0.214
6	A	4	4	1.	14	0.286
7	A	5	4	1.	14	0.286
8	A	6	4	1.	14	0.286
9	A	6	4	1.	16	0.25
10	A	4	3	1.	16	0.188
11	A	4	4	1.	16	0.25
12	A	2	1	1.	14	0.071
13	A	5	4	1.	16	0.25
14	A	5	5	1.	16	0.312

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
15	A	7	6	1.	16	0.375
16	A	12	4	1.	16	0.25
17	A	8	4	1.	16	0.25
18	A	6	4	1.	16	0.25
19	A	3	3	1.	14	0.214
20	A	8	4	1.	16	0.25
21	A	8	4	1.	16	0.25
22	A	12	5	1.	16	0.312
23	A	8	3	1.	12	0.25
24	A	8	4	1.	12	0.333
25	A	3	2	1.	10	0.2
26	A	8	4	1.	12	0.333
27	A	8	4	1.	12	0.333
28	A	14	5	1.	12	0.417
29	A	9	5	1.	14	0.357
30	A	7	4	1.	14	0.286
31	A	5	3	1.	12	0.25
32	A	0	0	0.	0	0.
33	A	6	6	1.	16	0.375
34	A	5	5	1.	16	0.312
35	A	2	2	1.	14	0.143
36	A	0	0	0.	0	0.
37	A	15	8	1.	16	0.5
38	A	9	6	1.	16	0.375
39	A	6	4	1.	14	0.286
40	A	0	0	0.	0	0.
41	A	8	6	1.	16	0.375
42	A	7	6	1.	16	0.375
43	A	6	6	1.	16	0.375
44	A	5	5	1.	16	0.312
45	A	6	6	1.	16	0.375
46	A	7	6	1.	16	0.375
47	A	8	6	1.	16	0.375
48	A	10	9	1.	18	0.5
49	A	9	8	1.	18	0.444
50	A	8	7	1.	18	0.389
51	A	7	6	1.	18	0.333
52	A	7	7	1.	18	0.389
53	A	9	8	1.	18	0.444
54	A	9	9	1.	18	0.5
55	A	11	8	1.	18	0.444
56	A	23	8	1.	18	0.444
57	A	20	8	1.	18	0.444
58	A	14	7	1.	18	0.389

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
59	A	12	6	1.	18	0.333
60	A	12	6	1.	18	0.333
61	A	18	7	1.	18	0.389
62	A	19	8	1.	18	0.444
63	A	4	3	1.	8	0.375
64	A	3	3	1.	8	0.375
65	A	2	2	1.	8	0.25
66	A	3	3	1.	8	0.375
67	A	5	3	1.	16	0.188
68	A	4	3	1.	16	0.188
69	A	4	3	1.	16	0.188
70	A	3	2	1.	16	0.125
71	A	3	2	1.	16	0.125
72	A	4	3	1.	16	0.188
73	A	4	3	1.	16	0.188
74	A	5	3	1.	16	0.188
75	A	0	0	0.	0	0.
76	A	1	1	1.	10	0.1
77	A	0	0	0.	0	0.
78	A	0	0	0.	0	0.
79	A	2	2	1.	10	0.2
80	A	0	0	0.	0	0.
81	A	2	1	1.	28	0.036
82	A	0	0	0.	0	0.
83	A	0	0	0.	0	0.
84	A	1	1	1.	10	0.1
85	A	0	0	0.	0	0.
86	A	0	0	0.	0	0.
87	A	2	2	1.	10	0.2
88	A	0	0	0.	0	0.
89	A	2	1	1.	25	0.04
90	A	2	1	1.	17	0.059
91	A	2	1	1.	20	0.05
92	A	3	1	1.	20	0.05
93	A	3	2	1.	21	0.095
94	A	4	2	1.	20	0.1
95	A	4	2	1.	20	0.1
96	A	5	2	1.	20	0.1
97	A	7	5	1.	24	0.208
98	A	0	0	0.	0	0.
99	A	8	3	1.	16	0.188
100	A	5	3	1.	16	0.188
101	A	3	2	1.	14	0.143
102	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
103	A	0	0	0.	0	0.
104	A	3	2	1.	12	0.167
105	A	3	2	1.	12	0.167
106	A	3	2	1.	12	0.167
107	A	3	2	1.	10	0.2
108	A	3	2	1.	12	0.167
109	A	3	2	1.	12	0.167
110	A	3	2	1.	12	0.167
111	A	5	3	1.	14	0.214
112	A	5	3	1.	14	0.214
113	A	5	3	1.	14	0.214
114	A	5	3	1.	12	0.25
115	A	5	3	1.	14	0.214
116	A	5	3	1.	14	0.214
117	A	5	3	1.	14	0.214
118	A	6	3	1.	18	0.167
119	A	5	3	1.	18	0.167
120	A	4	3	1.	16	0.188
121	A	5	4	1.	18	0.222
122	A	6	5	1.	18	0.278
123	A	10	6	1.	20	0.3
124	A	9	7	1.	20	0.35
125	A	6	4	1.	18	0.222
126	A	9	5	1.	20	0.25
127	A	9	5	1.	20	0.25
128	A	7	7	1.	20	0.35
129	A	6	6	1.	20	0.3
130	A	3	3	1.	18	0.167
131	A	0	0	0.	0	0.
132	A	0	0	0.	0	0.
133	A	10	9	1.	20	0.45
134	A	9	9	1.	20	0.45
135	A	4	4	1.	18	0.222
136	A	0	0	0.	0	0.
137	A	0	0	0.	0	0.
138	A	7	7	1.	21	0.333
139	A	6	6	1.	21	0.286
140	A	3	3	1.	19	0.158
141	A	0	0	0.	0	0.
142	A	0	0	0.	0	0.
143	A	5	3	1.	18	0.167
144	A	4	3	1.	18	0.167
145	A	3	3	1.	16	0.188
146	A	1	1	1.	14	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
147	A	4	4	1.	18	0.222
148	A	5	5	1.	18	0.278
149	A	6	5	1.	18	0.278
150	A	5	3	1.	14	0.214
151	A	4	3	1.	14	0.214
152	A	3	3	1.	12	0.25
153	A	1	1	1.	10	0.1
154	A	2	2	1.	14	0.143
155	A	3	3	1.	14	0.214
156	A	4	3	1.	14	0.214
157	A	5	3	1.	15	0.2
158	A	4	3	1.	15	0.2
159	A	3	3	1.	13	0.231
160	A	1	1	1.	11	0.091
161	A	2	2	1.	15	0.133
162	A	3	3	1.	15	0.2
163	A	4	3	1.	15	0.2
164	A	9	5	1.	14	0.357
165	A	7	5	1.	14	0.357
166	A	4	4	1.	12	0.333
167	A	5	3	1.	14	0.214
168	A	5	3	1.	14	0.214
169	A	7	4	1.	14	0.286
170	A	10	6	1.	18	0.333
171	A	8	5	1.	18	0.278
172	A	6	4	1.	16	0.25
173	A	2	2	1.	14	0.143
174	A	0	0	0.	0	0.
175	A	10	6	1.	15	0.4
176	A	8	5	1.	15	0.333
177	A	6	4	1.	13	0.308
178	A	2	2	1.	11	0.182
179	A	0	0	0.	0	0.
180	A	16	9	1.	14	0.643
181	A	10	7	1.	14	0.5
182	A	7	5	1.	12	0.417
183	A	0	0	0.	0	0.
184	A	0	0	0.	0	0.
185	A	12	7	1.	12	0.583
186	A	10	6	1.	16	0.375
187	A	8	5	1.	14	0.357
188	A	0	0	0.	0	0.
189	A	11	8	1.	18	0.444

Chapter 3

Listing of integrals

3.1 $\int (c+dx)^4 \cos(a+bx)\,dx$

Optimal. Leaf size=91

$$-\frac{12d^2(c+dx)^2\sin(a+bx)}{b^3}-\frac{24d^3(c+dx)\cos(a+bx)}{b^4}+\frac{4d(c+dx)^3\cos(a+bx)}{b^2}+\frac{24d^4\sin(a+bx)}{b^5}+\frac{(c+dx)^4\sin(a+bx)}{b}$$

```
[Out] (-24*d^3*(c + d*x)*Cos[a + b*x])/b^4 + (4*d*(c + d*x)^3*Cos[a + b*x])/b^2 +
 (24*d^4*Sin[a + b*x])/b^5 - (12*d^2*(c + d*x)^2*Sin[a + b*x])/b^3 + ((c +
d*x)^4*Sin[a + b*x])/b
```

Rubi [A] time = 0.0932382, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3296, 2637}

$$-\frac{12d^2(c+dx)^2\sin(a+bx)}{b^3}-\frac{24d^3(c+dx)\cos(a+bx)}{b^4}+\frac{4d(c+dx)^3\cos(a+bx)}{b^2}+\frac{24d^4\sin(a+bx)}{b^5}+\frac{(c+dx)^4\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x],x]
```

```
[Out] (-24*d^3*(c + d*x)*Cos[a + b*x])/b^4 + (4*d*(c + d*x)^3*Cos[a + b*x])/b^2 +
 (24*d^4*Sin[a + b*x])/b^5 - (12*d^2*(c + d*x)^2*Sin[a + b*x])/b^3 + ((c +
d*x)^4*Sin[a + b*x])/b
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^4\cos(a+bx)\,dx = \frac{(c+dx)^4\sin(a+bx)}{b} - \frac{(4d)\int (c+dx)^3\sin(a+bx)\,dx}{b}$$
$$= \frac{4d(c+dx)^3\cos(a+bx)}{b^2} + \frac{(c+dx)^4\sin(a+bx)}{b} - \frac{(12d^2)\int (c+dx)^2\cos(a+bx)\,dx}{b^2}$$
$$= \frac{4d(c+dx)^3\cos(a+bx)}{b^2} - \frac{12d^2(c+dx)^2\sin(a+bx)}{b^3} + \frac{(c+dx)^4\sin(a+bx)}{b} + \frac{(24d^3)\int (c+$$
$$= -\frac{24d^3(c+dx)\cos(a+bx)}{b^4} + \frac{4d(c+dx)^3\cos(a+bx)}{b^2} - \frac{12d^2(c+dx)^2\sin(a+bx)}{b^3} + \frac{(c+dx$$
$$= -\frac{24d^3(c+dx)\cos(a+bx)}{b^4} + \frac{4d(c+dx)^3\cos(a+bx)}{b^2} + \frac{24d^4\sin(a+bx)}{b^5} - \frac{12d^2(c+dx)^2\text{si}}{b^3}$$

Mathematica [A] time = 0.337115, size = 76, normalized size = 0.84

$$\frac{\sin(a+bx)\left(-12b^2d^2(c+dx)^2 + b^4(c+dx)^4 + 24d^4\right) + 4bd(c+dx)\cos(a+bx)\left(b^2(c+dx)^2 - 6d^2\right)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x],x]
```

```
[Out] (4*b*d*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + (24*d^4 - 12*b^2
*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Sin[a + b*x])/b^5
```

Maple [B] time = 0.029, size = 539, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a),x)
```

```
[Out] 1/b*(1/b^4*d^4*((b*x+a)^4*sin(b*x+a)+4*(b*x+a)^3*cos(b*x+a)-12*(b*x+a)^2*si
n(b*x+a)+24*sin(b*x+a)-24*(b*x+a)*cos(b*x+a))-4/b^4*a*d^4*((b*x+a)^3*sin(b*
x+a)+3*(b*x+a)^2*cos(b*x+a)-6*cos(b*x+a)-6*(b*x+a)*sin(b*x+a))+4/b^3*c*d^3*
((b*x+a)^3*sin(b*x+a)+3*(b*x+a)^2*cos(b*x+a)-6*cos(b*x+a)-6*(b*x+a)*sin(b*x
+a))+6/b^4*a^2*d^4*((b*x+a)^2*sin(b*x+a)-2*sin(b*x+a)+2*(b*x+a)*cos(b*x+a))
-12/b^3*a*c*d^3*((b*x+a)^2*sin(b*x+a)-2*sin(b*x+a)+2*(b*x+a)*cos(b*x+a))+6/
b^2*c^2*d^2*((b*x+a)^2*sin(b*x+a)-2*sin(b*x+a)+2*(b*x+a)*cos(b*x+a))-4/b^4*
a^3*d^4*(cos(b*x+a)+(b*x+a)*sin(b*x+a))+12/b^3*a^2*c*d^3*(cos(b*x+a)+(b*x+a
)*sin(b*x+a))-12/b^2*a*c^2*d^2*(cos(b*x+a)+(b*x+a)*sin(b*x+a))+4/b*c^3*d*(c
os(b*x+a)+(b*x+a)*sin(b*x+a))+1/b^4*a^4*d^4*sin(b*x+a)-4/b^3*a^3*c*d^3*sin(
b*x+a)+6/b^2*a^2*c^2*d^2*sin(b*x+a)-4/b*a*c^3*d*sin(b*x+a)+c^4*sin(b*x+a))
```

Maxima [B] time = 1.1163, size = 649, normalized size = 7.13

$$c^4\sin(bx+a) - \frac{4ac^3d\sin(bx+a)}{b} + \frac{6a^2c^2d^2\sin(bx+a)}{b^2} - \frac{4a^3cd^3\sin(bx+a)}{b^3} + \frac{a^4d^4\sin(bx+a)}{b^4} + \frac{4((bx+a)\sin(bx+a)+\cos(bx+a))c^3d}{b} - \frac{12((bx+a)\text{si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] (c^4*sin(b*x + a) - 4*a*c^3*d*sin(b*x + a)/b + 6*a^2*c^2*d^2*sin(b*x + a)/b
^2 - 4*a^3*c*d^3*sin(b*x + a)/b^3 + a^4*d^4*sin(b*x + a)/b^4 + 4*((b*x + a)
*sin(b*x + a) + cos(b*x + a))*c^3*d/b - 12*((b*x + a)*sin(b*x + a) + cos(b*
x + a))*a*c^2*d^2/b^2 + 12*((b*x + a)*sin(b*x + a) + cos(b*x + a))*a^2*c*d^
3/b^3 - 4*((b*x + a)*sin(b*x + a) + cos(b*x + a))*a^3*d^4/b^4 + 6*(2*(b*x +
 a)*cos(b*x + a) + ((b*x + a)^2 - 2)*sin(b*x + a))*c^2*d^2/b^2 - 12*(2*(b*x
 + a)*cos(b*x + a) + ((b*x + a)^2 - 2)*sin(b*x + a))*a*c*d^3/b^3 + 6*(2*(b*
x + a)*cos(b*x + a) + ((b*x + a)^2 - 2)*sin(b*x + a))*a^2*d^4/b^4 + 4*(3*((
b*x + a)^2 - 2)*cos(b*x + a) + ((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*c*
d^3/b^3 - 4*(3*((b*x + a)^2 - 2)*cos(b*x + a) + ((b*x + a)^3 - 6*b*x - 6*a)
*sin(b*x + a))*a*d^4/b^4 + (4*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) + ((
b*x + a)^4 - 12*(b*x + a)^2 + 24)*sin(b*x + a))*d^4/b^4)/b
```

Fricas [A] time = 1.02636, size = 347, normalized size = 3.81

$$\frac{4\left(b^{3}d^{4}x^{3}+3\,b^{3}cd^{3}x^{2}+b^{3}c^{3}d-6\,bcd^{3}+3\left(b^{3}c^{2}d^{2}-2\,bd^{4}\right)x\right)\cos\left(bx+a\right)+\left(b^{4}d^{4}x^{4}+4\,b^{4}cd^{3}x^{3}+b^{4}c^{4}-12\,b^{2}c^{2}d^{2}+\right.}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] (4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d - 6*b*c*d^3 + 3*(b^3*c^2*d^2
- 2*b*d^4)*x)*cos(b*x + a) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*
b^2*c^2*d^2 + 24*d^4 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b
^2*c*d^3)*x)*sin(b*x + a))/b^5
```

Sympy [A] time = 2.87839, size = 311, normalized size = 3.42

$$\begin{cases} \frac{c^{4}\sin\left(a+bx\right)}{b}+\frac{4c^{3}dx\sin\left(a+bx\right)}{b}+\frac{6c^{2}d^{2}x^{2}\sin\left(a+bx\right)}{b}+\frac{4cd^{3}x^{3}\sin\left(a+bx\right)}{b}+\frac{d^{4}x^{4}\sin\left(a+bx\right)}{b}+\frac{4c^{3}d\cos\left(a+bx\right)}{b^{2}}+\frac{12c^{2}d^{2}x\cos\left(a+bx\right)}{b^{2}}+\frac{12cd^{3}x^{2}}{} \\ \left(c^{4}x+2c^{3}dx^{2}+2c^{2}d^{2}x^{3}+cd^{3}x^{4}+\frac{d^{4}x^{5}}{5}\right)\cos\left(a\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a),x)
```

```
[Out] Piecewise((c**4*sin(a + b*x)/b + 4*c**3*d*x*sin(a + b*x)/b + 6*c**2*d**2*x*
*2*sin(a + b*x)/b + 4*c*d**3*x**3*sin(a + b*x)/b + d**4*x**4*sin(a + b*x)/b
 + 4*c**3*d*cos(a + b*x)/b**2 + 12*c**2*d**2*x*cos(a + b*x)/b**2 + 12*c*d**
3*x**2*cos(a + b*x)/b**2 + 4*d**4*x**3*cos(a + b*x)/b**2 - 12*c**2*d**2*sin
(a + b*x)/b**3 - 24*c*d**3*x*sin(a + b*x)/b**3 - 12*d**4*x**2*sin(a + b*x)/
b**3 - 24*c*d**3*cos(a + b*x)/b**4 - 24*d**4*x*cos(a + b*x)/b**4 + 24*d**4*
sin(a + b*x)/b**5, Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 +
 c*d**3*x**4 + d**4*x**5/5)*cos(a), True))
```

Giac [A] time = 1.13352, size = 230, normalized size = 2.53

$$\frac{4\left(b^{3}d^{4}x^{3}+3\,b^{3}cd^{3}x^{2}+3\,b^{3}c^{2}d^{2}x+b^{3}c^{3}d-6\,bd^{4}x-6\,bcd^{3}\right)\cos\left(bx+a\right)}{b^{5}}+\frac{\left(b^{4}d^{4}x^{4}+4\,b^{4}cd^{3}x^{3}+6\,b^{4}c^{2}d^{2}x^{2}+4\,b^{4}c^{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a),x, algorithm="giac")
```

```
[Out] 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*b*d^4*x
- 6*b*c*d^3)*cos(b*x + a)/b^5 + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*
d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b^
2*c^2*d^2 + 24*d^4)*sin(b*x + a)/b^5
```

3.2 $\int (c+dx)^3 \cos(a+bx)\,dx$

Optimal. Leaf size=70

$$-\frac{6d^2(c+dx)\sin(a+bx)}{b^3}+\frac{3d(c+dx)^2\cos(a+bx)}{b^2}-\frac{6d^3\cos(a+bx)}{b^4}+\frac{(c+dx)^3\sin(a+bx)}{b}$$

```
[Out] (-6*d^3*Cos[a + b*x])/b^4 + (3*d*(c + d*x)^2*Cos[a + b*x])/b^2 - (6*d^2*(c
+ d*x)*Sin[a + b*x])/b^3 + ((c + d*x)^3*Sin[a + b*x])/b
```

Rubi [A] time = 0.0655573, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3296, 2638}

$$-\frac{6d^2(c+dx)\sin(a+bx)}{b^3}+\frac{3d(c+dx)^2\cos(a+bx)}{b^2}-\frac{6d^3\cos(a+bx)}{b^4}+\frac{(c+dx)^3\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x],x]
```

```
[Out] (-6*d^3*Cos[a + b*x])/b^4 + (3*d*(c + d*x)^2*Cos[a + b*x])/b^2 - (6*d^2*(c
+ d*x)*Sin[a + b*x])/b^3 + ((c + d*x)^3*Sin[a + b*x])/b
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^3\cos(a+bx)\,dx &= \frac{(c+dx)^3\sin(a+bx)}{b}-\frac{(3d)\int (c+dx)^2\sin(a+bx)\,dx}{b}\\
&= \frac{3d(c+dx)^2\cos(a+bx)}{b^2}+\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{\left(6d^2\right)\int (c+dx)\cos(a+bx)\,dx}{b^2}\\
&= \frac{3d(c+dx)^2\cos(a+bx)}{b^2}-\frac{6d^2(c+dx)\sin(a+bx)}{b^3}+\frac{(c+dx)^3\sin(a+bx)}{b}+\frac{\left(6d^3\right)\int \sin}{b}\\
&= -\frac{6d^3\cos(a+bx)}{b^4}+\frac{3d(c+dx)^2\cos(a+bx)}{b^2}-\frac{6d^2(c+dx)\sin(a+bx)}{b^3}+\frac{(c+dx)^3\sin(a}{b}
\end{aligned}$$

Mathematica [A] time = 0.22181, size = 61, normalized size = 0.87

$$\frac{b(c+dx)\sin(a+bx)\left(b^2(c+dx)^2-6d^2\right)+3d\cos(a+bx)\left(b^2(c+dx)^2-2d^2\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x],x]
```

```
[Out] (3*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + b*(c + d*x)*(-6*d^2 + b^2*(c
 + d*x)^2)*Sin[a + b*x])/b^4
```

Maple [B] time = 0.029, size = 302, normalized size = 4.3

$$\frac{1}{b}\left(\frac{d^3\left((bx+a)^3\sin(bx+a)+3(bx+a)^2\cos(bx+a)-6\cos(bx+a)-6(bx+a)\sin(bx+a)\right)}{b^3}-3\frac{ad^3\left((bx+a)^2\sin(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a),x)
```

```
[Out] 1/b*(1/b^3*d^3*((b*x+a)^3*sin(b*x+a)+3*(b*x+a)^2*cos(b*x+a)-6*cos(b*x+a)-6*
(b*x+a)*sin(b*x+a))-3/b^3*a*d^3*((b*x+a)^2*sin(b*x+a)-2*sin(b*x+a)+2*(b*x+a
)*cos(b*x+a))+3/b^2*c*d^2*((b*x+a)^2*sin(b*x+a)-2*sin(b*x+a)+2*(b*x+a)*cos(
b*x+a))+3/b^3*a^2*d^3*(cos(b*x+a)+(b*x+a)*sin(b*x+a))-6/b^2*a*c*d^2*(cos(b*
x+a)+(b*x+a)*sin(b*x+a))+3/b*c^2*d*(cos(b*x+a)+(b*x+a)*sin(b*x+a))-1/b^3*a^
3*d^3*sin(b*x+a)+3/b^2*a^2*c*d^2*sin(b*x+a)-3/b*a*c^2*d*sin(b*x+a)+c^3*sin(
b*x+a))
```

Maxima [B] time = 1.01193, size = 375, normalized size = 5.36

$$c^3\sin(bx+a)-\frac{3ac^2d\sin(bx+a)}{b}+\frac{3a^2cd^2\sin(bx+a)}{b^2}-\frac{a^3d^3\sin(bx+a)}{b^3}+\frac{3((bx+a)\sin(bx+a)+\cos(bx+a))c^2d}{b}-\frac{6((bx+a)\sin(bx+a)+\cos(bx+a))acd^2}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] (c^3*sin(b*x + a) - 3*a*c^2*d*sin(b*x + a)/b + 3*a^2*c*d^2*sin(b*x + a)/b^2
 - a^3*d^3*sin(b*x + a)/b^3 + 3*((b*x + a)*sin(b*x + a) + cos(b*x + a))*c^2
*d/b - 6*((b*x + a)*sin(b*x + a) + cos(b*x + a))*a*c*d^2/b^2 + 3*((b*x + a)
*sin(b*x + a) + cos(b*x + a))*a^2*d^3/b^3 + 3*(2*(b*x + a)*cos(b*x + a) + (
(b*x + a)^2 - 2)*sin(b*x + a))*c*d^2/b^2 - 3*(2*(b*x + a)*cos(b*x + a) + ((
b*x + a)^2 - 2)*sin(b*x + a))*a*d^3/b^3 + (3*((b*x + a)^2 - 2)*cos(b*x + a)
 + ((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*d^3/b^3)/b
```

Fricas [A] time = 1.06501, size = 228, normalized size = 3.26

$$\frac{3\left(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d-2\,d^3\right)\cos(bx+a)+\left(b^3d^3x^3+3\,b^3cd^2x^2+b^3c^3-6\,bcd^2+3\left(b^3c^2d-2\,bd^3\right)x\right)\sin(bx+a)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] (3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a) + (b^3*d^
3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*
```

```
sin(b*x + a))/b^4
```

Sympy [A] time = 1.31311, size = 202, normalized size = 2.89

$$\begin{cases} \frac{c^3 \sin(a+bx)}{b} + \frac{3c^2dx\sin(a+bx)}{b} + \frac{3cd^2x^2\sin(a+bx)}{b} + \frac{d^3x^3\sin(a+bx)}{b} + \frac{3c^2d\cos(a+bx)}{b^2} + \frac{6cd^2x\cos(a+bx)}{b^2} + \frac{3d^3x^2\cos(a+bx)}{b^2} - \frac{6cd^2\sin(a+}{b^3} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a),x)
```

```
[Out] Piecewise((c**3*sin(a + b*x)/b + 3*c**2*d*x*sin(a + b*x)/b + 3*c*d**2*x**2*
sin(a + b*x)/b + d**3*x**3*sin(a + b*x)/b + 3*c**2*d*cos(a + b*x)/b**2 + 6*
c*d**2*x*cos(a + b*x)/b**2 + 3*d**3*x**2*cos(a + b*x)/b**2 - 6*c*d**2*sin(a
 + b*x)/b**3 - 6*d**3*x*sin(a + b*x)/b**3 - 6*d**3*cos(a + b*x)/b**4, Ne(b,
 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*cos(a), True)
)
```

Giac [A] time = 1.12317, size = 149, normalized size = 2.13

$$\frac{3\left(b^2d^3x^2 + 2b^2cd^2x + b^2c^2d - 2d^3\right)\cos(bx+a)}{b^4} + \frac{\left(b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + b^3c^3 - 6bd^3x - 6bcd^2\right)\sin(bx+}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a),x, algorithm="giac")
```

```
[Out] 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a)/b^4 + (b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^
2)*sin(b*x + a)/b^4
```

3.3 $\int (c+dx)^2 \cos(a+bx)\,dx$

Optimal. Leaf size=49

$$\frac{2d(c+dx)\cos(a+bx)}{b^2}-\frac{2d^2\sin(a+bx)}{b^3}+\frac{(c+dx)^2\sin(a+bx)}{b}$$

```
[Out] (2*d*(c + d*x)*Cos[a + b*x])/b^2 - (2*d^2*Sin[a + b*x])/b^3 + ((c + d*x)^2*
Sin[a + b*x])/b
```

Rubi [A] time = 0.0406137, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3296, 2637}

$$\frac{2d(c+dx)\cos(a+bx)}{b^2}-\frac{2d^2\sin(a+bx)}{b^3}+\frac{(c+dx)^2\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x],x]
```

```
[Out] (2*d*(c + d*x)*Cos[a + b*x])/b^2 - (2*d^2*Sin[a + b*x])/b^3 + ((c + d*x)^2*
Sin[a + b*x])/b
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\cos(a+bx)\,dx &= \frac{(c+dx)^2\sin(a+bx)}{b}-\frac{(2d)\int (c+dx)\sin(a+bx)\,dx}{b}\\ &= \frac{2d(c+dx)\cos(a+bx)}{b^2}+\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{\left(2d^2\right)\int \cos(a+bx)\,dx}{b^2}\\ &= \frac{2d(c+dx)\cos(a+bx)}{b^2}-\frac{2d^2\sin(a+bx)}{b^3}+\frac{(c+dx)^2\sin(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.166961, size = 44, normalized size = 0.9

$$\frac{\sin(a+bx)\left(b^2(c+dx)^2-2d^2\right)+2bd(c+dx)\cos(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x],x]
```

```
[Out] (2*b*d*(c + d*x)*Cos[a + b*x] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a + b*x])/b^
3
```

Maple [B] time = 0.026, size = 143, normalized size = 2.9

$$\frac{1}{b}\left(\frac{d^2\left((bx+a)^2\sin(bx+a)-2\sin(bx+a)+2(bx+a)\cos(bx+a)\right)}{b^2}-2\frac{ad^2(\cos(bx+a)+(bx+a)\sin(bx+a))}{b^2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a),x)
```

```
[Out] 1/b*(1/b^2*d^2*((b*x+a)^2*sin(b*x+a)-2*sin(b*x+a)+2*(b*x+a)*cos(b*x+a))-2/b
^2*a*d^2*(cos(b*x+a)+(b*x+a)*sin(b*x+a))+2/b*c*d*(cos(b*x+a)+(b*x+a)*sin(b*
x+a))+1/b^2*a^2*d^2*sin(b*x+a)-2/b*a*c*d*sin(b*x+a)+c^2*sin(b*x+a))
```

Maxima [B] time = 1.01029, size = 184, normalized size = 3.76

$$\frac{c^2\sin(bx+a)-\frac{2acd\sin(bx+a)}{b}+\frac{a^2d^2\sin(bx+a)}{b^2}+\frac{2((bx+a)\sin(bx+a)+\cos(bx+a))cd}{b}-\frac{2((bx+a)\sin(bx+a)+\cos(bx+a))ad^2}{b^2}+\frac{(2(bx+a)\cos(b\ldots}{}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] (c^2*sin(b*x + a) - 2*a*c*d*sin(b*x + a)/b + a^2*d^2*sin(b*x + a)/b^2 + 2*(
(b*x + a)*sin(b*x + a) + cos(b*x + a))*c*d/b - 2*((b*x + a)*sin(b*x + a) +
cos(b*x + a))*a*d^2/b^2 + (2*(b*x + a)*cos(b*x + a) + ((b*x + a)^2 - 2)*sin
(b*x + a))*d^2/b^2)/b
```

Fricas [A] time = 1.12931, size = 136, normalized size = 2.78

$$\frac{2\left(bd^2x+bcd\right)\cos(bx+a)+\left(b^2d^2x^2+2b^2cdx+b^2c^2-2d^2\right)\sin(bx+a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] (2*(b*d^2*x + b*c*d)*cos(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 -
2*d^2)*sin(b*x + a))/b^3
```

Sympy [A] time = 0.637776, size = 112, normalized size = 2.29

$$\begin{cases}\frac{c^2\sin(a+bx)}{b}+\frac{2cdx\sin(a+bx)}{b}+\frac{d^2x^2\sin(a+bx)}{b}+\frac{2cd\cos(a+bx)}{b^2}+\frac{2d^2x\cos(a+bx)}{b^2}-\frac{2d^2\sin(a+bx)}{b^3} & \text{for } b\neq 0\\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\cos(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a),x)
```

```
[Out] Piecewise((c**2*sin(a + b*x)/b + 2*c*d*x*sin(a + b*x)/b + d**2*x**2*sin(a +
 b*x)/b + 2*c*d*cos(a + b*x)/b**2 + 2*d**2*x*cos(a + b*x)/b**2 - 2*d**2*sin
(a + b*x)/b**3, Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*cos(a), True)
)
```

Giac [A] time = 1.10074, size = 86, normalized size = 1.76

$$\frac{2\left(bd^2x + bcd\right)\cos\left(bx + a\right)}{b^3} + \frac{\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 - 2\,d^2\right)\sin\left(bx + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(b*d^2*x + b*c*d)*cos(b*x + a)/b^3 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
 - 2*d^2)*sin(b*x + a)/b^3
```

3.4 $\int (c+dx)\cos(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}$$

```
[Out] (d*Cos[a + b*x])/b^2 + ((c + d*x)*Sin[a + b*x])/b
```

Rubi [A] time = 0.0164785, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3296, 2638}

$$\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x],x]
```

```
[Out] (d*Cos[a + b*x])/b^2 + ((c + d*x)*Sin[a + b*x])/b
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cos(a+bx)\,dx &= \frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int \sin(a+bx)\,dx}{b}\\ &= \frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0561993, size = 26, normalized size = 0.96

$$\frac{b(c+dx)\sin(a+bx)+d\cos(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x],x]
```

```
[Out] (d*Cos[a + b*x] + b*(c + d*x)*Sin[a + b*x])/b^2
```

Maple [A] time = 0.029, size = 51, normalized size = 1.9

$$\frac{1}{b}\left(\frac{d(\cos(bx+a)+(bx+a)\sin(bx+a))}{b}-\frac{da\sin(bx+a)}{b}+c\sin(bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a),x)
```

```
[Out] 1/b*(1/b*d*(cos(b*x+a)+(b*x+a)*sin(b*x+a))-a*d/b*sin(b*x+a)+c*sin(b*x+a))
```

Maxima [A] time = 0.959584, size = 68, normalized size = 2.52

$$\frac{c\sin(bx+a)-\frac{ad\sin(bx+a)}{b}+\frac{((bx+a)\sin(bx+a)+\cos(bx+a))d}{b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] (c*sin(b*x + a) - a*d*sin(b*x + a)/b + ((b*x + a)*sin(b*x + a) + cos(b*x +
a))*d/b)/b
```

Fricas [A] time = 1.05773, size = 69, normalized size = 2.56

$$\frac{d\cos(bx+a)+(bdx+bc)\sin(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] (d*cos(b*x + a) + (b*d*x + b*c)*sin(b*x + a))/b^2
```

Sympy [A] time = 0.233663, size = 46, normalized size = 1.7

$$\begin{cases}\frac{c\sin(a+bx)}{b}+\frac{dx\sin(a+bx)}{b}+\frac{d\cos(a+bx)}{b^2} & \text{for } b\neq 0\\ \left(cx+\frac{dx^2}{2}\right)\cos(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a),x)
```

```
[Out] Piecewise((c*sin(a + b*x)/b + d*x*sin(a + b*x)/b + d*cos(a + b*x)/b**2, Ne(
b, 0)), ((c*x + d*x**2/2)*cos(a), True))
```

Giac [A] time = 1.09986, size = 41, normalized size = 1.52

$$\frac{d\cos(bx+a)}{b^2}+\frac{(bdx+bc)\sin(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] d*cos(b*x + a)/b^2 + (b*d*x + b*c)*sin(b*x + a)/b^2
```

3.5 $\int \frac{\cos(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=52

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] (Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d - (Sin[a - (b*c)/d]*SinInte
gral[(b*c)/d + b*x])/d
```

Rubi [A] time = 0.0983092, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3303, 3299, 3302}

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x),x]
```

```
[Out] (Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d - (Sin[a - (b*c)/d]*SinInte
gral[(b*c)/d + b*x])/d
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)}{c+dx}\,dx &= \cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\\ &= \frac{\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.0992167, size = 50, normalized size = 0.96

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)-\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x),x]
```

```
[Out] (Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x] - Sin[a - (b*c)/d]*SinIntegral
[(b*c)/d + b*x])/d
```

Maple [A] time = 0.027, size = 72, normalized size = 1.4

$$\frac{1}{d}\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)+\frac{1}{d}\mathrm{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c),x)
```

```
[Out] Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d
+b*c)/d)/d
```

Maxima [C] time = 1.21845, size = 192, normalized size = 3.69

$$-\frac{b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b\left(i\,E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin(}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integra
l_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b*(I*exp_
integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(1, -(I*
b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/(b*d)
```

Fricas [A] time = 1.06019, size = 200, normalized size = 3.85

$$\frac{\left(\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-2\,\sin\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*((cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*cos(-
(b*c - a*d)/d) - 2*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x), x)
```

Giac [C] time = 1.18177, size = 779, normalized size = 14.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + r
eal_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 2*imag
_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*imag_part(
cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 4*sin_integral((b
*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*imag_part(cos_integral(b*x +
 b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(-b*x - b*c/
d))*tan(1/2*a)*tan(1/2*b*c/d)^2 + 4*sin_integral((b*d*x + b*c)/d)*tan(1/2*a
)*tan(1/2*b*c/d)^2 - real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2 - re
al_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 + 4*real_part(cos_integral
(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + 4*real_part(cos_integral(-b*x -
b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) - real_part(cos_integral(b*x + b*c/d))*ta
n(1/2*b*c/d)^2 - real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 - 2
*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a) + 2*imag_part(cos_integral
(-b*x - b*c/d))*tan(1/2*a) - 4*sin_integral((b*d*x + b*c)/d)*tan(1/2*a) + 2
*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d) - 2*imag_part(cos_inte
gral(-b*x - b*c/d))*tan(1/2*b*c/d) + 4*sin_integral((b*d*x + b*c)/d)*tan(1/
2*b*c/d) + real_part(cos_integral(b*x + b*c/d)) + real_part(cos_integral(-b
*x - b*c/d)))/(d*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(1/2*a)^2 + d*tan(1/2
*b*c/d)^2 + d)
```

3.6 $\int \frac{\cos(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=73

$$-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{\cos(a+bx)}{d(c+dx)}$$

```
[Out] -(Cos[a + b*x]/(d*(c + d*x))) - (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)
/d])/d^2 - (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2
```

Rubi [A] time = 0.109797, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3297, 3303, 3299, 3302}

$$-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{\cos(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x)^2,x]
```

```
[Out] -(Cos[a + b*x]/(d*(c + d*x))) - (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)
/d])/d^2 - (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{(c+dx)^2}dx = -\frac{\cos(a+bx)}{d(c+dx)} - \frac{b\int \frac{\sin(a+bx)}{c+dx}dx}{d}$$
$$= -\frac{\cos(a+bx)}{d(c+dx)} - \frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx}{d} - \frac{\left(b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx}{d}$$
$$= -\frac{\cos(a+bx)}{d(c+dx)} - \frac{b\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^2} - \frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}$$

Mathematica [A] time = 0.393225, size = 65, normalized size = 0.89

$$-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)+\frac{d\cos(a+bx)}{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^2,x]
```

```
[Out] -(((d*Cos[a + b*x])/(c + d*x) + b*CosIntegral[b*(c/d + x)]*Sin[a - (b*c)/d]
 + b*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)])/d^2)
```

Maple [A] time = 0.03, size = 109, normalized size = 1.5

$$b\left(-\frac{\cos(bx+a)}{((bx+a)d-da+cb)d}-\frac{1}{d}\left(\frac{1}{d}\text{Si}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)-\frac{1}{d}\text{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^2,x)
```

```
[Out] b*(-cos(b*x+a)/((b*x+a)*d-d*a+c*b)/d-(Si(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)
/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)
```

Maxima [C] time = 1.31494, size = 224, normalized size = 3.07

$$-\frac{8b^2\left(E_2\left(\frac{ibc+i(bx+a)d-iad}{d}\right)+E_2\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^2\left(8iE_2\left(\frac{ibc+i(bx+a)d-iad}{d}\right)-8iE_2\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\text{s}}{16\left(bcd+(bx+a)d^2-ad^2\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(8*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_in
tegral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b^2*
(8*I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - 8*I*exp_integra
l_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/((b*c*d +
(b*x + a)*d^2 - a*d^2)*b)
```

Fricas [A] time = 1.10738, size = 302, normalized size = 4.14

$$-\frac{2\,(bdx+bc)\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)+2\,d\cos\left(bx+a\right)+\left((bdx+bc)\,\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+(bdx+bc)\,\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*d*x + b*c)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 2
*d*cos(b*x + a) + ((b*d*x + b*c)*cos_integral((b*d*x + b*c)/d) + (b*d*x + b
*c)*cos_integral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/d))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x)**2, x)
```

Giac [C] time = 1.49013, size = 4298, normalized size = 58.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^
2*tan(1/2*b*c/d)^2 - b*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*
x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b*d*x*sin_integral((b*d*x + b*c)/d)*
tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b*d*x*real_part(cos_integr
al(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b*d*x*real_
part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)
 - 2*b*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*t
an(1/2*b*c/d)^2 - 2*b*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x
)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + b*c*imag_part(cos_integral(b*x + b*c/d))*
tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b*c*imag_part(cos_integral(-
b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b*c*sin_inte
gral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b*d*x*
imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*d*x*im
ag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b*d*x*s
in_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2 + 4*b*d*x*imag_par
t(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) - 4*b
*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/
2*b*c/d) + 8*b*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*
tan(1/2*b*c/d) + 2*b*c*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*
tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b*c*real_part(cos_integral(-b*x - b*c/d))*t
an(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - b*d*x*imag_part(cos_integral(b*
```

```
x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + b*d*x*imag_part(cos_integral(
-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 2*b*d*x*sin_integral((b*d*
x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 2*b*c*real_part(cos_integral(
b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b*c*real_part(
cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + b*
d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b*
d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2
*b*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b*d*
x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) + 2*b*d*x*
real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) - b*c*imag_
part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*c*imag_part
(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b*c*sin_integr
al((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b*d*x*real_part(cos_int
egral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) - 2*b*d*x*real_part(cos_i
ntegral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) + 4*b*c*imag_part(cos_
integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) - 4*b*c*ima
g_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)
 + 8*b*c*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*
c/d) + 2*b*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*
c/d) + 2*b*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b
*c/d) - b*c*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c
/d)^2 + b*c*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*
c/d)^2 - 2*b*c*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*b*c/d)^
2 - 2*b*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^
2 - 2*b*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)
^2 + b*c*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2
 - b*c*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2
+ 2*b*c*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*d*t
an(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + b*d*x*imag_part(cos_integral(
b*x + b*c/d))*tan(1/2*b*x)^2 - b*d*x*imag_part(cos_integral(-b*x - b*c/d))*
tan(1/2*b*x)^2 + 2*b*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2 + 2*b
*c*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) + 2*b*c*r
eal_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) - b*d*x*imag
_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2 + b*d*x*imag_part(cos_integra
l(-b*x - b*c/d))*tan(1/2*a)^2 - 2*b*d*x*sin_integral((b*d*x + b*c)/d)*tan(1
/2*a)^2 - 2*b*c*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2
*b*c/d) - 2*b*c*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/
2*b*c/d) + 4*b*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*
b*c/d) - 4*b*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b
*c/d) + 8*b*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d) + 2
*b*c*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b
*c*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - b*d*
x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d)^2 + b*d*x*imag_part(c
os_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 - 2*b*d*x*sin_integral((b*d*x +
 b*c)/d)*tan(1/2*b*c/d)^2 - 2*b*c*real_part(cos_integral(b*x + b*c/d))*tan(
1/2*a)*tan(1/2*b*c/d)^2 - 2*b*c*real_part(cos_integral(-b*x - b*c/d))*tan(1
/2*a)*tan(1/2*b*c/d)^2 + b*c*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b
*x)^2 - b*c*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2 + 2*b*c*si
n_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2 + 2*b*d*x*real_part(cos_integral
(b*x + b*c/d))*tan(1/2*a) + 2*b*d*x*real_part(cos_integral(-b*x - b*c/d))*t
an(1/2*a) - b*c*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2 + b*c*ima
g_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 - 2*b*c*sin_integral((b*d*x
 + b*c)/d)*tan(1/2*a)^2 + 2*d*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b*d*x*real_pa
rt(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d) - 2*b*d*x*real_part(cos_integr
al(-b*x - b*c/d))*tan(1/2*b*c/d) + 4*b*c*imag_part(cos_integral(b*x + b*c/d
))*tan(1/2*a)*tan(1/2*b*c/d) - 4*b*c*imag_part(cos_integral(-b*x - b*c/d))*
tan(1/2*a)*tan(1/2*b*c/d) + 8*b*c*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*
tan(1/2*b*c/d) - b*c*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d)^2
+ b*c*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 - 2*b*c*sin_in
```

```
tegral((b*d*x + b*c)/d)*tan(1/2*b*c/d)^2 - 2*d*tan(1/2*b*x)^2*tan(1/2*b*c/d
)^2 - 8*d*tan(1/2*b*x)*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*d*tan(1/2*a)^2*tan(1
/2*b*c/d)^2 + b*d*x*imag_part(cos_integral(b*x + b*c/d)) - b*d*x*imag_part(
cos_integral(-b*x - b*c/d)) + 2*b*d*x*sin_integral((b*d*x + b*c)/d) + 2*b*c
*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a) + 2*b*c*real_part(cos_inte
gral(-b*x - b*c/d))*tan(1/2*a) - 2*b*c*real_part(cos_integral(b*x + b*c/d))
*tan(1/2*b*c/d) - 2*b*c*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d
) + b*c*imag_part(cos_integral(b*x + b*c/d)) - b*c*imag_part(cos_integral(-
b*x - b*c/d)) + 2*b*c*sin_integral((b*d*x + b*c)/d) - 2*d*tan(1/2*b*x)^2 -
8*d*tan(1/2*b*x)*tan(1/2*a) - 2*d*tan(1/2*a)^2 + 2*d*tan(1/2*b*c/d)^2 + 2*d
)/(d^3*x*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + c*d^2*tan(1/2*b*x)^
2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d^3*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + d^3*x
*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + d^3*x*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + c*
d^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + c*d^2*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + c
*d^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d^3*x*tan(1/2*b*x)^2 + d^3*x*tan(1/2*a
)^2 + d^3*x*tan(1/2*b*c/d)^2 + c*d^2*tan(1/2*b*x)^2 + c*d^2*tan(1/2*a)^2 +
c*d^2*tan(1/2*b*c/d)^2 + d^3*x + c*d^2)
```

3.7 $\int \frac{\cos(a+bx)}{(c+dx)^3} dx$

Optimal. Leaf size=104

$$-\frac{b^2 \cos\left(a-\frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{2d^3}+\frac{b^2 \sin\left(a-\frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d}+bx\right)}{2d^3}+\frac{b\sin(a+bx)}{2d^2(c+dx)}-\frac{\cos(a+bx)}{2d(c+dx)^2}$$

```
[Out] -Cos[a + b*x]/(2*d*(c + d*x)^2) - (b^2*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d
 + b*x])/(2*d^3) + (b*Sin[a + b*x])/(2*d^2*(c + d*x)) + (b^2*Sin[a - (b*c)/
d]*SinIntegral[(b*c)/d + b*x])/(2*d^3)
```

Rubi [A] time = 0.138254, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3297, 3303, 3299, 3302}

$$-\frac{b^2 \cos\left(a-\frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{2d^3}+\frac{b^2 \sin\left(a-\frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d}+bx\right)}{2d^3}+\frac{b\sin(a+bx)}{2d^2(c+dx)}-\frac{\cos(a+bx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x)^3,x]
```

```
[Out] -Cos[a + b*x]/(2*d*(c + d*x)^2) - (b^2*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d
 + b*x])/(2*d^3) + (b*Sin[a + b*x])/(2*d^2*(c + d*x)) + (b^2*Sin[a - (b*c)/
d]*SinIntegral[(b*c)/d + b*x])/(2*d^3)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx = -\frac{\cos(a+bx)}{2d(c+dx)^2} - \frac{b\int \frac{\sin(a+bx)}{(c+dx)^2}dx}{2d}$$
$$= -\frac{\cos(a+bx)}{2d(c+dx)^2} + \frac{b\sin(a+bx)}{2d^2(c+dx)} - \frac{b^2\int \frac{\cos(a+bx)}{c+dx}dx}{2d^2}$$
$$= -\frac{\cos(a+bx)}{2d(c+dx)^2} + \frac{b\sin(a+bx)}{2d^2(c+dx)} - \frac{\left(b^2\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx}{2d^2} + \frac{\left(b^2\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}}{2d^2}$$
$$= -\frac{\cos(a+bx)}{2d(c+dx)^2} - \frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{2d^3} + \frac{b\sin(a+bx)}{2d^2(c+dx)} + \frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{2d^3}$$

Mathematica [A] time = 0.608522, size = 89, normalized size = 0.86

$$\frac{b^2\left(-\cos\left(a-\frac{bc}{d}\right)\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)+\frac{d(b(c+dx)\sin(a+bx)-d\cos(a+bx))}{(c+dx)^2}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^3,x]
```

```
[Out] (-(b^2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)]) + (d*(-(d*Cos[a + b*x]) +
 b*(c + d*x)*Sin[a + b*x]))/(c + d*x)^2 + b^2*Sin[a - (b*c)/d]*SinIntegral[
b*(c/d + x)])/(2*d^3)
```

Maple [A] time = 0.033, size = 143, normalized size = 1.4

$$b^2\left(-\frac{\cos(bx+a)}{2\left((bx+a)d-da+cb\right)^2 d} - \frac{1}{2d}\left(-\frac{\sin(bx+a)}{((bx+a)d-da+cb)d} + \frac{1}{d}\left(\frac{1}{d}\text{Si}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right) + \frac{1}{d}\text{C}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^3,x)
```

```
[Out] b^2*(-1/2*cos(b*x+a)/((b*x+a)*d-d*a+c*b)^2/d-1/2*(-sin(b*x+a)/((b*x+a)*d-d*
a+c*b)/d+(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)
*cos((-a*d+b*c)/d)/d)/d)/d)
```

Maxima [C] time = 1.60081, size = 271, normalized size = 2.61

$$-\frac{8\,b^3\left(E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^3\left(8i\,E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-8i\,E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right.\right.}{16\left(b^2c^2d-2\,abcd^2+(bx+a)^2d^3+a^2d^3+2\left(bcd^2-ad^3\right)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(8*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_in
tegral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b^3*
```

```
(8*I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - 8*I*exp_integra
l_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/((b^2*c^2*
d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a)
)*b)
```

Fricas [B] time = 1.15994, size = 471, normalized size = 4.53

$$-\frac{2\,d^2\cos\left(bx+a\right)-2\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\sin\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)+\left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+\left(b^2d^2\right.\right.}{4\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d^2*cos(b*x + a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-(b*
c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*cos_integral((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2)*cos_integral(-(b*d*x + b*c)/d))*cos(-(b*c - a*d)/d) - 2*(b*d^2*x + b*c*d
)*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(a+bx\right)}{\left(c+dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)**3,x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x)**3, x)
```

Giac [C] time = 1.53793, size = 7449, normalized size = 71.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*(b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1
/2*a)^2*tan(1/2*b*c/d)^2 + b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d)
)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(co
s_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b^2
*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*
tan(1/2*b*c/d) - 4*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2
*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b^2*d^2*x^2*imag_part(cos_integral(b*x + b
*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*imag_part
(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 4
*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/
2*b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^
2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(-b*x -
```

```
 b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b^2*d^2*x^2*real_pa
rt(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 - b^2*d^2*x^2*rea
l_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 4*b^2*d^2*
x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*
b*c/d) + 4*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2
*tan(1/2*a)*tan(1/2*b*c/d) - 4*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d
))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 4*b^2*c*d*x*imag_part(cos_i
ntegral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 8*b^2*c
*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/
d) - b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/
2*b*c/d)^2 - b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)
^2*tan(1/2*b*c/d)^2 + 4*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(
1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 4*b^2*c*d*x*imag_part(cos_integral
(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 8*b^2*c*d*x*si
n_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + b^
2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^
2 + b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*
b*c/d)^2 + b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(
1/2*a)^2*tan(1/2*b*c/d)^2 + b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*t
an(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_i
ntegral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) + 2*b^2*d^2*x^2*imag_part(c
os_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) - 4*b^2*d^2*x^2*sin_in
tegral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a) - 2*b^2*c*d*x*real_part(c
os_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^2*c*d*x*real_pa
rt(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b^2*d^2*x^2*
imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) - 2*b^2*
d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)
 + 4*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*b*c/d
) + 8*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2
*a)*tan(1/2*b*c/d) + 8*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(
1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) - 2*b^2*d^2*x^2*imag_part(cos_integral
(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b^2*d^2*x^2*imag_part(cos_in
tegral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 4*b^2*d^2*x^2*sin_integ
ral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*b^2*c^2*imag_part(cos_
integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b^2*c
^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/
2*b*c/d) - 4*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a
)^2*tan(1/2*b*c/d) - 2*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1
/2*b*x)^2*tan(1/2*b*c/d)^2 - 2*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/
d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^2*x^2*imag_part(cos_integral(
b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_int
egral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 + 4*b^2*d^2*x^2*sin_integr
al((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d)^2 + 2*b^2*c^2*imag_part(cos_i
ntegral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^2*c^
2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b
*c/d)^2 + 4*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)
*tan(1/2*b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/
2*a)^2*tan(1/2*b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))
*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + b^2*d^2*x^2*real_part(cos_integral(b*x + b
*c/d))*tan(1/2*b*x)^2 + b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*t
an(1/2*b*x)^2 - 4*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*
x)^2*tan(1/2*a) + 4*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2
*b*x)^2*tan(1/2*a) - 8*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)
^2*tan(1/2*a) - b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)
^2 - b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 - b^2*c
^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 - b^2*c
^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 4*b^
2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)
- 4*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*
```

```
b*c/d) + 8*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*b
*c/d) + 4*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1
/2*b*c/d) + 4*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*
tan(1/2*b*c/d) + 4*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x
)^2*tan(1/2*a)*tan(1/2*b*c/d) + 4*b^2*c^2*real_part(cos_integral(-b*x - b*c
/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) - 4*b^2*c*d*x*imag_part(cos_i
ntegral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 4*b^2*c*d*x*imag_part(c
os_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 8*b^2*c*d*x*sin_in
tegral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d) - b^2*d^2*x^2*real_part
(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d)^2 - b^2*d^2*x^2*real_part(cos_in
tegral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 - b^2*c^2*real_part(cos_integral(b*x
 + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - b^2*c^2*real_part(cos_integral
(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + 4*b^2*c*d*x*imag_part(cos
_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 4*b^2*c*d*x*imag_part
(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 + 8*b^2*c*d*x*sin_
integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d)^2 + 4*b*d^2*x*tan(1/2*b
*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + b^2*c^2*real_part(cos_integral(b*x + b*
c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + b^2*c^2*real_part(cos_integral(-b*x -
 b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*b*d^2*x*tan(1/2*b*x)*tan(1/2*a)^
2*tan(1/2*b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1
/2*b*x)^2 + 2*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^
2 - 2*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a) + 2*b^2*d
^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a) - 4*b^2*d^2*x^2*sin
_integral((b*d*x + b*c)/d)*tan(1/2*a) - 2*b^2*c^2*imag_part(cos_integral(b*
x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) + 2*b^2*c^2*imag_part(cos_integral(-b
*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) - 4*b^2*c^2*sin_integral((b*d*x + b*
c)/d)*tan(1/2*b*x)^2*tan(1/2*a) - 2*b^2*c*d*x*real_part(cos_integral(b*x +
b*c/d))*tan(1/2*a)^2 - 2*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*ta
n(1/2*a)^2 + 2*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c
/d) - 2*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d) +
4*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*c/d) + 2*b^2*c^2*imag
_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) - 2*b^2*c^2*
imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) + 4*b^2
*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*b*c/d) + 8*b^2*c*
d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + 8*b^2*
c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) - 2*b
^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2
*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)
- 4*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*b
^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d)^2 - 2*b^2*c*d*
x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 + 2*b^2*c^2*imag_p
art(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^2*c^2*imag
_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 + 4*b^2*c^2*s
in_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d)^2 + 4*b*c*d*tan(1/2*
b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 4*b*c*d*tan(1/2*b*x)*tan(1/2*a)^2*tan(
1/2*b*c/d)^2 + 2*d^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + b^2*d^2
*x^2*real_part(cos_integral(b*x + b*c/d)) + b^2*d^2*x^2*real_part(cos_integ
ral(-b*x - b*c/d)) + b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b
*x)^2 + b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2 - 4*b^
2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a) + 4*b^2*c*d*x*imag_
part(cos_integral(-b*x - b*c/d))*tan(1/2*a) - 8*b^2*c*d*x*sin_integral((b*d
*x + b*c)/d)*tan(1/2*a) + 4*b*d^2*x*tan(1/2*b*x)^2*tan(1/2*a) - b^2*c^2*rea
l_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2 - b^2*c^2*real_part(cos_inte
gral(-b*x - b*c/d))*tan(1/2*a)^2 + 4*b*d^2*x*tan(1/2*b*x)*tan(1/2*a)^2 + 4*
b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d) - 4*b^2*c*d*x
*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d) + 8*b^2*c*d*x*sin_int
egral((b*d*x + b*c)/d)*tan(1/2*b*c/d) + 4*b^2*c^2*real_part(cos_integral(b*
x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + 4*b^2*c^2*real_part(cos_integral(-b
*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) - b^2*c^2*real_part(cos_integral(b*x
```

```
 + b*c/d))*tan(1/2*b*c/d)^2 - b^2*c^2*real_part(cos_integral(-b*x - b*c/d))
*tan(1/2*b*c/d)^2 - 4*b*d^2*x*tan(1/2*b*x)*tan(1/2*b*c/d)^2 - 4*b*d^2*x*tan
(1/2*a)*tan(1/2*b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))
 + 2*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d)) - 2*b^2*c^2*imag_part(
cos_integral(b*x + b*c/d))*tan(1/2*a) + 2*b^2*c^2*imag_part(cos_integral(-b
*x - b*c/d))*tan(1/2*a) - 4*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a
) + 4*b*c*d*tan(1/2*b*x)^2*tan(1/2*a) + 4*b*c*d*tan(1/2*b*x)*tan(1/2*a)^2 +
 2*d^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b^2*c^2*imag_part(cos_integral(b*x +
 b*c/d))*tan(1/2*b*c/d) - 2*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*t
an(1/2*b*c/d) + 4*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*c/d) - 4*
b*c*d*tan(1/2*b*x)*tan(1/2*b*c/d)^2 - 2*d^2*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2
 - 4*b*c*d*tan(1/2*a)*tan(1/2*b*c/d)^2 - 8*d^2*tan(1/2*b*x)*tan(1/2*a)*tan(
1/2*b*c/d)^2 - 2*d^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + b^2*c^2*real_part(cos_
integral(b*x + b*c/d)) + b^2*c^2*real_part(cos_integral(-b*x - b*c/d)) - 4*
b*d^2*x*tan(1/2*b*x) - 4*b*d^2*x*tan(1/2*a) - 4*b*c*d*tan(1/2*b*x) - 2*d^2*
tan(1/2*b*x)^2 - 4*b*c*d*tan(1/2*a) - 8*d^2*tan(1/2*b*x)*tan(1/2*a) - 2*d^2
*tan(1/2*a)^2 + 2*d^2*tan(1/2*b*c/d)^2 + 2*d^2)/(d^5*x^2*tan(1/2*b*x)^2*tan
(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*
b*c/d)^2 + d^5*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + d^5*x^2*tan(1/2*b*x)^2*tan
(1/2*b*c/d)^2 + d^5*x^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(1/2*b*x
)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2*b*x)^2*tan(1/2*a)^2 +
 2*c*d^4*x*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2*a)^2*tan(1/2
*b*c/d)^2 + d^5*x^2*tan(1/2*b*x)^2 + d^5*x^2*tan(1/2*a)^2 + c^2*d^3*tan(1/2
*b*x)^2*tan(1/2*a)^2 + d^5*x^2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(1/2*b*x)^2*ta
n(1/2*b*c/d)^2 + c^2*d^3*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2*
b*x)^2 + 2*c*d^4*x*tan(1/2*a)^2 + 2*c*d^4*x*tan(1/2*b*c/d)^2 + d^5*x^2 + c^
2*d^3*tan(1/2*b*x)^2 + c^2*d^3*tan(1/2*a)^2 + c^2*d^3*tan(1/2*b*c/d)^2 + 2*
c*d^4*x + c^2*d^3)
```

3.8 $\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=127

$$\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{6d^4}+\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{6d^4}+\frac{b^2\cos(a+bx)}{6d^3(c+dx)}+\frac{b\sin(a+bx)}{6d^2(c+dx)^2}-\frac{\cos(a+bx)}{3d(c+dx)^3}$$

```
[Out] -Cos[a + b*x]/(3*d*(c + d*x)^3) + (b^2*Cos[a + b*x])/(6*d^3*(c + d*x)) + (b
^3*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(6*d^4) + (b*Sin[a + b*x])/
(6*d^2*(c + d*x)^2) + (b^3*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(6*
d^4)
```

Rubi [A] time = 0.159249, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3297, 3303, 3299, 3302}

$$\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{6d^4}+\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{6d^4}+\frac{b^2\cos(a+bx)}{6d^3(c+dx)}+\frac{b\sin(a+bx)}{6d^2(c+dx)^2}-\frac{\cos(a+bx)}{3d(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x)^4,x]
```

```
[Out] -Cos[a + b*x]/(3*d*(c + d*x)^3) + (b^2*Cos[a + b*x])/(6*d^3*(c + d*x)) + (b
^3*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(6*d^4) + (b*Sin[a + b*x])/
(6*d^2*(c + d*x)^2) + (b^3*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(6*
d^4)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule steps

$$\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx = -\frac{\cos(a+bx)}{3d(c+dx)^3} - \frac{b\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx}{3d}$$
$$= -\frac{\cos(a+bx)}{3d(c+dx)^3} + \frac{b\sin(a+bx)}{6d^2(c+dx)^2} - \frac{b^2\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{6d^2}$$
$$= -\frac{\cos(a+bx)}{3d(c+dx)^3} + \frac{b^2\cos(a+bx)}{6d^3(c+dx)} + \frac{b\sin(a+bx)}{6d^2(c+dx)^2} + \frac{b^3\int \frac{\sin(a+bx)}{c+dx}\,dx}{6d^3}$$
$$= -\frac{\cos(a+bx)}{3d(c+dx)^3} + \frac{b^2\cos(a+bx)}{6d^3(c+dx)} + \frac{b\sin(a+bx)}{6d^2(c+dx)^2} + \frac{\left(b^3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{6d^3} + \frac{\left(b^3\sin\left(a-\right.\right.}{}$$
$$= -\frac{\cos(a+bx)}{3d(c+dx)^3} + \frac{b^2\cos(a+bx)}{6d^3(c+dx)} + \frac{b^3\mathrm{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{6d^4} + \frac{b\sin(a+bx)}{6d^2(c+dx)^2} + \frac{b^3\cos\left(a-\frac{bc}{d}\right)\mathrm{S}}{6d^4}$$

Mathematica [A] time = 0.552071, size = 144, normalized size = 1.13

$$\frac{b^3(c+dx)^3\left(\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)\right)+d\cos(bx)\left(\cos(a)\left(b^2(c+dx)^2-2d^2\right.\right.}{6d^4(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^4,x]
```

```
[Out] (d*Cos[b*x]*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*d*(c + d*x)*Sin[a]) + d*
(b*d*(c + d*x)*Cos[a] - (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x] + b^3*(
c + d*x)^3*(CosIntegral[b*(c/d + x)]*Sin[a - (b*c)/d] + Cos[a - (b*c)/d]*Si
nIntegral[b*(c/d + x)]))/(6*d^4*(c + d*x)^3)
```

Maple [A] time = 0.032, size = 179, normalized size = 1.4

$$b^3\left(-\frac{\cos(bx+a)}{3\left((bx+a)d-da+cb\right)^3d}-\frac{1}{3d}\left(-\frac{\sin(bx+a)}{2\left((bx+a)d-da+cb\right)^2d}+\frac{1}{2d}\left(-\frac{\cos(bx+a)}{\left((bx+a)d-da+cb\right)d}-\frac{1}{d}\left(\frac{1}{d}\mathrm{Si}\left(bx+a\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^4,x)
```

```
[Out] b^3*(-1/3*cos(b*x+a)/((b*x+a)*d-d*a+c*b)^3/d-1/3*(-1/2*sin(b*x+a)/((b*x+a)*
d-d*a+c*b)^2/d+1/2*(-cos(b*x+a)/((b*x+a)*d-d*a+c*b)/d-(Si(b*x+a+(-a*d+b*c)/
d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)/d)/d)
```

Maxima [C] time = 1.93391, size = 339, normalized size = 2.67

$$-\frac{8b^4\left(E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^4\left(8i\,E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-8i\,E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right.\right.}{16\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3+(bx+a)^3d^4-a^3d^4+3\left(bcd^3-ad^4\right)(bx+a)^2+3\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/16*(8*b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_in
tegral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b^4*
(8*I*exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - 8*I*exp_integra
l_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/((b^3*c^3*
d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^
3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))
*b)
```

Fricas [B] time = 1.17408, size = 639, normalized size = 5.03

$$\frac{2\left(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3\right)\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right) + 2\left(b^2d^3x^2 + 2\,b^2cd^2x + b^2c^2d - 2\,d^3\right)\cos\left(bx+a\right) + 2}{12\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-(b*c
 - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x +
 b^2*c^2*d - 2*d^3)*cos(b*x + a) + 2*(b*d^3*x + b*c*d^2)*sin(b*x + a) + ((b
^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral((b*d*x
 + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_
integral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3
*c^2*d^5*x + c^3*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(a + bx\right)}{\left(c + dx\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)**4,x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x)**4, x)
```

Giac [C] time = 1.88635, size = 11310, normalized size = 89.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/12*(b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1
/2*a)^2*tan(1/2*b*c/d)^2 - b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d)
)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^3*d^3*x^3*sin_integral
((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^3*d^3*
x^3*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/
2*b*c/d) + 2*b^3*d^3*x^3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)
^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*b^3*d^3*x^3*real_part(cos_integral(b*x +
```

```
b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*real_pa
rt(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 +
 3*b^3*c*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/
2*a)^2*tan(1/2*b*c/d)^2 - 3*b^3*c*d^2*x^2*imag_part(cos_integral(-b*x - b*c
/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 6*b^3*c*d^2*x^2*sin_int
egral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b^3*d
^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + b
^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^
2 - 2*b^3*d^3*x^3*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2
 + 4*b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/
2*a)*tan(1/2*b*c/d) - 4*b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d))*t
an(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 8*b^3*d^3*x^3*sin_integral((b*d*x
 + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 6*b^3*c*d^2*x^2*real_
part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)
+ 6*b^3*c*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(
1/2*a)^2*tan(1/2*b*c/d) - b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*
tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + b^3*d^3*x^3*imag_part(cos_integral(-b*x -
 b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*sin_integral((b*d*
x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 6*b^3*c*d^2*x^2*real_part(cos
_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 6*b^3*
c*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*t
an(1/2*b*c/d)^2 + b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*
a)^2*tan(1/2*b*c/d)^2 - b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d))*t
an(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^3*d^3*x^3*sin_integral((b*d*x + b*c)/d)*
tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^3*c^2*d*x*imag_part(cos_integral(b*x +
b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 3*b^3*c^2*d*x*imag_p
art(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^
2 + 6*b^3*c^2*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2
*tan(1/2*b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(b*x + b*c/d))*tan(
1/2*b*x)^2*tan(1/2*a) + 2*b^3*d^3*x^3*real_part(cos_integral(-b*x - b*c/d))
*tan(1/2*b*x)^2*tan(1/2*a) - 3*b^3*c*d^2*x^2*imag_part(cos_integral(b*x + b
*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 3*b^3*c*d^2*x^2*imag_part(cos_integral
(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 - 6*b^3*c*d^2*x^2*sin_integral(
(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^3*d^3*x^3*real_part(cos_
integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) - 2*b^3*d^3*x^3*real_p
art(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) + 12*b^3*c*d^
2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/
2*b*c/d) - 12*b^3*c*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b
*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 24*b^3*c*d^2*x^2*sin_integral((b*d*x + b*
c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 2*b^3*d^3*x^3*real_part(co
s_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b^3*d^3*x^3*real_p
art(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*b^3*c^2*d*x
*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b
*c/d) + 6*b^3*c^2*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*
tan(1/2*a)^2*tan(1/2*b*c/d) - 3*b^3*c*d^2*x^2*imag_part(cos_integral(b*x +
b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + 3*b^3*c*d^2*x^2*imag_part(cos_int
egral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 6*b^3*c*d^2*x^2*sin_
integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*r
eal_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^3*d^3
*x^3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 6*
b^3*c^2*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*
tan(1/2*b*c/d)^2 - 6*b^3*c^2*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(
1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 3*b^3*c*d^2*x^2*imag_part(cos_inte
gral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 3*b^3*c*d^2*x^2*imag_par
t(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 6*b^3*c*d^2*x
^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^3*
x^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + b^3*c^3*imag_part(cos_in
tegral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b^3*c^3
*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*
```

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b*c/d)^2 + 2*b^3*c^3*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a
)^2*tan(1/2*b*c/d)^2 + b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan
(1/2*b*x)^2 - b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x
)^2 + 2*b^3*d^3*x^3*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2 + 6*b^3*c*
d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) + 6*
b^3*c*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*
a) - b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2 + b^3*d^
3*x^3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 - 2*b^3*d^3*x^3*si
n_integral((b*d*x + b*c)/d)*tan(1/2*a)^2 - 3*b^3*c^2*d*x*imag_part(cos_inte
gral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 3*b^3*c^2*d*x*imag_part(co
s_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 - 6*b^3*c^2*d*x*sin_i
ntegral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2 - 6*b^3*c*d^2*x^2*real
_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) - 6*b^3*c*d^
2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) +
 4*b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/
d) - 4*b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2
*b*c/d) + 8*b^3*d^3*x^3*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*
c/d) + 12*b^3*c^2*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*t
an(1/2*a)*tan(1/2*b*c/d) - 12*b^3*c^2*d*x*imag_part(cos_integral(-b*x - b*c
/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 24*b^3*c^2*d*x*sin_integral
((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 6*b^3*c*d^2*x^
2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*b^3*
c*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)
 + 2*b^3*c^3*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)
^2*tan(1/2*b*c/d) + 2*b^3*c^3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2
*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - b^3*d^3*x^3*imag_part(cos_integral(b*
x + b*c/d))*tan(1/2*b*c/d)^2 + b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*
c/d))*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*sin_integral((b*d*x + b*c)/d)*tan(1/
2*b*c/d)^2 - 3*b^3*c^2*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x
)^2*tan(1/2*b*c/d)^2 + 3*b^3*c^2*d*x*imag_part(cos_integral(-b*x - b*c/d))*
tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 6*b^3*c^2*d*x*sin_integral((b*d*x + b*c)/
d)*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 6*b^3*c*d^2*x^2*real_part(cos_integral
(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 6*b^3*c*d^2*x^2*real_part(cos_
integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^3*c^3*real_part(c
os_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^
3*c^3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1
/2*b*c/d)^2 + 3*b^3*c^2*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)
^2*tan(1/2*b*c/d)^2 - 3*b^3*c^2*d*x*imag_part(cos_integral(-b*x - b*c/d))*t
an(1/2*a)^2*tan(1/2*b*c/d)^2 + 6*b^3*c^2*d*x*sin_integral((b*d*x + b*c)/d)*
tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*b^2*c*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^2*t
an(1/2*b*c/d)^2 + 3*b^3*c*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(
1/2*b*x)^2 - 3*b^3*c*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*
b*x)^2 + 6*b^3*c*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2 + 2*b
^3*d^3*x^3*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a) + 2*b^3*d^3*x^3*
real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a) + 6*b^3*c^2*d*x*real_part(
cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) + 6*b^3*c^2*d*x*real_p
art(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) - 3*b^3*c*d^2*x^2
*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2 + 3*b^3*c*d^2*x^2*imag_p
art(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 - 6*b^3*c*d^2*x^2*sin_integral
((b*d*x + b*c)/d)*tan(1/2*a)^2 + 2*b^2*d^3*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2
- b^3*c^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2
+ b^3*c^3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2
 - 2*b^3*c^3*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*
b^3*d^3*x^3*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d) - 2*b^3*d^3
*x^3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d) - 6*b^3*c^2*d*x*r
eal_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) - 6*b^3*c
^2*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)
+ 12*b^3*c*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*
b*c/d) - 12*b^3*c*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*
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tan(1/2*b*c/d) + 24*b^3*c*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*
tan(1/2*b*c/d) + 4*b^3*c^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x
)^2*tan(1/2*a)*tan(1/2*b*c/d) - 4*b^3*c^3*imag_part(cos_integral(-b*x - b*c
/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 8*b^3*c^3*sin_integral((b*d
*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 6*b^3*c^2*d*x*real_
part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*b^3*c^2*d*x
*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 3*b^3*
c*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d)^2 + 3*b^3*c*d
^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 - 6*b^3*c*d^2
*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*c/d)^2 - 2*b^2*d^3*x^2*tan(1/2
*b*x)^2*tan(1/2*b*c/d)^2 - b^3*c^3*imag_part(cos_integral(b*x + b*c/d))*tan
(1/2*b*x)^2*tan(1/2*b*c/d)^2 + b^3*c^3*imag_part(cos_integral(-b*x - b*c/d)
)*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 2*b^3*c^3*sin_integral((b*d*x + b*c)/d)
*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 6*b^3*c^2*d*x*real_part(cos_integral(b*x
 + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 6*b^3*c^2*d*x*real_part(cos_integr
al(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 8*b^2*d^3*x^2*tan(1/2*b*x)*
tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^2*d^3*x^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 +
 b^3*c^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2
 - b^3*c^3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d
)^2 + 2*b^3*c^3*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d)^2
 + 2*b^2*c^2*d*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + b^3*d^3*x^3*i
mag_part(cos_integral(b*x + b*c/d)) - b^3*d^3*x^3*imag_part(cos_integral(-b
*x - b*c/d)) + 2*b^3*d^3*x^3*sin_integral((b*d*x + b*c)/d) + 3*b^3*c^2*d*x*
imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2 - 3*b^3*c^2*d*x*imag_pa
rt(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2 + 6*b^3*c^2*d*x*sin_integral(
(b*d*x + b*c)/d)*tan(1/2*b*x)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(b*
x + b*c/d))*tan(1/2*a) + 6*b^3*c*d^2*x^2*real_part(cos_integral(-b*x - b*c/
d))*tan(1/2*a) + 2*b^3*c^3*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x
)^2*tan(1/2*a) + 2*b^3*c^3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*
x)^2*tan(1/2*a) - 3*b^3*c^2*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/
2*a)^2 + 3*b^3*c^2*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 -
 6*b^3*c^2*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2 + 4*b^2*c*d^2*x*t
an(1/2*b*x)^2*tan(1/2*a)^2 - 6*b^3*c*d^2*x^2*real_part(cos_integral(b*x + b
*c/d))*tan(1/2*b*c/d) - 6*b^3*c*d^2*x^2*real_part(cos_integral(-b*x - b*c/d
))*tan(1/2*b*c/d) - 2*b^3*c^3*real_part(cos_integral(b*x + b*c/d))*tan(1/2*
b*x)^2*tan(1/2*b*c/d) - 2*b^3*c^3*real_part(cos_integral(-b*x - b*c/d))*tan
(1/2*b*x)^2*tan(1/2*b*c/d) + 12*b^3*c^2*d*x*imag_part(cos_integral(b*x + b*
c/d))*tan(1/2*a)*tan(1/2*b*c/d) - 12*b^3*c^2*d*x*imag_part(cos_integral(-b*
x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + 24*b^3*c^2*d*x*sin_integral((b*d*x
+ b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d) + 2*b^3*c^3*real_part(cos_integral(b*x
+ b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b^3*c^3*real_part(cos_integral(-b
*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 3*b^3*c^2*d*x*imag_part(cos_inte
gral(b*x + b*c/d))*tan(1/2*b*c/d)^2 + 3*b^3*c^2*d*x*imag_part(cos_integral(
-b*x - b*c/d))*tan(1/2*b*c/d)^2 - 6*b^3*c^2*d*x*sin_integral((b*d*x + b*c)/
d)*tan(1/2*b*c/d)^2 - 4*b^2*c*d^2*x*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 2*b^3
*c^3*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b
^3*c^3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 -
16*b^2*c*d^2*x*tan(1/2*b*x)*tan(1/2*a)*tan(1/2*b*c/d)^2 - 4*b*d^3*x*tan(1/2
*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 4*b^2*c*d^2*x*tan(1/2*a)^2*tan(1/2*b*
c/d)^2 - 4*b*d^3*x*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^3*c*d^2
*x^2*imag_part(cos_integral(b*x + b*c/d)) - 3*b^3*c*d^2*x^2*imag_part(cos_i
ntegral(-b*x - b*c/d)) + 6*b^3*c*d^2*x^2*sin_integral((b*d*x + b*c)/d) - 2*
b^2*d^3*x^2*tan(1/2*b*x)^2 + b^3*c^3*imag_part(cos_integral(b*x + b*c/d))*t
an(1/2*b*x)^2 - b^3*c^3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^
2 + 2*b^3*c^3*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2 + 6*b^3*c^2*d*x*
real_part(cos_integral(b*x + b*c/d))*tan(1/2*a) + 6*b^3*c^2*d*x*real_part(c
os_integral(-b*x - b*c/d))*tan(1/2*a) - 8*b^2*d^3*x^2*tan(1/2*b*x)*tan(1/2*
a) - 2*b^2*d^3*x^2*tan(1/2*a)^2 - b^3*c^3*imag_part(cos_integral(b*x + b*c/
d))*tan(1/2*a)^2 + b^3*c^3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)
```

```
^2 - 2*b^3*c^3*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2 + 2*b^2*c^2*d*tan
(1/2*b*x)^2*tan(1/2*a)^2 - 6*b^3*c^2*d*x*real_part(cos_integral(b*x + b*c/d
))*tan(1/2*b*c/d) - 6*b^3*c^2*d*x*real_part(cos_integral(-b*x - b*c/d))*tan
(1/2*b*c/d) + 4*b^3*c^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan
(1/2*b*c/d) - 4*b^3*c^3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*ta
n(1/2*b*c/d) + 8*b^3*c^3*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b
*c/d) + 2*b^2*d^3*x^2*tan(1/2*b*c/d)^2 - b^3*c^3*imag_part(cos_integral(b*x
 + b*c/d))*tan(1/2*b*c/d)^2 + b^3*c^3*imag_part(cos_integral(-b*x - b*c/d))
*tan(1/2*b*c/d)^2 - 2*b^3*c^3*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*c/d)^
2 - 2*b^2*c^2*d*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 8*b^2*c^2*d*tan(1/2*b*x)*
tan(1/2*a)*tan(1/2*b*c/d)^2 - 4*b*c*d^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b
*c/d)^2 - 2*b^2*c^2*d*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 4*b*c*d^2*tan(1/2*b*x
)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 4*d^3*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2
*b*c/d)^2 + 3*b^3*c^2*d*x*imag_part(cos_integral(b*x + b*c/d)) - 3*b^3*c^2*
d*x*imag_part(cos_integral(-b*x - b*c/d)) + 6*b^3*c^2*d*x*sin_integral((b*d
*x + b*c)/d) - 4*b^2*c*d^2*x*tan(1/2*b*x)^2 + 2*b^3*c^3*real_part(cos_integ
ral(b*x + b*c/d))*tan(1/2*a) + 2*b^3*c^3*real_part(cos_integral(-b*x - b*c/
d))*tan(1/2*a) - 16*b^2*c*d^2*x*tan(1/2*b*x)*tan(1/2*a) - 4*b*d^3*x*tan(1/2
*b*x)^2*tan(1/2*a) - 4*b^2*c*d^2*x*tan(1/2*a)^2 - 4*b*d^3*x*tan(1/2*b*x)*ta
n(1/2*a)^2 - 2*b^3*c^3*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d)
- 2*b^3*c^3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d) + 4*b^2*c*
d^2*x*tan(1/2*b*c/d)^2 + 4*b*d^3*x*tan(1/2*b*x)*tan(1/2*b*c/d)^2 + 4*b*d^3*
x*tan(1/2*a)*tan(1/2*b*c/d)^2 + 2*b^2*d^3*x^2 + b^3*c^3*imag_part(cos_integ
ral(b*x + b*c/d)) - b^3*c^3*imag_part(cos_integral(-b*x - b*c/d)) + 2*b^3*c
^3*sin_integral((b*d*x + b*c)/d) - 2*b^2*c^2*d*tan(1/2*b*x)^2 - 8*b^2*c^2*d
*tan(1/2*b*x)*tan(1/2*a) - 4*b*c*d^2*tan(1/2*b*x)^2*tan(1/2*a) - 2*b^2*c^2*
d*tan(1/2*a)^2 - 4*b*c*d^2*tan(1/2*b*x)*tan(1/2*a)^2 - 4*d^3*tan(1/2*b*x)^2
*tan(1/2*a)^2 + 2*b^2*c^2*d*tan(1/2*b*c/d)^2 + 4*b*c*d^2*tan(1/2*b*x)*tan(1
/2*b*c/d)^2 + 4*d^3*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + 4*b*c*d^2*tan(1/2*a)*
tan(1/2*b*c/d)^2 + 16*d^3*tan(1/2*b*x)*tan(1/2*a)*tan(1/2*b*c/d)^2 + 4*d^3*
tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*b^2*c*d^2*x + 4*b*d^3*x*tan(1/2*b*x) + 4*
b*d^3*x*tan(1/2*a) + 2*b^2*c^2*d + 4*b*c*d^2*tan(1/2*b*x) + 4*d^3*tan(1/2*b
*x)^2 + 4*b*c*d^2*tan(1/2*a) + 16*d^3*tan(1/2*b*x)*tan(1/2*a) + 4*d^3*tan(1
/2*a)^2 - 4*d^3*tan(1/2*b*c/d)^2 - 4*d^3)/(d^7*x^3*tan(1/2*b*x)^2*tan(1/2*a
)^2*tan(1/2*b*c/d)^2 + 3*c*d^6*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/
d)^2 + d^7*x^3*tan(1/2*b*x)^2*tan(1/2*a)^2 + d^7*x^3*tan(1/2*b*x)^2*tan(1/2
*b*c/d)^2 + d^7*x^3*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*c^2*d^5*x*tan(1/2*b*x
)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*c*d^6*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2
 + 3*c*d^6*x^2*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + 3*c*d^6*x^2*tan(1/2*a)^2*t
an(1/2*b*c/d)^2 + c^3*d^4*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d^
7*x^3*tan(1/2*b*x)^2 + d^7*x^3*tan(1/2*a)^2 + 3*c^2*d^5*x*tan(1/2*b*x)^2*ta
n(1/2*a)^2 + d^7*x^3*tan(1/2*b*c/d)^2 + 3*c^2*d^5*x*tan(1/2*b*x)^2*tan(1/2*
b*c/d)^2 + 3*c^2*d^5*x*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*c*d^6*x^2*tan(1/2*
b*x)^2 + 3*c*d^6*x^2*tan(1/2*a)^2 + c^3*d^4*tan(1/2*b*x)^2*tan(1/2*a)^2 + 3
*c*d^6*x^2*tan(1/2*b*c/d)^2 + c^3*d^4*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + c^3
*d^4*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d^7*x^3 + 3*c^2*d^5*x*tan(1/2*b*x)^2 +
 3*c^2*d^5*x*tan(1/2*a)^2 + 3*c^2*d^5*x*tan(1/2*b*c/d)^2 + 3*c*d^6*x^2 + c^
3*d^4*tan(1/2*b*x)^2 + c^3*d^4*tan(1/2*a)^2 + c^3*d^4*tan(1/2*b*c/d)^2 + 3*
c^2*d^5*x + c^3*d^4)
```

3.9 $\int (c+dx)^4 \cos^2(a+bx)\,dx$

Optimal. Leaf size=161

$$-\frac{3d^3(c+dx)\cos^2(a+bx)}{2b^4}-\frac{3d^2(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b^3}+\frac{d(c+dx)^3\cos^2(a+bx)}{b^2}+\frac{3d^4\sin(a+bx)\cos(a+}{4b^5}$$

```
[Out] (3*d^4*x)/(4*b^4) - (d*(c + d*x)^3)/(2*b^2) + (c + d*x)^5/(10*d) - (3*d^3*(
c + d*x)*Cos[a + b*x]^2)/(2*b^4) + (d*(c + d*x)^3*Cos[a + b*x]^2)/b^2 + (3*
d^4*Cos[a + b*x]*Sin[a + b*x])/(4*b^5) - (3*d^2*(c + d*x)^2*Cos[a + b*x]*Si
n[a + b*x])/(2*b^3) + ((c + d*x)^4*Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rubi [A] time = 0.101873, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3311, 32, 2635, 8}

$$-\frac{3d^3(c+dx)\cos^2(a+bx)}{2b^4}-\frac{3d^2(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b^3}+\frac{d(c+dx)^3\cos^2(a+bx)}{b^2}+\frac{3d^4\sin(a+bx)\cos(a+}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^2,x]
```

```
[Out] (3*d^4*x)/(4*b^4) - (d*(c + d*x)^3)/(2*b^2) + (c + d*x)^5/(10*d) - (3*d^3*(
c + d*x)*Cos[a + b*x]^2)/(2*b^4) + (d*(c + d*x)^3*Cos[a + b*x]^2)/b^2 + (3*
d^4*Cos[a + b*x]*Sin[a + b*x])/(4*b^5) - (3*d^2*(c + d*x)^2*Cos[a + b*x]*Si
n[a + b*x])/(2*b^3) + ((c + d*x)^4*Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (c+dx)^4 \cos^2(a+bx)\,dx = \frac{d(c+dx)^3\cos^2(a+bx)}{b^2} + \frac{(c+dx)^4\cos(a+bx)\sin(a+bx)}{2b} + \frac{1}{2}\int (c+dx)^4\,dx - \frac{\left(3d^2\right)\int (}{}$$
$$= \frac{(c+dx)^5}{10d} - \frac{3d^3(c+dx)\cos^2(a+bx)}{2b^4} + \frac{d(c+dx)^3\cos^2(a+bx)}{b^2} - \frac{3d^2(c+dx)^2\cos(a+bx)}{2b^3}$$
$$= -\frac{d(c+dx)^3}{2b^2} + \frac{(c+dx)^5}{10d} - \frac{3d^3(c+dx)\cos^2(a+bx)}{2b^4} + \frac{d(c+dx)^3\cos^2(a+bx)}{b^2} + \frac{3d^4\cos(}{}$$
$$= \frac{3d^4x}{4b^4} - \frac{d(c+dx)^3}{2b^2} + \frac{(c+dx)^5}{10d} - \frac{3d^3(c+dx)\cos^2(a+bx)}{2b^4} + \frac{d(c+dx)^3\cos^2(a+bx)}{b^2} + \frac{3d}{}$$

Mathematica [A] time = 0.657682, size = 132, normalized size = 0.82

$$\frac{10\sin(2(a+bx))\left(-6b^2d^2(c+dx)^2+2b^4(c+dx)^4+3d^4\right)+20bd(c+dx)\cos(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)+8b^5x\left(10c\right.}{80b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^2,x]
```

```
[Out] (8*b^5*x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4) + 20
*b*d*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 10*(3*d^4 -
6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Sin[2*(a + b*x)])/(80*b^5)
```

Maple [B] time = 0.047, size = 1027, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^4*d^4*((b*x+a)^4*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+(b*x+a)
^3*cos(b*x+a)^2-3*(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/2*(
b*x+a)*cos(b*x+a)^2+3/4*cos(b*x+a)*sin(b*x+a)+3/4*b*x+3/4*a+(b*x+a)^3-2/5*(
b*x+a)^5)-4/b^4*a*d^4*((b*x+a)^3*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+
3/4*(b*x+a)^2*cos(b*x+a)^2-3/2*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1
/2*a)+3/8*(b*x+a)^2+3/8*sin(b*x+a)^2-3/8*(b*x+a)^4)+4/b^3*c*d^3*((b*x+a)^3*
(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)^2*cos(b*x+a)^2-3/2*(b
*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/8*(b*x+a)^2+3/8*sin(b*x+a
)^2-3/8*(b*x+a)^4)+6/b^4*a^2*d^4*((b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*
b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a
-1/3*(b*x+a)^3)-12/b^3*a*c*d^3*((b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*
x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1
/3*(b*x+a)^3)+6/b^2*c^2*d^2*((b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1
/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*
(b*x+a)^3)-4/b^4*a^3*d^4*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)
-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+12/b^3*a^2*c*d^3*((b*x+a)*(1/2*cos(b*x+a)*
sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)-12/b^2*a*c^2*d^2*
((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*
x+a)^2)+4/b*c^3*d*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b
*x+a)^2-1/4*sin(b*x+a)^2)+1/b^4*a^4*d^4*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+
1/2*a)-4/b^3*a^3*c*d^3*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+6/b^2*a^2*
c^2*d^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-4/b*a*c^3*d*(1/2*cos(b*x+
```

```
a)*sin(b*x+a)+1/2*b*x+1/2*a)+c^4*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))
```

Maxima [B] time = 1.18895, size = 968, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/40*(10*(2*b*x + 2*a + sin(2*b*x + 2*a))*c^4 - 40*(2*b*x + 2*a + sin(2*b*x
 + 2*a))*a*c^3*d/b + 60*(2*b*x + 2*a + sin(2*b*x + 2*a))*a^2*c^2*d^2/b^2 -
40*(2*b*x + 2*a + sin(2*b*x + 2*a))*a^3*c*d^3/b^3 + 10*(2*b*x + 2*a + sin(2
*b*x + 2*a))*a^4*d^4/b^4 + 20*(2*(b*x + a)^2 + 2*(b*x + a)*sin(2*b*x + 2*a)
 + cos(2*b*x + 2*a))*c^3*d/b - 60*(2*(b*x + a)^2 + 2*(b*x + a)*sin(2*b*x +
2*a) + cos(2*b*x + 2*a))*a*c^2*d^2/b^2 + 60*(2*(b*x + a)^2 + 2*(b*x + a)*si
n(2*b*x + 2*a) + cos(2*b*x + 2*a))*a^2*c*d^3/b^3 - 20*(2*(b*x + a)^2 + 2*(b
*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*a^3*d^4/b^4 + 10*(4*(b*x + a)^
3 + 6*(b*x + a)*cos(2*b*x + 2*a) + 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*
c^2*d^2/b^2 - 20*(4*(b*x + a)^3 + 6*(b*x + a)*cos(2*b*x + 2*a) + 3*(2*(b*x
+ a)^2 - 1)*sin(2*b*x + 2*a))*a*c*d^3/b^3 + 10*(4*(b*x + a)^3 + 6*(b*x + a)
*cos(2*b*x + 2*a) + 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a^2*d^4/b^4 + 1
0*(2*(b*x + a)^4 + 3*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) + 2*(2*(b*x + a)^
3 - 3*b*x - 3*a)*sin(2*b*x + 2*a))*c*d^3/b^3 - 10*(2*(b*x + a)^4 + 3*(2*(b*
x + a)^2 - 1)*cos(2*b*x + 2*a) + 2*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b*x
+ 2*a))*a*d^4/b^4 + (4*(b*x + a)^5 + 10*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2
*b*x + 2*a) + 5*(2*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*sin(2*b*x + 2*a))*d^4/b
^4)/b
```

Fricas [A] time = 1.16394, size = 593, normalized size = 3.68

$$\frac{2\,b^5d^4x^5 + 10\,b^5cd^3x^4 + 10\left(2\,b^5c^2d^2 - b^3d^4\right)x^3 + 10\left(2\,b^5c^3d - 3\,b^3cd^3\right)x^2 + 10\left(2\,b^3d^4x^3 + 6\,b^3cd^3x^2 + 2\,b^3c^3d - 3\,bcd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/20*(2*b^5*d^4*x^5 + 10*b^5*c*d^3*x^4 + 10*(2*b^5*c^2*d^2 - b^3*d^4)*x^3 +
 10*(2*b^5*c^3*d - 3*b^3*c*d^3)*x^2 + 10*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 +
 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^2 + 5*
(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2
*b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)
*sin(b*x + a) + 5*(2*b^5*c^4 - 6*b^3*c^2*d^2 + 3*b*d^4)*x)/b^5
```

Sympy [A] time = 5.98816, size = 660, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**2,x)
```

```
[Out] Piecewise((c**4*x*sin(a + b*x)**2/2 + c**4*x*cos(a + b*x)**2/2 + c**3*d*x**
2*sin(a + b*x)**2 + c**3*d*x**2*cos(a + b*x)**2 + c**2*d**2*x**3*sin(a + b*
x)**2 + c**2*d**2*x**3*cos(a + b*x)**2 + c*d**3*x**4*sin(a + b*x)**2/2 + c*
d**3*x**4*cos(a + b*x)**2/2 + d**4*x**5*sin(a + b*x)**2/10 + d**4*x**5*cos(
a + b*x)**2/10 + c**4*sin(a + b*x)*cos(a + b*x)/(2*b) + 2*c**3*d*x*sin(a +
b*x)*cos(a + b*x)/b + 3*c**2*d**2*x**2*sin(a + b*x)*cos(a + b*x)/b + 2*c*d*
*3*x**3*sin(a + b*x)*cos(a + b*x)/b + d**4*x**4*sin(a + b*x)*cos(a + b*x)/(
2*b) + c**3*d*cos(a + b*x)**2/b**2 - 3*c**2*d**2*x*sin(a + b*x)**2/(2*b**2)
 + 3*c**2*d**2*x*cos(a + b*x)**2/(2*b**2) - 3*c*d**3*x**2*sin(a + b*x)**2/(
2*b**2) + 3*c*d**3*x**2*cos(a + b*x)**2/(2*b**2) - d**4*x**3*sin(a + b*x)**
2/(2*b**2) + d**4*x**3*cos(a + b*x)**2/(2*b**2) - 3*c**2*d**2*sin(a + b*x)*
cos(a + b*x)/(2*b**3) - 3*c*d**3*x*sin(a + b*x)*cos(a + b*x)/b**3 - 3*d**4*
x**2*sin(a + b*x)*cos(a + b*x)/(2*b**3) - 3*c*d**3*cos(a + b*x)**2/(2*b**4)
 + 3*d**4*x*sin(a + b*x)**2/(4*b**4) - 3*d**4*x*cos(a + b*x)**2/(4*b**4) +
3*d**4*sin(a + b*x)*cos(a + b*x)/(4*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x
**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*cos(a)**2, True))
```

Giac [A] time = 1.10143, size = 300, normalized size = 1.86

$$\frac{1}{10}\,d^4x^5 + \frac{1}{2}\,cd^3x^4 + c^2d^2x^3 + c^3dx^2 + \frac{1}{2}\,c^4x + \frac{\left(2\,b^3d^4x^3 + 6\,b^3cd^3x^2 + 6\,b^3c^2d^2x + 2\,b^3c^3d - 3\,bd^4x - 3\,bcd^3\right)\cos\left(2\,bx + \right.}{4\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/10*d^4*x^5 + 1/2*c*d^3*x^4 + c^2*d^2*x^3 + c^3*d*x^2 + 1/2*c^4*x + 1/4*(2
*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x + 2*b^3*c^3*d - 3*b*d^4*x
- 3*b*c*d^3)*cos(2*b*x + 2*a)/b^5 + 1/8*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 +
12*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d
^3*x - 6*b^2*c^2*d^2 + 3*d^4)*sin(2*b*x + 2*a)/b^5
```

3.10 $\int (c+dx)^3 \cos^2(a+bx)\,dx$

Optimal. Leaf size=134

$$-\frac{3d^2(c+dx)\sin(a+bx)\cos(a+bx)}{4b^3}+\frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2}-\frac{3d^3\cos^2(a+bx)}{8b^4}+\frac{(c+dx)^3\sin(a+bx)\cos(a+b}{2b}$$

```
[Out] (-3*c*d^2*x)/(4*b^2) - (3*d^3*x^2)/(8*b^2) + (c + d*x)^4/(8*d) - (3*d^3*Cos
[a + b*x]^2)/(8*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]^2)/(4*b^2) - (3*d^2*(c
 + d*x)*Cos[a + b*x]*Sin[a + b*x])/(4*b^3) + ((c + d*x)^3*Cos[a + b*x]*Sin[
a + b*x])/(2*b)
```

Rubi [A] time = 0.073885, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3311, 32, 3310}

$$-\frac{3d^2(c+dx)\sin(a+bx)\cos(a+bx)}{4b^3}+\frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2}-\frac{3d^3\cos^2(a+bx)}{8b^4}+\frac{(c+dx)^3\sin(a+bx)\cos(a+b}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^2,x]
```

```
[Out] (-3*c*d^2*x)/(4*b^2) - (3*d^3*x^2)/(8*b^2) + (c + d*x)^4/(8*d) - (3*d^3*Cos
[a + b*x]^2)/(8*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]^2)/(4*b^2) - (3*d^2*(c
 + d*x)*Cos[a + b*x]*Sin[a + b*x])/(4*b^3) + ((c + d*x)^3*Cos[a + b*x]*Sin[
a + b*x])/(2*b)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)^3\cos^2(a+bx)\,dx = \frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2} + \frac{(c+dx)^3\cos(a+bx)\sin(a+bx)}{2b} + \frac{1}{2}\int (c+dx)^3\,dx - \frac{(3d^2)\int$$

$$= \frac{(c+dx)^4}{8d} - \frac{3d^3\cos^2(a+bx)}{8b^4} + \frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2} - \frac{3d^2(c+dx)\cos(a+bx)\sin(a+}{4b^3}$$

$$= -\frac{3cd^2x}{4b^2} - \frac{3d^3x^2}{8b^2} + \frac{(c+dx)^4}{8d} - \frac{3d^3\cos^2(a+bx)}{8b^4} + \frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2} - \frac{3d^2(c+dx)}{}$$

Mathematica [A] time = 0.450567, size = 106, normalized size = 0.79

$$\frac{2b(c+dx)\sin(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)+3d\cos(2(a+bx))\left(2b^2(c+dx)^2-d^2\right)+2b^4x\left(6c^2dx+4c^3+4cd^2x^2+d^3\right.}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^2,x]
```

```
[Out] (2*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 3*d*(-d^2 + 2*b^2*(c
 + d*x)^2)*Cos[2*(a + b*x)] + 2*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Si
n[2*(a + b*x)])/(16*b^4)
```

Maple [B] time = 0.027, size = 587, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^3*d^3*((b*x+a)^3*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/4*(b*
x+a)^2*cos(b*x+a)^2-3/2*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3
/8*(b*x+a)^2+3/8*sin(b*x+a)^2-3/8*(b*x+a)^4)-3/b^3*a*d^3*((b*x+a)^2*(1/2*co
s(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*
sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3)+3/b^2*c*d^2*((b*x+a)^2*(1/2*cos(b*x
+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b
*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3)+3/b^3*a^2*d^3*((b*x+a)*(1/2*cos(b*x+a)*s
in(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)-6/b^2*a*c*d^2*((b*
x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)
^2)+3/b*c^2*d*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a
)^2-1/4*sin(b*x+a)^2)-1/b^3*a^3*d^3*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*
a)+3/b^2*a^2*c*d^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/b*a*c^2*d*(1
/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+c^3*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*
b*x+1/2*a))
```

Maxima [B] time = 1.11375, size = 578, normalized size = 4.31

$$\frac{4\,(2\,bx+2\,a+\sin(2\,bx+2\,a))c^3 - \frac{12\,(2\,bx+2\,a+\sin(2\,bx+2\,a))ac^2d}{b} + \frac{12\,(2\,bx+2\,a+\sin(2\,bx+2\,a))a^2cd^2}{b^2} - \frac{4\,(2\,bx+2\,a+\sin(2\,bx+2\,a))a^3d^3}{b^3} + \frac{6\,(2}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*(2*b*x + 2*a + sin(2*b*x + 2*a))*c^3 - 12*(2*b*x + 2*a + sin(2*b*x
+ 2*a))*a*c^2*d/b + 12*(2*b*x + 2*a + sin(2*b*x + 2*a))*a^2*c*d^2/b^2 - 4*(
2*b*x + 2*a + sin(2*b*x + 2*a))*a^3*d^3/b^3 + 6*(2*(b*x + a)^2 + 2*(b*x + a
)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*c^2*d/b - 12*(2*(b*x + a)^2 + 2*(b*x
 + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*a*c*d^2/b^2 + 6*(2*(b*x + a)^2 +
 2*(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*a^2*d^3/b^3 + 2*(4*(b*x +
 a)^3 + 6*(b*x + a)*cos(2*b*x + 2*a) + 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*
a))*c*d^2/b^2 - 2*(4*(b*x + a)^3 + 6*(b*x + a)*cos(2*b*x + 2*a) + 3*(2*(b*x
 + a)^2 - 1)*sin(2*b*x + 2*a))*a*d^3/b^3 + (2*(b*x + a)^4 + 3*(2*(b*x + a)^
2 - 1)*cos(2*b*x + 2*a) + 2*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b*x + 2*a))
*d^3/b^3)/b
```

Fricas [A] time = 1.15807, size = 394, normalized size = 2.94

$$\frac{b^4d^3x^4 + 4\,b^4cd^2x^3 + 3\left(2\,b^4c^2d - b^2d^3\right)x^2 + 3\left(2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d - d^3\right)\cos\left(bx+a\right)^2 + 2\left(2\,b^3d^3x^3 + 6\,b^3c\cdots\right.}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 3*(2*b^4*c^2*d - b^2*d^3)*x^2 + 3*(2*b
^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 + 2*(2*b^3*d
^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*
x)*cos(b*x + a)*sin(b*x + a) + 2*(2*b^4*c^3 - 3*b^2*c*d^2)*x)/b^4
```

Sympy [A] time = 3.2626, size = 456, normalized size = 3.4

$$\begin{cases} \frac{c^3x\sin^2(a+bx)}{2} + \frac{c^3x\cos^2(a+bx)}{2} + \frac{3c^2dx^2\sin^2(a+bx)}{4} + \frac{3c^2dx^2\cos^2(a+bx)}{4} + \frac{cd^2x^3\sin^2(a+bx)}{2} + \frac{cd^2x^3\cos^2(a+bx)}{2} + \frac{d^3x^4\sin^2(a+bx)}{8} + \frac{d^3x\cdots}{} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**2,x)
```

```
[Out] Piecewise((c**3*x*sin(a + b*x)**2/2 + c**3*x*cos(a + b*x)**2/2 + 3*c**2*d*x
**2*sin(a + b*x)**2/4 + 3*c**2*d*x**2*cos(a + b*x)**2/4 + c*d**2*x**3*sin(a
 + b*x)**2/2 + c*d**2*x**3*cos(a + b*x)**2/2 + d**3*x**4*sin(a + b*x)**2/8
+ d**3*x**4*cos(a + b*x)**2/8 + c**3*sin(a + b*x)*cos(a + b*x)/(2*b) + 3*c*
*2*d*x*sin(a + b*x)*cos(a + b*x)/(2*b) + 3*c*d**2*x**2*sin(a + b*x)*cos(a +
 b*x)/(2*b) + d**3*x**3*sin(a + b*x)*cos(a + b*x)/(2*b) + 3*c**2*d*cos(a +
b*x)**2/(4*b**2) - 3*c*d**2*x*sin(a + b*x)**2/(4*b**2) + 3*c*d**2*x*cos(a +
 b*x)**2/(4*b**2) - 3*d**3*x**2*sin(a + b*x)**2/(8*b**2) + 3*d**3*x**2*cos(
a + b*x)**2/(8*b**2) - 3*c*d**2*sin(a + b*x)*cos(a + b*x)/(4*b**3) - 3*d**3
*x*sin(a + b*x)*cos(a + b*x)/(4*b**3) - 3*d**3*cos(a + b*x)**2/(8*b**4), Ne
(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*cos(a)**2,
 True))
```

Giac [A] time = 1.1253, size = 207, normalized size = 1.54

$$\frac{1}{8}d^3x^4 + \frac{1}{2}cd^2x^3 + \frac{3}{4}c^2dx^2 + \frac{1}{2}c^3x + \frac{3\left(2b^2d^3x^2 + 4b^2cd^2x + 2b^2c^2d - d^3\right)\cos\left(2bx + 2a\right)}{16b^4} + \frac{\left(2b^3d^3x^3 + 6b^3cd^2x^2 + 6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*d^3*x^4 + 1/2*c*d^2*x^3 + 3/4*c^2*d*x^2 + 1/2*c^3*x + 3/16*(2*b^2*d^3*x
^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(2*b*x + 2*a)/b^4 + 1/8*(2*b^3*d
^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^
2)*sin(2*b*x + 2*a)/b^4
```

3.11 $\int (c+dx)^2 \cos^2(a+bx)\,dx$

Optimal. Leaf size=95

$$\frac{d(c+dx)\cos^2(a+bx)}{2b^2}-\frac{d^2\sin(a+bx)\cos(a+bx)}{4b^3}+\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}-\frac{d^2x}{4b^2}+\frac{(c+dx)^3}{6d}$$

```
[Out] -(d^2*x)/(4*b^2) + (c + d*x)^3/(6*d) + (d*(c + d*x)*Cos[a + b*x]^2)/(2*b^2)
 - (d^2*Cos[a + b*x]*Sin[a + b*x])/(4*b^3) + ((c + d*x)^2*Cos[a + b*x]*Sin[
a + b*x])/(2*b)
```

Rubi [A] time = 0.0527312, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3311, 32, 2635, 8}

$$\frac{d(c+dx)\cos^2(a+bx)}{2b^2}-\frac{d^2\sin(a+bx)\cos(a+bx)}{4b^3}+\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}-\frac{d^2x}{4b^2}+\frac{(c+dx)^3}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^2,x]
```


```
[Out] -(d^2*x)/(4*b^2) + (c + d*x)^3/(6*d) + (d*(c + d*x)*Cos[a + b*x]^2)/(2*b^2)
 - (d^2*Cos[a + b*x]*Sin[a + b*x])/(4*b^3) + ((c + d*x)^2*Cos[a + b*x]*Sin[
a + b*x])/(2*b)
```


Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (c+dx)^2\cos^2(a+bx)\,dx = \frac{d(c+dx)\cos^2(a+bx)}{2b^2} + \frac{(c+dx)^2\cos(a+bx)\sin(a+bx)}{2b} + \frac{1}{2}\int (c+dx)^2\,dx - \frac{d^2\int \cos^2(}{2}$$
$$= \frac{(c+dx)^3}{6d} + \frac{d(c+dx)\cos^2(a+bx)}{2b^2} - \frac{d^2\cos(a+bx)\sin(a+bx)}{4b^3} + \frac{(c+dx)^2\cos(a+bx)\sin}{2b}$$
$$= -\frac{d^2x}{4b^2} + \frac{(c+dx)^3}{6d} + \frac{d(c+dx)\cos^2(a+bx)}{2b^2} - \frac{d^2\cos(a+bx)\sin(a+bx)}{4b^3} + \frac{(c+dx)^2\cos(a}{}$$

Mathematica [A] time = 0.312399, size = 77, normalized size = 0.81

$$\frac{3\sin(2(a+bx))\left(2b^2(c+dx)^2-d^2\right)+6bd(c+dx)\cos(2(a+bx))+4b^3x\left(3c^2+3cdx+d^2x^2\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^2,x]
```

```
[Out] (4*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) + 6*b*d*(c + d*x)*Cos[2*(a + b*x)] + 3
*(-d^2 + 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(24*b^3)
```

Maple [B] time = 0.026, size = 289, normalized size = 3.

$$\frac{1}{b}\left(\frac{d^2}{b^2}\left((bx+a)^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)+\frac{(bx+a)(\cos(bx+a))^2}{2}-\frac{\cos(bx+a)\sin(bx+a)}{4}-\frac{bx}{4}-\frac{a}{4}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^2*d^2*((b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*
x+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3)-2/
b^2*a*d^2*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-
1/4*sin(b*x+a)^2)+2/b*c*d*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a
)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+1/b^2*a^2*d^2*(1/2*cos(b*x+a)*sin(b*x+a)+
1/2*b*x+1/2*a)-2/b*a*c*d*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+c^2*(1/2
*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))
```

Maxima [B] time = 1.06933, size = 300, normalized size = 3.16

$$\frac{6\,(2\,bx+2\,a+\sin(2\,bx+2\,a))c^2-\frac{12\,(2\,bx+2\,a+\sin(2\,bx+2\,a))acd}{b}+\frac{6\,(2\,bx+2\,a+\sin(2\,bx+2\,a))a^2d^2}{b^2}+\frac{6\left(2\,(bx+a)^2+2\,(bx+a)\sin(2\,bx+2\,a)+\cos(2\right.}{b}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(6*(2*b*x + 2*a + sin(2*b*x + 2*a))*c^2 - 12*(2*b*x + 2*a + sin(2*b*x
+ 2*a))*a*c*d/b + 6*(2*b*x + 2*a + sin(2*b*x + 2*a))*a^2*d^2/b^2 + 6*(2*(b*
x + a)^2 + 2*(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*c*d/b - 6*(2*(b
*x + a)^2 + 2*(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*a*d^2/b^2 + (4
*(b*x + a)^3 + 6*(b*x + a)*cos(2*b*x + 2*a) + 3*(2*(b*x + a)^2 - 1)*sin(2*b
```

```
*x + 2*a))*d^2/b^2)/b
```

Fricas [A] time = 1.09403, size = 247, normalized size = 2.6

$$\frac{2\,b^3d^2x^3 + 6\,b^3cdx^2 + 6\left(bd^2x + bcd\right)\cos\left(bx + a\right)^2 + 3\left(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 - d^2\right)\cos\left(bx + a\right)\sin\left(bx + a\right) + 3}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*d^2*x^3 + 6*b^3*c*d*x^2 + 6*(b*d^2*x + b*c*d)*cos(b*x + a)^2 +
3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(b*x + a)*sin(b*x + a)
 + 3*(2*b^3*c^2 - b*d^2)*x)/b^3
```

Sympy [A] time = 1.42399, size = 264, normalized size = 2.78

$$\begin{cases} \frac{c^2x\sin^2\left(a+bx\right)}{2} + \frac{c^2x\cos^2\left(a+bx\right)}{2} + \frac{cdx^2\sin^2\left(a+bx\right)}{2} + \frac{cdx^2\cos^2\left(a+bx\right)}{2} + \frac{d^2x^3\sin^2\left(a+bx\right)}{6} + \frac{d^2x^3\cos^2\left(a+bx\right)}{6} + \frac{c^2\sin\left(a+bx\right)\cos\left(a+bx\right)}{2b} + cd \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\cos^2\left(a\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**2,x)
```

```
[Out] Piecewise((c**2*x*sin(a + b*x)**2/2 + c**2*x*cos(a + b*x)**2/2 + c*d*x**2*s
in(a + b*x)**2/2 + c*d*x**2*cos(a + b*x)**2/2 + d**2*x**3*sin(a + b*x)**2/6
 + d**2*x**3*cos(a + b*x)**2/6 + c**2*sin(a + b*x)*cos(a + b*x)/(2*b) + c*d
*x*sin(a + b*x)*cos(a + b*x)/b + d**2*x**2*sin(a + b*x)*cos(a + b*x)/(2*b)
- c*d*sin(a + b*x)**2/(2*b**2) - d**2*x*sin(a + b*x)**2/(4*b**2) + d**2*x*c
os(a + b*x)**2/(4*b**2) - d**2*sin(a + b*x)*cos(a + b*x)/(4*b**3), Ne(b, 0)
), ((c**2*x + c*d*x**2 + d**2*x**3/3)*cos(a)**2, True))
```

Giac [A] time = 1.1218, size = 127, normalized size = 1.34

$$\frac{1}{6}\,d^2x^3 + \frac{1}{2}\,cdx^2 + \frac{1}{2}\,c^2x + \frac{\left(bd^2x + bcd\right)\cos\left(2\,bx + 2\,a\right)}{4\,b^3} + \frac{\left(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 - d^2\right)\sin\left(2\,bx + 2\,a\right)}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*d^2*x^3 + 1/2*c*d*x^2 + 1/2*c^2*x + 1/4*(b*d^2*x + b*c*d)*cos(2*b*x + 2
*a)/b^3 + 1/8*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*sin(2*b*x + 2
*a)/b^3
```

3.12 $\int (c+dx)\cos^2(a+bx)\,dx$

Optimal. Leaf size=55

$$\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{cx}{2}+\frac{dx^2}{4}$$

```
[Out] (c*x)/2 + (d*x^2)/4 + (d*Cos[a + b*x]^2)/(4*b^2) + ((c + d*x)*Cos[a + b*x]*
Sin[a + b*x])/(2*b)
```

Rubi [A] time = 0.0246872, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3310}

$$\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{cx}{2}+\frac{dx^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^2,x]
```

```
[Out] (c*x)/2 + (d*x^2)/4 + (d*Cos[a + b*x]^2)/(4*b^2) + ((c + d*x)*Cos[a + b*x]*
Sin[a + b*x])/(2*b)
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cos^2(a+bx)\,dx &= \frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\cos(a+bx)\sin(a+bx)}{2b}+\frac{1}{2}\int (c+dx)\,dx\\ &= \frac{cx}{2}+\frac{dx^2}{4}+\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\cos(a+bx)\sin(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.160211, size = 50, normalized size = 0.91

$$\frac{2b((c+dx)\sin(2(a+bx))+2ac+bx(2c+dx))+d\cos(2(a+bx))}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^2,x]
```

```
[Out] (d*Cos[2*(a + b*x)] + 2*b*(2*a*c + b*x*(2*c + d*x) + (c + d*x)*Sin[2*(a + b
*x)]))/(8*b^2)
```

Maple [B] time = 0.028, size = 112, normalized size = 2.

$$\frac{1}{b}\left(\frac{d}{b}\left((bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)^2}{4}-\frac{(\sin(bx+a))^2}{4}\right)-\frac{da}{b}\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b*d*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2
-1/4*sin(b*x+a)^2)-a*d/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+c*(1/2*c
os(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))
```

Maxima [A] time = 1.05426, size = 122, normalized size = 2.22

$$\frac{2\,(2\,bx+2\,a+\sin(2\,bx+2\,a))c-\frac{2\,(2\,bx+2\,a+\sin(2\,bx+2\,a))ad}{b}+\frac{\left(2\,(bx+a)^2+2\,(bx+a)\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a)\right)d}{b}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*(2*b*x + 2*a + sin(2*b*x + 2*a))*c - 2*(2*b*x + 2*a + sin(2*b*x + 2*
a))*a*d/b + (2*(b*x + a)^2 + 2*(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a
))*d/b)/b
```

Fricas [A] time = 1.02594, size = 130, normalized size = 2.36

$$\frac{b^2dx^2+2\,b^2cx+d\cos(bx+a)^2+2\,(bdx+bc)\cos(bx+a)\sin(bx+a)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d*x^2 + 2*b^2*c*x + d*cos(b*x + a)^2 + 2*(b*d*x + b*c)*cos(b*x + a
)*sin(b*x + a))/b^2
```

Sympy [A] time = 0.61916, size = 126, normalized size = 2.29

$$\begin{cases}\frac{cx\sin^2(a+bx)}{2}+\frac{cx\cos^2(a+bx)}{2}+\frac{dx^2\sin^2(a+bx)}{4}+\frac{dx^2\cos^2(a+bx)}{4}+\frac{c\sin(a+bx)\cos(a+bx)}{2b}+\frac{dx\sin(a+bx)\cos(a+bx)}{2b}-\frac{d\sin^2(a+bx)}{4b^2} & \text{for} \\ \left(cx+\frac{dx^2}{2}\right)\cos^2(a) & \text{oth}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**2,x)
```

```
[Out] Piecewise((c*x*sin(a + b*x)**2/2 + c*x*cos(a + b*x)**2/2 + d*x**2*sin(a + b
*x)**2/4 + d*x**2*cos(a + b*x)**2/4 + c*sin(a + b*x)*cos(a + b*x)/(2*b) + d
*x*sin(a + b*x)*cos(a + b*x)/(2*b) - d*sin(a + b*x)**2/(4*b**2), Ne(b, 0)),
```

```
((c*x + d*x**2/2)*cos(a)**2, True))
```

Giac [A] time = 1.10369, size = 65, normalized size = 1.18

$$\frac{1}{4}dx^2 + \frac{1}{2}cx + \frac{d\cos(2bx + 2a)}{8b^2} + \frac{(bdx + bc)\sin(2bx + 2a)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*d*x^2 + 1/2*c*x + 1/8*d*cos(2*b*x + 2*a)/b^2 + 1/4*(b*d*x + b*c)*sin(2*
b*x + 2*a)/b^2
```

3.13 $\int \frac{\cos^2(a+bx)}{c+dx} dx$

Optimal. Leaf size=78

$$\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d}-\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}$$

```
[Out] (Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Log[c + d*x]/
(2*d) - (Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

Rubi [A] time = 0.153384, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3312, 3303, 3299, 3302}

$$\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d}-\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/(c + d*x),x]
```

```
[Out] (Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Log[c + d*x]/
(2*d) - (Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(a+bx)}{c+dx}\,dx &= \int \left(\frac{1}{2(c+dx)} + \frac{\cos(2a+2bx)}{2(c+dx)}\right) dx \\ &= \frac{\log(c+dx)}{2d} + \frac{1}{2}\int \frac{\cos(2a+2bx)}{c+dx}\,dx \\ &= \frac{\log(c+dx)}{2d} + \frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx - \frac{1}{2}\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx \\ &= \frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d} + \frac{\log(c+dx)}{2d} - \frac{\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}\end{aligned}$$

Mathematica [A] time = 0.130859, size = 65, normalized size = 0.83

$$\frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) - \sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2b(c+dx)}{d}\right) + \log(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/(c + d*x),x]
```

```
[Out] (Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + Log[c + d*x] - Sin[2
*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/(2*d)
```

Maple [A] time = 0.029, size = 105, normalized size = 1.4

$$\frac{1}{2d}\mathrm{Si}\left(2bx+2a+2\frac{-da+cb}{d}\right)\sin\left(2\frac{-da+cb}{d}\right) + \frac{1}{2d}\mathrm{Ci}\left(2bx+2a+2\frac{-da+cb}{d}\right)\cos\left(2\frac{-da+cb}{d}\right) + \frac{\ln((bx+a)d-d}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/(d*x+c),x)
```

```
[Out] 1/2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+1/2*Ci(2*b*x+2*a+2*(
-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d+1/2*ln((b*x+a)*d-d*a+c*b)/d
```

Maxima [C] time = 1.20739, size = 217, normalized size = 2.78

$$-\frac{b\left(E_1\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+E_1\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right) - b\left(i\,E_1\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right) - i\,E_1\left(-\frac{2i\,bc+2i\,(bx+a)d-}{d}\right.\right.}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/4*(b*(exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) + exp_i
ntegral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c - a*d)/
d) - b*(I*exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) - I*ex
p_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2*(b*c - a*
d)/d) - 2*b*log(b*c + (b*x + a)*d - a*d))/(b*d)
```

Fricas [A] time = 1.16209, size = 236, normalized size = 3.03

$$\frac{\left(\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-2\,\sin\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)+2\,\log\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*((cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d))*c
os(-2*(b*c - a*d)/d) - 2*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)
/d) + 2*log(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)**2/(c + d*x), x)
```

Giac [C] time = 1.2034, size = 824, normalized size = 10.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(2*log(abs(d*x + c))*tan(a)^2*tan(b*c/d)^2 + real_part(cos_integral(2*b
*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + real_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(a)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*ta
n(a)^2*tan(b*c/d) + 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*ta
n(b*c/d) - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d) + 2*imag_p
art(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*imag_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 4*sin_integral(2*(b*d*x + b
*c)/d)*tan(a)*tan(b*c/d)^2 + 2*log(abs(d*x + c))*tan(a)^2 - real_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(a)^2 - real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(a)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)
 + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 2*log(ab
s(d*x + c))*tan(b*c/d)^2 - real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c
/d)^2 - real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 2*imag_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 2*imag_part(cos_integral(-2*b*x -
 2*b*c/d))*tan(a) - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(a) + 2*imag_part(
cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*imag_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(b*c/d) + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) + 2*
log(abs(d*x + c)) + real_part(cos_integral(2*b*x + 2*b*c/d)) + real_part(co
s_integral(-2*b*x - 2*b*c/d)))/(d*tan(a)^2*tan(b*c/d)^2 + d*tan(a)^2 + d*ta
n(b*c/d)^2 + d)
```

3.14 $\int \frac{\cos^2(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=83

$$-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{\cos^2(a+bx)}{d(c+dx)}$$

```
[Out] -(Cos[a + b*x]^2/(d*(c + d*x))) - (b*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a
 - (2*b*c)/d])/d^2 - (b*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x]
)/d^2
```

Rubi [A] time = 0.134264, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3313, 12, 3303, 3299, 3302}

$$-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{\cos^2(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] -(Cos[a + b*x]^2/(d*(c + d*x))) - (b*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a
 - (2*b*c)/d])/d^2 - (b*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x]
)/d^2
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx &= -\frac{\cos^2(a+bx)}{d(c+dx)}+\frac{(2b)\int -\frac{\sin(2a+2bx)}{2(c+dx)}\,dx}{d}\\ &= -\frac{\cos^2(a+bx)}{d(c+dx)}-\frac{b\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d}\\ &= -\frac{\cos^2(a+bx)}{d(c+dx)}-\frac{\left(b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d}-\frac{\left(b\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d}\\ &= -\frac{\cos^2(a+bx)}{d(c+dx)}-\frac{b\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^2}-\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}\end{aligned}$$

Mathematica [A] time = 0.64013, size = 75, normalized size = 0.9

$$-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)+b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)+\frac{d\cos^2(a+bx)}{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] -(((d*Cos[a + b*x]^2)/(c + d*x) + b*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a
- (2*b*c)/d] + b*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d^2)
```

Maple [A] time = 0.03, size = 156, normalized size = 1.9

$$\frac{1}{b}\left(\frac{b^2}{4}\left(-2\frac{\cos(2bx+2a)}{((bx+a)d-da+cb)d}-2\frac{1}{d}\left(2\frac{1}{d}\text{Si}\left(2bx+2a+2\frac{-da+cb}{d}\right)\cos\left(2\frac{-da+cb}{d}\right)-2\frac{1}{d}\text{Ci}\left(2bx+2a+2\frac{-d}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] 1/b*(1/4*b^2*(-2*cos(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)/d-2*(2*Si(2*b*x+2*a+2*(
-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a
*d+b*c)/d)/d)/d)-1/2*b^2/((b*x+a)*d-d*a+c*b)/d)
```

Maxima [C] time = 1.31382, size = 231, normalized size = 2.78

$$-\frac{16b^2\left(E_2\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+E_2\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)-b^2\left(16i\,E_2\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-16i\,E_2\left(-\frac{2}{}\right.\right.}{64\left(bcd+(bx+a)d^2-ad^2\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/64*(16*b^2*(exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) +
 exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c -
 a*d)/d) - b^2*(16*I*exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d
)/d) - 16*I*exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*si
n(-2*(b*c - a*d)/d) + 32*b^2)/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Fricas [A] time = 1.15683, size = 319, normalized size = 3.84

$$-\frac{2\,d\cos\left(bx+a\right)^{2}+2\left(bdx+bc\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)+\left(\left(bdx+bc\right)\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+\left(bdx+bc\right)\mathrm{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\mathrm{s}}{2\left(d^{3}x+cd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*cos(b*x + a)^2 + 2*(b*d*x + b*c)*cos(-2*(b*c - a*d)/d)*sin_integr
al(2*(b*d*x + b*c)/d) + ((b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d) + (b
*d*x + b*c)*cos_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d))/(d^3*x
 + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^{2}\left(a+bx\right)}{\left(c+dx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)**2/(c + d*x)**2, x)
```

Giac [C] time = 1.34117, size = 3976, normalized size = 47.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*ta
n(b*c/d)^2 - b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan
(a)^2*tan(b*c/d)^2 + 2*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan
(a)^2*tan(b*c/d)^2 + 2*b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b
*x)^2*tan(a)^2*tan(b*c/d) + 2*b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d
))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b*d*x*real_part(cos_integral(2*b*x +
2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 2*b*d*x*real_part(cos_integral(-
2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b*c*imag_part(cos_integr
al(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b*c*imag_part(cos_i
ntegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b*c*sin_int
egral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b*d*x*imag_part
(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 + b*d*x*imag_part(cos_i
```

```
ntegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 2*b*d*x*sin_integral(2*(b*
d*x + b*c)/d)*tan(b*x)^2*tan(a)^2 + 4*b*d*x*imag_part(cos_integral(2*b*x +
2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 4*b*d*x*imag_part(cos_integral(-2*
b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 8*b*d*x*sin_integral(2*(b*d*
x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d) + 2*b*c*real_part(cos_integral(2*b
*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b*c*real_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - b*d*x*imag_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + b*d*x*imag_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 2*b*d*x*sin_integral(2*(
b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d)^2 - 2*b*c*real_part(cos_integral(2*b*
x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 2*b*c*real_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b*d*x*imag_part(cos_in
tegral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - b*d*x*imag_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b*d*x*sin_integral(2*(b*d*x
 + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + 2*b*d*x*real_part(cos_integral(2*b*x + 2
*b*c/d))*tan(b*x)^2*tan(a) + 2*b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(b*x)^2*tan(a) - b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(a)^2 + b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*ta
n(a)^2 - 2*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2 - 2*b*d*
x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) - 2*b*d*x*
real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d) + 4*b*c*ima
g_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 4*b*c*
imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 8*
b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d) + 2*b*d*x*
real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d) + 2*b*d*x*real
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d) - b*c*imag_part(c
os_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + b*c*imag_part(cos_i
ntegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 2*b*c*sin_integral(2*(
b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d)^2 - 2*b*d*x*real_part(cos_integral(2*
b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b*d*x*real_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + b*c*imag_part(cos_integral(2*b*x + 2*b*c
/d))*tan(a)^2*tan(b*c/d)^2 - b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*
tan(a)^2*tan(b*c/d)^2 + 2*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(
b*c/d)^2 + 2*d*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + b*d*x*imag_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(b*x)^2 - b*d*x*imag_part(cos_integral(-2*b*x - 2*
b*c/d))*tan(b*x)^2 + 2*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2 + 2
*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 2*b*c*rea
l_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - b*d*x*imag_part(
cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 + b*d*x*imag_part(cos_integral(-2*b
*x - 2*b*c/d))*tan(a)^2 - 2*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2
- 2*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) - 2*
b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d) + 4*b*d
*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) - 4*b*d*x*ima
g_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 8*b*d*x*sin_inte
gral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d) + 2*b*c*real_part(cos_integral(2*
b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d) + 2*b*c*real_part(cos_integral(-2*b*x -
 2*b*c/d))*tan(a)^2*tan(b*c/d) - b*d*x*imag_part(cos_integral(2*b*x + 2*b*c
/d))*tan(b*c/d)^2 + b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c
/d)^2 - 2*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 - 2*b*c*real_p
art(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b*c*real_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + b*c*imag_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(b*x)^2 - b*c*imag_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(b*x)^2 + 2*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2 + 2*b*d
*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 2*b*d*x*real_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(a) - b*c*imag_part(cos_integral(2*b*x + 2*b
*c/d))*tan(a)^2 + b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 -
2*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2 + 2*d*tan(b*x)^2*tan(a)^2 -
2*b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*b*d*x*real_
part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + 4*b*c*imag_part(cos_integ
```

```
ral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) - 4*b*c*imag_part(cos_integral(-2*b
*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 8*b*c*sin_integral(2*(b*d*x + b*c)/d)*ta
n(a)*tan(b*c/d) - b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2
 + b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 2*b*c*sin_i
ntegral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 - 4*d*tan(b*x)*tan(a)*tan(b*c/d)^2
+ b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d)) - b*d*x*imag_part(cos_inte
gral(-2*b*x - 2*b*c/d)) + 2*b*d*x*sin_integral(2*(b*d*x + b*c)/d) + 2*b*c*r
eal_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 2*b*c*real_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(a) - 2*b*c*real_part(cos_integral(2*b*x + 2*b*c/d
))*tan(b*c/d) - 2*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)
+ b*c*imag_part(cos_integral(2*b*x + 2*b*c/d)) - b*c*imag_part(cos_integral
(-2*b*x - 2*b*c/d)) + 2*b*c*sin_integral(2*(b*d*x + b*c)/d) - 4*d*tan(b*x)*
tan(a) + 2*d*tan(b*c/d)^2 + 2*d)/(d^3*x*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 +
c*d^2*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + d^3*x*tan(b*x)^2*tan(a)^2 + d^3*x*
tan(b*x)^2*tan(b*c/d)^2 + d^3*x*tan(a)^2*tan(b*c/d)^2 + c*d^2*tan(b*x)^2*ta
n(a)^2 + c*d^2*tan(b*x)^2*tan(b*c/d)^2 + c*d^2*tan(a)^2*tan(b*c/d)^2 + d^3*
x*tan(b*x)^2 + d^3*x*tan(a)^2 + d^3*x*tan(b*c/d)^2 + c*d^2*tan(b*x)^2 + c*d
^2*tan(a)^2 + c*d^2*tan(b*c/d)^2 + d^3*x + c*d^2)
```

3.15 $\int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=112

$$-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)}-\frac{\cos^2(a+}{2d(c+}$$

```
[Out] -Cos[a + b*x]^2/(2*d*(c + d*x)^2) - (b^2*Cos[2*a - (2*b*c)/d]*CosIntegral[(
2*b*c)/d + 2*b*x])/d^3 + (b*Cos[a + b*x]*Sin[a + b*x])/(d^2*(c + d*x)) + (b
^2*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^3
```

Rubi [A] time = 0.196853, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3314, 31, 3312, 3303, 3299, 3302}

$$-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)}-\frac{\cos^2(a+}{2d(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/(c + d*x)^3,x]
```

```
[Out] -Cos[a + b*x]^2/(2*d*(c + d*x)^2) - (b^2*Cos[2*a - (2*b*c)/d]*CosIntegral[(
2*b*c)/d + 2*b*x])/d^3 + (b*Cos[a + b*x]*Sin[a + b*x])/(d^2*(c + d*x)) + (b
^2*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^3
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx = -\frac{\cos^2(a+bx)}{2d(c+dx)^2} + \frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{b^2\int\frac{1}{c+dx}\,dx}{d^2} - \frac{\left(2b^2\right)\int\frac{\cos^2(a+bx)}{c+dx}\,dx}{d^2}$$

$$= -\frac{\cos^2(a+bx)}{2d(c+dx)^2} + \frac{b^2\log(c+dx)}{d^3} + \frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} - \frac{\left(2b^2\right)\int\left(\frac{1}{2(c+dx)}+\frac{\cos(2a+2bx)}{2(c+dx)}\right)dx}{d^2}$$

$$= -\frac{\cos^2(a+bx)}{2d(c+dx)^2} + \frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} - \frac{b^2\int\frac{\cos(2a+2bx)}{c+dx}\,dx}{d^2}$$

$$= -\frac{\cos^2(a+bx)}{2d(c+dx)^2} + \frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} - \frac{\left(b^2\cos\left(2a-\frac{2bc}{d}\right)\right)\int\frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d^2} + \frac{\left(b^2\sin\left(2a-\frac{2\right.}{}\right.}{}$$

$$= -\frac{\cos^2(a+bx)}{2d(c+dx)^2} - \frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^3} + \frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\mathrm{S}}{d^3}$$

Mathematica [A] time = 0.924553, size = 102, normalized size = 0.91

$$\frac{-2b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)+2b^2\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2b(c+dx)}{d}\right)+\frac{d\left(b(c+dx)\sin(2(a+bx))-d\cos^2(a+bx)\right)}{(c+dx)^2}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/(c + d*x)^3,x]
```

```
[Out] (-2*b^2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + (d*(-(d*Cos[a
 + b*x]^2) + b*(c + d*x)*Sin[2*(a + b*x)]))/(c + d*x)^2 + 2*b^2*Sin[2*a - (
2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/(2*d^3)
```

Maple [A] time = 0.032, size = 193, normalized size = 1.7

$$\frac{1}{b}\left(\frac{b^3}{4}\left(-\frac{\cos(2bx+2a)}{\left((bx+a)d-da+cb\right)^2d}-\frac{1}{d}\left(-2\frac{\sin(2bx+2a)}{\left((bx+a)d-da+cb\right)d}+2\frac{1}{d}\left(2\frac{1}{d}\mathrm{Si}\left(2bx+2a+2\frac{-da+cb}{d}\right)\sin\left(2\frac{-da+cb}{d}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/(d*x+c)^3,x)
```

```
[Out] 1/b*(1/4*b^3*(-cos(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^2/d-(-2*sin(2*b*x+2*a)/((
b*x+a)*d-d*a+c*b)/d+2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d
+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)/d)-1/4*b^3/((b*x+
```

```
a)*d-d*a+c*b)^2/d)
```

Maxima [C] time = 1.61695, size = 278, normalized size = 2.48

$$-\frac{16\,b^3\left(E_3\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+E_3\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^3\left(16i\,E_3\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-16i\,E_3\left(-\frac{2}{\ldots}\right.\right.}{64\left(b^2c^2d-2\,abcd^2+(bx+a)^2d^3+a^2d^3+2\left(bcd^2-ad^3\right)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/64*(16*b^3*(exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) +
 exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c -
 a*d)/d) - b^3*(16*I*exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d
)/d) - 16*I*exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*si
n(-2*(b*c - a*d)/d) + 16*b^3)/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 +
 a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

Fricas [A] time = 1.42418, size = 502, normalized size = 4.48

$$-\frac{d^2\cos\left(bx+a\right)^2-2\left(bd^2x+bcd\right)\cos\left(bx+a\right)\sin\left(bx+a\right)-2\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)\ldots}{2\left(d^5x^2+2\,cd^4x+\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*cos(b*x + a)^2 - 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) -
2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-2*(b*c - a*d)/d)*sin_integral(
2*(b*d*x + b*c)/d) + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(2*
(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-2*(b
*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos^2\left(a+bx\right)}{\left(c+dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] Integral(cos(a + b*x)**2/(c + d*x)**3, x)
```

Giac [C] time = 1.97172, size = 6934, normalized size = 61.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a
)^2*tan(b*c/d)^2 + b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_integral(2*b*x
 + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*d^2*x^2*imag_part(cos_i
ntegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 4*b^2*d^2*x^2*s
in_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*d^2*x
^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2
- 2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)
*tan(b*c/d)^2 + 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*ta
n(a)*tan(b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*ta
n(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b^2*d^2*x^2*real_part(cos_in
tegral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - b^2*d^2*x^2*real_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 4*b^2*d^2*x^2*real_part(cos
_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 4*b^2*d^2*x^2*re
al_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 4*b^
2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*
c/d) + 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan
(a)^2*tan(b*c/d) - 8*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*t
an(a)^2*tan(b*c/d) - b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*t
an(b*x)^2*tan(b*c/d)^2 - b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(b*x)^2*tan(b*c/d)^2 + 4*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*
b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 4*b^2*c*d*x*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 8*b^2*c*d*x*sin_integr
al(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b^2*d^2*x^2*real_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b^2*d^2*x^2*real_p
art(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b^2*c^2*real_pa
rt(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + b^2*c^
2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^
2 - 2*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a
) + 2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(
a) - 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - 2*b^
2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 2*b^
2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 2*b
^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) -
 2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c
/d) + 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d) +
 8*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan
(b*c/d) + 8*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*
tan(a)*tan(b*c/d) - 2*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*
tan(a)^2*tan(b*c/d) + 2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d
))*tan(a)^2*tan(b*c/d) - 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(
a)^2*tan(b*c/d) - 2*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 4*b^2*c^2*sin_integral(2*(b*d*x + b*c)
/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b^2*c*d*x*real_part(cos_integral(2*b
*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 2*b^2*c*d*x*real_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*imag_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos
_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 4*b^2*d^2*x^2*sin_integr
al(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + 2*b^2*c^2*imag_part(cos_integra
l(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 2*b^2*c^2*imag_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 4*b^2*c^2*si
n_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 2*b^2*c*d*x*
real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b^2*c*d*
```

```
x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b^2*d^2
*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + b^2*d^2*x^2*real
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 - 4*b^2*c*d*x*imag_part(co
s_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 4*b^2*c*d*x*imag_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 8*b^2*c*d*x*sin_integral(2*
(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - b^2*d^2*x^2*real_part(cos_integral(2*b
*x + 2*b*c/d))*tan(a)^2 - b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(a)^2 - b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2
*tan(a)^2 - b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*ta
n(a)^2 + 4*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*ta
n(b*c/d) - 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2
*tan(b*c/d) + 8*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*
c/d) + 4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*
c/d) + 4*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b
*c/d) + 4*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a
)*tan(b*c/d) + 4*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)
^2*tan(a)*tan(b*c/d) - 4*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))
*tan(a)^2*tan(b*c/d) + 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(a)^2*tan(b*c/d) - 8*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^
2*tan(b*c/d) - b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c
/d)^2 - b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2
- b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2
- b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2
 + 4*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2
 - 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^
2 + 8*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + 2*b*d
^2*x*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b^2*c^2*real_part(cos_integral(2*b*x
+ 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b^2*c^2*real_part(cos_integral(-2*b*x -
 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b*d^2*x*tan(b*x)*tan(a)^2*tan(b*c/d)^2
 + 2*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + 2*b^2*
c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 - 2*b^2*d^2*x^2*
imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 2*b^2*d^2*x^2*imag_part(c
os_integral(-2*b*x - 2*b*c/d))*tan(a) - 4*b^2*d^2*x^2*sin_integral(2*(b*d*x
 + b*c)/d)*tan(a) - 2*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(
b*x)^2*tan(a) + 2*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x
)^2*tan(a) - 4*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) -
2*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 - 2*b^2*c*d*x
*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 + 2*b^2*d^2*x^2*imag_pa
rt(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*b^2*d^2*x^2*imag_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + 4*b^2*d^2*x^2*sin_integral(2*(b*d*
x + b*c)/d)*tan(b*c/d) + 2*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))
*tan(b*x)^2*tan(b*c/d) - 2*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(b*x)^2*tan(b*c/d) + 4*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x
)^2*tan(b*c/d) + 8*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a
)*tan(b*c/d) + 8*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)
*tan(b*c/d) - 2*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*t
an(b*c/d) + 2*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*ta
n(b*c/d) - 4*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d) -
2*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - 2*b^2*c
*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 + 2*b^2*c^2*ima
g_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b^2*c^2*imag_
part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 4*b^2*c^2*sin_in
tegral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + 2*b*c*d*tan(b*x)^2*tan(a)*t
an(b*c/d)^2 + 2*b*c*d*tan(b*x)*tan(a)^2*tan(b*c/d)^2 + d^2*tan(b*x)^2*tan(a
)^2*tan(b*c/d)^2 + b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d)) + b
^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d)) + b^2*c^2*real_part(co
s_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + b^2*c^2*real_part(cos_integral(-2
*b*x - 2*b*c/d))*tan(b*x)^2 - 4*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*
b*c/d))*tan(a) + 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
```

```
a) - 8*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(a) + 2*b*d^2*x*tan(b*x
)^2*tan(a) - b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 - b^
2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 + 2*b*d^2*x*tan(b*
x)*tan(a)^2 + 4*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/
d) - 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + 8*b
^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) + 4*b^2*c^2*real_part(c
os_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) + 4*b^2*c^2*real_part(cos_i
ntegral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) - b^2*c^2*real_part(cos_integr
al(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - b^2*c^2*real_part(cos_integral(-2*b*x -
 2*b*c/d))*tan(b*c/d)^2 - 2*b*d^2*x*tan(b*x)*tan(b*c/d)^2 - 2*b*d^2*x*tan(a
)*tan(b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d)) + 2*b
^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d)) - 2*b^2*c^2*imag_part(co
s_integral(2*b*x + 2*b*c/d))*tan(a) + 2*b^2*c^2*imag_part(cos_integral(-2*b
*x - 2*b*c/d))*tan(a) - 4*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(a) +
2*b*c*d*tan(b*x)^2*tan(a) + 2*b*c*d*tan(b*x)*tan(a)^2 + d^2*tan(b*x)^2*tan(
a)^2 + 2*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*b^
2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + 4*b^2*c^2*sin_
integral(2*(b*d*x + b*c)/d)*tan(b*c/d) - 2*b*c*d*tan(b*x)*tan(b*c/d)^2 - 2*
b*c*d*tan(a)*tan(b*c/d)^2 - 2*d^2*tan(b*x)*tan(a)*tan(b*c/d)^2 + b^2*c^2*re
al_part(cos_integral(2*b*x + 2*b*c/d)) + b^2*c^2*real_part(cos_integral(-2*
b*x - 2*b*c/d)) - 2*b*d^2*x*tan(b*x) - 2*b*d^2*x*tan(a) - 2*b*c*d*tan(b*x)
- 2*b*c*d*tan(a) - 2*d^2*tan(b*x)*tan(a) + d^2*tan(b*c/d)^2 + d^2)/(d^5*x^2
*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*c*d^4*x*tan(b*x)^2*tan(a)^2*tan(b*c/d
)^2 + d^5*x^2*tan(b*x)^2*tan(a)^2 + d^5*x^2*tan(b*x)^2*tan(b*c/d)^2 + d^5*x
^2*tan(a)^2*tan(b*c/d)^2 + c^2*d^3*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*c*d
^4*x*tan(b*x)^2*tan(a)^2 + 2*c*d^4*x*tan(b*x)^2*tan(b*c/d)^2 + 2*c*d^4*x*ta
n(a)^2*tan(b*c/d)^2 + d^5*x^2*tan(b*x)^2 + d^5*x^2*tan(a)^2 + c^2*d^3*tan(b
*x)^2*tan(a)^2 + d^5*x^2*tan(b*c/d)^2 + c^2*d^3*tan(b*x)^2*tan(b*c/d)^2 + c
^2*d^3*tan(a)^2*tan(b*c/d)^2 + 2*c*d^4*x*tan(b*x)^2 + 2*c*d^4*x*tan(a)^2 +
2*c*d^4*x*tan(b*c/d)^2 + d^5*x^2 + c^2*d^3*tan(b*x)^2 + c^2*d^3*tan(a)^2 +
c^2*d^3*tan(b*c/d)^2 + 2*c*d^4*x + c^2*d^3)
```

3.16 $\int (c+dx)^4 \cos^3(a+bx)\,dx$

Optimal. Leaf size=225

$$-\frac{80d^2(c+dx)^2\sin(a+bx)}{9b^3}-\frac{8d^3(c+dx)\cos^3(a+bx)}{27b^4}-\frac{160d^3(c+dx)\cos(a+bx)}{9b^4}-\frac{4d^2(c+dx)^2\sin(a+bx)\cos^2(a$$

```
[Out] (-160*d^3*(c + d*x)*Cos[a + b*x])/(9*b^4) + (8*d*(c + d*x)^3*Cos[a + b*x])/
(3*b^2) - (8*d^3*(c + d*x)*Cos[a + b*x]^3)/(27*b^4) + (4*d*(c + d*x)^3*Cos[
a + b*x]^3)/(9*b^2) + (488*d^4*Sin[a + b*x])/(27*b^5) - (80*d^2*(c + d*x)^2
*Sin[a + b*x])/(9*b^3) + (2*(c + d*x)^4*Sin[a + b*x])/(3*b) - (4*d^2*(c + d
*x)^2*Cos[a + b*x]^2*Sin[a + b*x])/(9*b^3) + ((c + d*x)^4*Cos[a + b*x]^2*Si
n[a + b*x])/(3*b) - (8*d^4*Sin[a + b*x]^3)/(81*b^5)
```

Rubi [A] time = 0.254349, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3311, 3296, 2637, 2633}

$$-\frac{80d^2(c+dx)^2\sin(a+bx)}{9b^3}-\frac{8d^3(c+dx)\cos^3(a+bx)}{27b^4}-\frac{160d^3(c+dx)\cos(a+bx)}{9b^4}-\frac{4d^2(c+dx)^2\sin(a+bx)\cos^2(a$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^3,x]
```


```
[Out] (-160*d^3*(c + d*x)*Cos[a + b*x])/(9*b^4) + (8*d*(c + d*x)^3*Cos[a + b*x])/
(3*b^2) - (8*d^3*(c + d*x)*Cos[a + b*x]^3)/(27*b^4) + (4*d*(c + d*x)^3*Cos[
a + b*x]^3)/(9*b^2) + (488*d^4*Sin[a + b*x])/(27*b^5) - (80*d^2*(c + d*x)^2
*Sin[a + b*x])/(9*b^3) + (2*(c + d*x)^4*Sin[a + b*x])/(3*b) - (4*d^2*(c + d
*x)^2*Cos[a + b*x]^2*Sin[a + b*x])/(9*b^3) + ((c + d*x)^4*Cos[a + b*x]^2*Si
n[a + b*x])/(3*b) - (8*d^4*Sin[a + b*x]^3)/(81*b^5)
```


Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (c+dx)^4\cos^3(a+bx)\,dx = \frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^4\cos^2(a+bx)\sin(a+bx)}{3b}+\frac{2}{3}\int (c+dx)^4\cos(a+bx)$$
$$= -\frac{8d^3(c+dx)\cos^3(a+bx)}{27b^4}+\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{2(c+dx)^4\sin(a+bx)}{3b}-\frac{4d^2(c+$$
$$= \frac{8d(c+dx)^3\cos(a+bx)}{3b^2}-\frac{8d^3(c+dx)\cos^3(a+bx)}{27b^4}+\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}-\frac{8d^2(c+$$
$$= -\frac{16d^3(c+dx)\cos(a+bx)}{9b^4}+\frac{8d(c+dx)^3\cos(a+bx)}{3b^2}-\frac{8d^3(c+dx)\cos^3(a+bx)}{27b^4}+\frac{4d(c+$$
$$= -\frac{160d^3(c+dx)\cos(a+bx)}{9b^4}+\frac{8d(c+dx)^3\cos(a+bx)}{3b^2}-\frac{8d^3(c+dx)\cos^3(a+bx)}{27b^4}+\frac{4d(c$$
$$= -\frac{160d^3(c+dx)\cos(a+bx)}{9b^4}+\frac{8d(c+dx)^3\cos(a+bx)}{3b^2}-\frac{8d^3(c+dx)\cos^3(a+bx)}{27b^4}+\frac{4d(c$$

Mathematica [A] time = 1.03027, size = 385, normalized size = 1.71

$$\frac{1458b^4c^2d^2x^2\sin(a+bx)+162b^4c^2d^2x^2\sin(3(a+bx))-2916b^2c^2d^2\sin(a+bx)-36b^2c^2d^2\sin(3(a+bx))+972b^4c^3dx\,s}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^3,x]
```

```
[Out] (972*b*d*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + 12*b*d*(c + d*
x)*(-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 243*b^4*c^4*Sin[a + b*x]
 - 2916*b^2*c^2*d^2*Sin[a + b*x] + 5832*d^4*Sin[a + b*x] + 972*b^4*c^3*d*x*
Sin[a + b*x] - 5832*b^2*c*d^3*x*Sin[a + b*x] + 1458*b^4*c^2*d^2*x^2*Sin[a +
 b*x] - 2916*b^2*d^4*x^2*Sin[a + b*x] + 972*b^4*c*d^3*x^3*Sin[a + b*x] + 24
3*b^4*d^4*x^4*Sin[a + b*x] + 27*b^4*c^4*Sin[3*(a + b*x)] - 36*b^2*c^2*d^2*S
in[3*(a + b*x)] + 8*d^4*Sin[3*(a + b*x)] + 108*b^4*c^3*d*x*Sin[3*(a + b*x)]
 - 72*b^2*c*d^3*x*Sin[3*(a + b*x)] + 162*b^4*c^2*d^2*x^2*Sin[3*(a + b*x)] -
 36*b^2*d^4*x^2*Sin[3*(a + b*x)] + 108*b^4*c*d^3*x^3*Sin[3*(a + b*x)] + 27*
b^4*d^4*x^4*Sin[3*(a + b*x)])/(324*b^5)
```

Maple [B] time = 0.071, size = 1023, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^4*d^4*(1/3*(b*x+a)^4*(2+cos(b*x+a)^2)*sin(b*x+a)+8/3*(b*x+a)^3*cos
(b*x+a)-8*(b*x+a)^2*sin(b*x+a)+160/9*sin(b*x+a)-160/9*(b*x+a)*cos(b*x+a)+4/
9*(b*x+a)^3*cos(b*x+a)^3-4/9*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-8/27*(b*
x+a)*cos(b*x+a)^3+8/81*(2+cos(b*x+a)^2)*sin(b*x+a))-4/b^4*a*d^4*(1/3*(b*x+a
)^3*(2+cos(b*x+a)^2)*sin(b*x+a)+2*(b*x+a)^2*cos(b*x+a)-40/9*cos(b*x+a)-4*(b
*x+a)*sin(b*x+a)+1/3*(b*x+a)^2*cos(b*x+a)^3-2/9*(b*x+a)*(2+cos(b*x+a)^2)*si
n(b*x+a)-2/27*cos(b*x+a)^3)+4/b^3*c*d^3*(1/3*(b*x+a)^3*(2+cos(b*x+a)^2)*sin
```

```
(b*x+a)+2*(b*x+a)^2*cos(b*x+a)-40/9*cos(b*x+a)-4*(b*x+a)*sin(b*x+a)+1/3*(b*
x+a)^2*cos(b*x+a)^3-2/9*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)-2/27*cos(b*x+a)
^3)+6/b^4*a^2*d^4*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/3*sin(b*x+a)
+4/3*(b*x+a)*cos(b*x+a)+2/9*(b*x+a)*cos(b*x+a)^3-2/27*(2+cos(b*x+a)^2)*sin(
b*x+a))-12/b^3*a*c*d^3*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/3*sin(b
*x+a)+4/3*(b*x+a)*cos(b*x+a)+2/9*(b*x+a)*cos(b*x+a)^3-2/27*(2+cos(b*x+a)^2)
*sin(b*x+a))+6/b^2*c^2*d^2*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/3*s
in(b*x+a)+4/3*(b*x+a)*cos(b*x+a)+2/9*(b*x+a)*cos(b*x+a)^3-2/27*(2+cos(b*x+a
)^2)*sin(b*x+a))-4/b^4*a^3*d^4*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/9
*cos(b*x+a)^3+2/3*cos(b*x+a))+12/b^3*a^2*c*d^3*(1/3*(b*x+a)*(2+cos(b*x+a)^2
)*sin(b*x+a)+1/9*cos(b*x+a)^3+2/3*cos(b*x+a))-12/b^2*a*c^2*d^2*(1/3*(b*x+a)
*(2+cos(b*x+a)^2)*sin(b*x+a)+1/9*cos(b*x+a)^3+2/3*cos(b*x+a))+4/b*c^3*d*(1/
3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/9*cos(b*x+a)^3+2/3*cos(b*x+a))+1/3/
b^4*a^4*d^4*(2+cos(b*x+a)^2)*sin(b*x+a)-4/3/b^3*a^3*c*d^3*(2+cos(b*x+a)^2)*
sin(b*x+a)+2/b^2*a^2*c^2*d^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/3/b*a*c^3*d*(2+c
os(b*x+a)^2)*sin(b*x+a)+1/3*c^4*(2+cos(b*x+a)^2)*sin(b*x+a))
```

Maxima [B] time = 1.2231, size = 1249, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/324*(108*(sin(b*x + a)^3 - 3*sin(b*x + a))*c^4 - 432*(sin(b*x + a)^3 - 3
*sin(b*x + a))*a*c^3*d/b + 648*(sin(b*x + a)^3 - 3*sin(b*x + a))*a^2*c^2*d^
2/b^2 - 432*(sin(b*x + a)^3 - 3*sin(b*x + a))*a^3*c*d^3/b^3 + 108*(sin(b*x
+ a)^3 - 3*sin(b*x + a))*a^4*d^4/b^4 - 36*(3*(b*x + a)*sin(3*b*x + 3*a) + 2
7*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) + 27*cos(b*x + a))*c^3*d/b + 10
8*(3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3
*a) + 27*cos(b*x + a))*a*c^2*d^2/b^2 - 108*(3*(b*x + a)*sin(3*b*x + 3*a) +
27*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) + 27*cos(b*x + a))*a^2*c*d^3/b
^3 + 36*(3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x + a)*sin(b*x + a) + cos(3*b
*x + 3*a) + 27*cos(b*x + a))*a^3*d^4/b^4 - 18*(6*(b*x + a)*cos(3*b*x + 3*a)
 + 162*(b*x + a)*cos(b*x + a) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 81*(
(b*x + a)^2 - 2)*sin(b*x + a))*c^2*d^2/b^2 + 36*(6*(b*x + a)*cos(3*b*x + 3*
a) + 162*(b*x + a)*cos(b*x + a) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 81
*((b*x + a)^2 - 2)*sin(b*x + a))*a*c*d^3/b^3 - 18*(6*(b*x + a)*cos(3*b*x +
3*a) + 162*(b*x + a)*cos(b*x + a) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) +
81*((b*x + a)^2 - 2)*sin(b*x + a))*a^2*d^4/b^4 - 12*((9*(b*x + a)^2 - 2)*co
s(3*b*x + 3*a) + 243*((b*x + a)^2 - 2)*cos(b*x + a) + 3*(3*(b*x + a)^3 - 2*
b*x - 2*a)*sin(3*b*x + 3*a) + 81*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*
c*d^3/b^3 + 12*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 243*((b*x + a)^2 - 2
)*cos(b*x + a) + 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) + 81*((b*
x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*a*d^4/b^4 - (12*(3*(b*x + a)^3 - 2*b*
x - 2*a)*cos(3*b*x + 3*a) + 972*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) +
(27*(b*x + a)^4 - 36*(b*x + a)^2 + 8)*sin(3*b*x + 3*a) + 243*((b*x + a)^4 -
 12*(b*x + a)^2 + 24)*sin(b*x + a))*d^4/b^4)/b
```

Fricas [A] time = 1.54615, size = 759, normalized size = 3.37

$$12\left(3\,b^3d^4x^3 + 9\,b^3cd^3x^2 + 3\,b^3c^3d - 2\,bcd^3 + \left(9\,b^3c^2d^2 - 2\,bd^4\right)x\right)\cos\left(bx + a\right)^3 + 72\left(3\,b^3d^4x^3 + 9\,b^3cd^3x^2 + 3\,b^3c^3d -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/81*(12*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d - 2*b*c*d^3 + (9*b^
3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^3 + 72*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^
2 + 3*b^3*c^3*d - 20*b*c*d^3 + (9*b^3*c^2*d^2 - 20*b*d^4)*x)*cos(b*x + a) +
 (54*b^4*d^4*x^4 + 216*b^4*c*d^3*x^3 + 54*b^4*c^4 - 720*b^2*c^2*d^2 + 1456*
d^4 + 36*(9*b^4*c^2*d^2 - 20*b^2*d^4)*x^2 + (27*b^4*d^4*x^4 + 108*b^4*c*d^3
*x^3 + 27*b^4*c^4 - 36*b^2*c^2*d^2 + 8*d^4 + 18*(9*b^4*c^2*d^2 - 2*b^2*d^4)
*x^2 + 36*(3*b^4*c^3*d - 2*b^2*c*d^3)*x)*cos(b*x + a)^2 + 72*(3*b^4*c^3*d -
 20*b^2*c*d^3)*x)*sin(b*x + a))/b^5
```

Sympy [A] time = 10.4196, size = 772, normalized size = 3.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**3,x)
```

```
[Out] Piecewise((2*c**4*sin(a + b*x)**3/(3*b) + c**4*sin(a + b*x)*cos(a + b*x)**2
/b + 8*c**3*d*x*sin(a + b*x)**3/(3*b) + 4*c**3*d*x*sin(a + b*x)*cos(a + b*x
)**2/b + 4*c**2*d**2*x**2*sin(a + b*x)**3/b + 6*c**2*d**2*x**2*sin(a + b*x)
*cos(a + b*x)**2/b + 8*c*d**3*x**3*sin(a + b*x)**3/(3*b) + 4*c*d**3*x**3*si
n(a + b*x)*cos(a + b*x)**2/b + 2*d**4*x**4*sin(a + b*x)**3/(3*b) + d**4*x**
4*sin(a + b*x)*cos(a + b*x)**2/b + 8*c**3*d*sin(a + b*x)**2*cos(a + b*x)/(3
*b**2) + 28*c**3*d*cos(a + b*x)**3/(9*b**2) + 8*c**2*d**2*x*sin(a + b*x)**2
*cos(a + b*x)/b**2 + 28*c**2*d**2*x*cos(a + b*x)**3/(3*b**2) + 8*c*d**3*x**
2*sin(a + b*x)**2*cos(a + b*x)/b**2 + 28*c*d**3*x**2*cos(a + b*x)**3/(3*b**
2) + 8*d**4*x**3*sin(a + b*x)**2*cos(a + b*x)/(3*b**2) + 28*d**4*x**3*cos(a
 + b*x)**3/(9*b**2) - 80*c**2*d**2*sin(a + b*x)**3/(9*b**3) - 28*c**2*d**2*
sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 160*c*d**3*x*sin(a + b*x)**3/(9*b**
3) - 56*c*d**3*x*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 80*d**4*x**2*sin(a
 + b*x)**3/(9*b**3) - 28*d**4*x**2*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) -
160*c*d**3*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 488*c*d**3*cos(a + b*x)*
*3/(27*b**4) - 160*d**4*x*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 488*d**4*
x*cos(a + b*x)**3/(27*b**4) + 1456*d**4*sin(a + b*x)**3/(81*b**5) + 488*d**
4*sin(a + b*x)*cos(a + b*x)**2/(27*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x*
*2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*cos(a)**3, True))
```

Giac [A] time = 1.15112, size = 474, normalized size = 2.11

$$\frac{\left(3\,b^3d^4x^3+9\,b^3cd^3x^2+9\,b^3c^2d^2x+3\,b^3c^3d-2\,bd^4x-2\,bcd^3\right)\cos\left(3\,bx+3\,a\right)}{27\,b^5}+\frac{3\left(b^3d^4x^3+3\,b^3cd^3x^2+3\,b^3c^2d^2x+b^3c\right.}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b
*d^4*x - 2*b*c*d^3)*cos(3*b*x + 3*a)/b^5 + 3*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2
 + 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*cos(b*x + a)/b^5 +
1/324*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*c
^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 + 8*
d^4)*sin(3*b*x + 3*a)/b^5 + 3/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*
```

```
d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b^
2*c^2*d^2 + 24*d^4)*sin(b*x + a)/b^5
```

3.17 $\int (c+dx)^3 \cos^3(a+bx)\,dx$

Optimal. Leaf size=175

$$-\frac{40d^2(c+dx)\sin(a+bx)}{9b^3}-\frac{2d^2(c+dx)\sin(a+bx)\cos^2(a+bx)}{9b^3}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2d(c+dx)^2\cos(a+bx)}{b^2}-$$

```
[Out] (-40*d^3*Cos[a + b*x])/(9*b^4) + (2*d*(c + d*x)^2*Cos[a + b*x])/b^2 - (2*d^
3*Cos[a + b*x]^3)/(27*b^4) + (d*(c + d*x)^2*Cos[a + b*x]^3)/(3*b^2) - (40*d
^2*(c + d*x)*Sin[a + b*x])/(9*b^3) + (2*(c + d*x)^3*Sin[a + b*x])/(3*b) - (
2*d^2*(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(9*b^3) + ((c + d*x)^3*Cos[a +
 b*x]^2*Sin[a + b*x])/(3*b)
```

Rubi [A] time = 0.156228, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3311, 3296, 2638, 3310}

$$-\frac{40d^2(c+dx)\sin(a+bx)}{9b^3}-\frac{2d^2(c+dx)\sin(a+bx)\cos^2(a+bx)}{9b^3}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2d(c+dx)^2\cos(a+bx)}{b^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^3,x]
```

```
[Out] (-40*d^3*Cos[a + b*x])/(9*b^4) + (2*d*(c + d*x)^2*Cos[a + b*x])/b^2 - (2*d^
3*Cos[a + b*x]^3)/(27*b^4) + (d*(c + d*x)^2*Cos[a + b*x]^3)/(3*b^2) - (40*d
^2*(c + d*x)*Sin[a + b*x])/(9*b^3) + (2*(c + d*x)^3*Sin[a + b*x])/(3*b) - (
2*d^2*(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(9*b^3) + ((c + d*x)^3*Cos[a +
 b*x]^2*Sin[a + b*x])/(3*b)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
```

]

Rubi steps

$$\int (c+dx)^3\cos^3(a+bx)\,dx = \frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{(c+dx)^3\cos^2(a+bx)\sin(a+bx)}{3b}+\frac{2}{3}\int (c+dx)^3\cos(a+b$$

$$= -\frac{2d^3\cos^3(a+bx)}{27b^4}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2(c+dx)^3\sin(a+bx)}{3b}-\frac{2d^2(c+dx)\text{co}}{}$$

$$= \frac{2d(c+dx)^2\cos(a+bx)}{b^2}-\frac{2d^3\cos^3(a+bx)}{27b^4}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}-\frac{4d^2(c+dx)\sin}{9b^3}$$

$$= -\frac{4d^3\cos(a+bx)}{9b^4}+\frac{2d(c+dx)^2\cos(a+bx)}{b^2}-\frac{2d^3\cos^3(a+bx)}{27b^4}+\frac{d(c+dx)^2\cos^3(a+b}{3b^2}$$

$$= -\frac{40d^3\cos(a+bx)}{9b^4}+\frac{2d(c+dx)^2\cos(a+bx)}{b^2}-\frac{2d^3\cos^3(a+bx)}{27b^4}+\frac{d(c+dx)^2\cos^3(a+}{3b^2}$$

Mathematica [A] time = 0.968956, size = 121, normalized size = 0.69

$$\frac{243d\cos(a+bx)\left(b^2(c+dx)^2-2d^2\right)+d\cos(3(a+bx))\left(9b^2(c+dx)^2-2d^2\right)+6b(c+dx)\sin(a+bx)\left(\cos(2(a+bx))\right.}{108b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^3,x]
```

```
[Out] (243*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + d*(-2*d^2 + 9*b^2*(c + d*x
)^2)*Cos[3*(a + b*x)] + 6*b*(c + d*x)*(-82*d^2 + 15*b^2*(c + d*x)^2 + (-2*d
^2 + 3*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/(108*b^4)
```

Maple [B] time = 0.029, size = 560, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^3*d^3*(1/3*(b*x+a)^3*(2+cos(b*x+a)^2)*sin(b*x+a)+2*(b*x+a)^2*cos(b
*x+a)-40/9*cos(b*x+a)-4*(b*x+a)*sin(b*x+a)+1/3*(b*x+a)^2*cos(b*x+a)^3-2/9*(
b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)-2/27*cos(b*x+a)^3)-3/b^3*a*d^3*(1/3*(b*x
+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/3*sin(b*x+a)+4/3*(b*x+a)*cos(b*x+a)+2/9
*(b*x+a)*cos(b*x+a)^3-2/27*(2+cos(b*x+a)^2)*sin(b*x+a))+3/b^2*c*d^2*(1/3*(b
*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/3*sin(b*x+a)+4/3*(b*x+a)*cos(b*x+a)+2
/9*(b*x+a)*cos(b*x+a)^3-2/27*(2+cos(b*x+a)^2)*sin(b*x+a))+3/b^3*a^2*d^3*(1/
3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/9*cos(b*x+a)^3+2/3*cos(b*x+a))-6/b^
2*a*c*d^2*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/9*cos(b*x+a)^3+2/3*cos
(b*x+a))+3/b*c^2*d*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/9*cos(b*x+a)^
3+2/3*cos(b*x+a))-1/3/b^3*a^3*d^3*(2+cos(b*x+a)^2)*sin(b*x+a)+1/b^2*a^2*c*d
^2*(2+cos(b*x+a)^2)*sin(b*x+a)-1/b*a*c^2*d*(2+cos(b*x+a)^2)*sin(b*x+a)+1/3*
c^3*(2+cos(b*x+a)^2)*sin(b*x+a))
```

Maxima [B] time = 1.11295, size = 722, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/108*(36*(sin(b*x + a)^3 - 3*sin(b*x + a))*c^3 - 108*(sin(b*x + a)^3 - 3*
sin(b*x + a))*a*c^2*d/b + 108*(sin(b*x + a)^3 - 3*sin(b*x + a))*a^2*c*d^2/b
^2 - 36*(sin(b*x + a)^3 - 3*sin(b*x + a))*a^3*d^3/b^3 - 9*(3*(b*x + a)*sin(
3*b*x + 3*a) + 27*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) + 27*cos(b*x +
a))*c^2*d/b + 18*(3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x + a)*sin(b*x + a)
+ cos(3*b*x + 3*a) + 27*cos(b*x + a))*a*c*d^2/b^2 - 9*(3*(b*x + a)*sin(3*b*
x + 3*a) + 27*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) + 27*cos(b*x + a))*
a^2*d^3/b^3 - 3*(6*(b*x + a)*cos(3*b*x + 3*a) + 162*(b*x + a)*cos(b*x + a)
+ (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 81*((b*x + a)^2 - 2)*sin(b*x + a))
*c*d^2/b^2 + 3*(6*(b*x + a)*cos(3*b*x + 3*a) + 162*(b*x + a)*cos(b*x + a) +
 (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 81*((b*x + a)^2 - 2)*sin(b*x + a))*
a*d^3/b^3 - ((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 243*((b*x + a)^2 - 2)*c
os(b*x + a) + 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) + 81*((b*x +
 a)^3 - 6*b*x - 6*a)*sin(b*x + a))*d^3/b^3)/b
```

Fricas [A] time = 1.50399, size = 494, normalized size = 2.82

$$\frac{\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)^3+6\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-20\,d^3\right)\cos\left(bx+a\right)+3\left(6\,b^3d^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/27*((9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a)^3
 + 6*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 20*d^3)*cos(b*x + a) +
 3*(6*b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 + 6*b^3*c^3 - 40*b*c*d^2 + (3*b^3*d^3*
x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 2*b*c*d^2 + (9*b^3*c^2*d - 2*b*d^3)*x)*
cos(b*x + a)^2 + 2*(9*b^3*c^2*d - 20*b*d^3)*x)*sin(b*x + a))/b^4
```

Sympy [A] time = 5.38698, size = 495, normalized size = 2.83

$$\begin{cases} \frac{2c^3\sin^3(a+bx)}{3b}+\frac{c^3\sin(a+bx)\cos^2(a+bx)}{b}+\frac{2c^2dx\sin^3(a+bx)}{b}+\frac{3c^2dx\sin(a+bx)\cos^2(a+bx)}{b}+\frac{2cd^2x^2\sin^3(a+bx)}{b}+\frac{3cd^2x^2\sin(a+bx)\cos^2(a+bx)}{b} \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\cos^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**3,x)
```

```
[Out] Piecewise((2*c**3*sin(a + b*x)**3/(3*b) + c**3*sin(a + b*x)*cos(a + b*x)**2
/b + 2*c**2*d*x*sin(a + b*x)**3/b + 3*c**2*d*x*sin(a + b*x)*cos(a + b*x)**2
/b + 2*c*d**2*x**2*sin(a + b*x)**3/b + 3*c*d**2*x**2*sin(a + b*x)*cos(a + b
*x)**2/b + 2*d**3*x**3*sin(a + b*x)**3/(3*b) + d**3*x**3*sin(a + b*x)*cos(a
 + b*x)**2/b + 2*c**2*d*sin(a + b*x)**2*cos(a + b*x)/b**2 + 7*c**2*d*cos(a
+ b*x)**3/(3*b**2) + 4*c*d**2*x*sin(a + b*x)**2*cos(a + b*x)/b**2 + 14*c*d*
```

```
*2*x*cos(a + b*x)**3/(3*b**2) + 2*d**3*x**2*sin(a + b*x)**2*cos(a + b*x)/b*
*2 + 7*d**3*x**2*cos(a + b*x)**3/(3*b**2) - 40*c*d**2*sin(a + b*x)**3/(9*b*
*3) - 14*c*d**2*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 40*d**3*x*sin(a + b
*x)**3/(9*b**3) - 14*d**3*x*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 40*d**3
*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 122*d**3*cos(a + b*x)**3/(27*b**4)
, Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*cos(a)
**3, True))
```

Giac [A] time = 1.1231, size = 312, normalized size = 1.78

$$\frac{\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3\right)\cos\left(3\,bx+3\,a\right)}{108\,b^4}+\frac{9\left(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)}{4\,b^4}+\frac{\left(3\,b^3d^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/108*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(3*b*x + 3*
a)/b^4 + 9/4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a)
/b^4 + 1/36*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 -
2*b*d^3*x - 2*b*c*d^2)*sin(3*b*x + 3*a)/b^4 + 3/4*(b^3*d^3*x^3 + 3*b^3*c*d^
2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(b*x + a)/b^4
```

3.18 $\int (c+dx)^2 \cos^3(a+bx)\,dx$

Optimal. Leaf size=123

$$\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{4d(c+dx)\cos(a+bx)}{3b^2}+\frac{2d^2\sin^3(a+bx)}{27b^3}-\frac{14d^2\sin(a+bx)}{9b^3}+\frac{2(c+dx)^2\sin(a+bx)}{3b}+\frac{(c+$$

```
[Out] (4*d*(c + d*x)*Cos[a + b*x])/(3*b^2) + (2*d*(c + d*x)*Cos[a + b*x]^3)/(9*b^
2) - (14*d^2*Sin[a + b*x])/(9*b^3) + (2*(c + d*x)^2*Sin[a + b*x])/(3*b) + (
(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x])/(3*b) + (2*d^2*Sin[a + b*x]^3)/(27
*b^3)
```

Rubi [A] time = 0.0967742, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3311, 3296, 2637, 2633}

$$\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{4d(c+dx)\cos(a+bx)}{3b^2}+\frac{2d^2\sin^3(a+bx)}{27b^3}-\frac{14d^2\sin(a+bx)}{9b^3}+\frac{2(c+dx)^2\sin(a+bx)}{3b}+\frac{(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^3,x]
```

```
[Out] (4*d*(c + d*x)*Cos[a + b*x])/(3*b^2) + (2*d*(c + d*x)*Cos[a + b*x]^3)/(9*b^
2) - (14*d^2*Sin[a + b*x])/(9*b^3) + (2*(c + d*x)^2*Sin[a + b*x])/(3*b) + (
(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x])/(3*b) + (2*d^2*Sin[a + b*x]^3)/(27
*b^3)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (c+dx)^2\cos^3(a+bx)\,dx = \frac{2d(c+dx)\cos^3(a+bx)}{9b^2} + \frac{(c+dx)^2\cos^2(a+bx)\sin(a+bx)}{3b} + \frac{2}{3}\int (c+dx)^2\cos(a+b$$

$$= \frac{2d(c+dx)\cos^3(a+bx)}{9b^2} + \frac{2(c+dx)^2\sin(a+bx)}{3b} + \frac{(c+dx)^2\cos^2(a+bx)\sin(a+bx)}{3b}$$

$$= \frac{4d(c+dx)\cos(a+bx)}{3b^2} + \frac{2d(c+dx)\cos^3(a+bx)}{9b^2} - \frac{2d^2\sin(a+bx)}{9b^3} + \frac{2(c+dx)^2\sin(a}{3b}$$

$$= \frac{4d(c+dx)\cos(a+bx)}{3b^2} + \frac{2d(c+dx)\cos^3(a+bx)}{9b^2} - \frac{14d^2\sin(a+bx)}{9b^3} + \frac{2(c+dx)^2\sin(a}{3b}$$

Mathematica [A] time = 0.602845, size = 93, normalized size = 0.76

$$\frac{2\sin(a+bx)\left(\cos(2(a+bx))\left(9b^2(c+dx)^2-2d^2\right)+45b^2(c+dx)^2-82d^2\right)+162bd(c+dx)\cos(a+bx)+6bd(c+dx)}{108b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^3,x]
```

```
[Out] (162*b*d*(c + d*x)*Cos[a + b*x] + 6*b*d*(c + d*x)*Cos[3*(a + b*x)] + 2*(-82
*d^2 + 45*b^2*(c + d*x)^2 + (-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*
Sin[a + b*x])/(108*b^3)
```

Maple [B] time = 0.029, size = 265, normalized size = 2.2

$$\frac{1}{b}\left(\frac{d^2}{b^2}\left(\frac{(bx+a)^2\left(2+(\cos(bx+a))^2\right)\sin(bx+a)}{3} - \frac{4\sin(bx+a)}{3} + \frac{(4bx+4a)\cos(bx+a)}{3} + \frac{(2bx+2a)(\cos(bx}{9}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^2*d^2*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/3*sin(b*x+a)+4/
3*(b*x+a)*cos(b*x+a)+2/9*(b*x+a)*cos(b*x+a)^3-2/27*(2+cos(b*x+a)^2)*sin(b*x
+a))-2/b^2*a*d^2*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/9*cos(b*x+a)^3+
2/3*cos(b*x+a))+2/b*c*d*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/9*cos(b*
x+a)^3+2/3*cos(b*x+a))+1/3/b^2*a^2*d^2*(2+cos(b*x+a)^2)*sin(b*x+a)-2/3/b*a*
c*d*(2+cos(b*x+a)^2)*sin(b*x+a)+1/3*c^2*(2+cos(b*x+a)^2)*sin(b*x+a))
```

Maxima [B] time = 1.06168, size = 360, normalized size = 2.93

$$-\frac{36\left(\sin(bx+a)^3-3\sin(bx+a)\right)c^2 - \frac{72\left(\sin(bx+a)^3-3\sin(bx+a)\right)acd}{b} + \frac{36\left(\sin(bx+a)^3-3\sin(bx+a)\right)a^2d^2}{b^2} - \frac{6(3(bx+a)\sin(3bx+3a)+2}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/108*(36*(sin(b*x + a)^3 - 3*sin(b*x + a))*c^2 - 72*(sin(b*x + a)^3 - 3*s
in(b*x + a))*a*c*d/b + 36*(sin(b*x + a)^3 - 3*sin(b*x + a))*a^2*d^2/b^2 - 6
```

```
*(3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*
a) + 27*cos(b*x + a))*c*d/b + 6*(3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x + a
)*sin(b*x + a) + cos(3*b*x + 3*a) + 27*cos(b*x + a))*a*d^2/b^2 - (6*(b*x +
a)*cos(3*b*x + 3*a) + 162*(b*x + a)*cos(b*x + a) + (9*(b*x + a)^2 - 2)*sin(
3*b*x + 3*a) + 81*((b*x + a)^2 - 2)*sin(b*x + a))*d^2/b^2)/b
```

Fricas [A] time = 1.47013, size = 297, normalized size = 2.41

$$\frac{6\left(bd^2x + bcd\right)\cos(bx+a)^3 + 36\left(bd^2x + bcd\right)\cos(bx+a) + \left(18\,b^2d^2x^2 + 36\,b^2cdx + 18\,b^2c^2 + \left(9\,b^2d^2x^2 + 18\,b^2cdx + 9\right.\right.}{27\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/27*(6*(b*d^2*x + b*c*d)*cos(b*x + a)^3 + 36*(b*d^2*x + b*c*d)*cos(b*x + a
) + (18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^2*c^2 + (9*b^2*d^2*x^2 + 18*b^2*c
*d*x + 9*b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - 40*d^2)*sin(b*x + a))/b^3
```

Sympy [A] time = 3.02884, size = 284, normalized size = 2.31

$$\begin{cases} \frac{2c^2\sin^3(a+bx)}{3b} + \frac{c^2\sin(a+bx)\cos^2(a+bx)}{b} + \frac{4cdx\sin^3(a+bx)}{3b} + \frac{2cdx\sin(a+bx)\cos^2(a+bx)}{b} + \frac{2d^2x^2\sin^3(a+bx)}{3b} + \frac{d^2x^2\sin(a+bx)\cos^2(a+bx)}{b} + \frac{4c}{} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\cos^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**3,x)
```

```
[Out] Piecewise((2*c**2*sin(a + b*x)**3/(3*b) + c**2*sin(a + b*x)*cos(a + b*x)**2
/b + 4*c*d*x*sin(a + b*x)**3/(3*b) + 2*c*d*x*sin(a + b*x)*cos(a + b*x)**2/b
 + 2*d**2*x**2*sin(a + b*x)**3/(3*b) + d**2*x**2*sin(a + b*x)*cos(a + b*x)*
*2/b + 4*c*d*sin(a + b*x)**2*cos(a + b*x)/(3*b**2) + 14*c*d*cos(a + b*x)**3
/(9*b**2) + 4*d**2*x*sin(a + b*x)**2*cos(a + b*x)/(3*b**2) + 14*d**2*x*cos(
a + b*x)**3/(9*b**2) - 40*d**2*sin(a + b*x)**3/(27*b**3) - 14*d**2*sin(a +
b*x)*cos(a + b*x)**2/(9*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3
)*cos(a)**3, True))
```

Giac [A] time = 1.20051, size = 185, normalized size = 1.5

$$\frac{\left(bd^2x + bcd\right)\cos(3\,bx + 3\,a)}{18\,b^3} + \frac{3\left(bd^2x + bcd\right)\cos(bx+a)}{2\,b^3} + \frac{\left(9\,b^2d^2x^2 + 18\,b^2cdx + 9\,b^2c^2 - 2\,d^2\right)\sin(3\,bx + 3\,a)}{108\,b^3} + \frac{3\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/18*(b*d^2*x + b*c*d)*cos(3*b*x + 3*a)/b^3 + 3/2*(b*d^2*x + b*c*d)*cos(b*x
 + a)/b^3 + 1/108*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*sin(3*
b*x + 3*a)/b^3 + 3/4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(b*x
+ a)/b^3
```

3.19 $\int (c+dx)\cos^3(a+bx)\,dx$

Optimal. Leaf size=75

$$\frac{d\cos^3(a+bx)}{9b^2}+\frac{2d\cos(a+bx)}{3b^2}+\frac{2(c+dx)\sin(a+bx)}{3b}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}$$

```
[Out] (2*d*Cos[a + b*x])/(3*b^2) + (d*Cos[a + b*x]^3)/(9*b^2) + (2*(c + d*x)*Sin[
a + b*x])/(3*b) + ((c + d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(3*b)
```

Rubi [A] time = 0.0417872, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3310, 3296, 2638}

$$\frac{d\cos^3(a+bx)}{9b^2}+\frac{2d\cos(a+bx)}{3b^2}+\frac{2(c+dx)\sin(a+bx)}{3b}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^3,x]
```

```
[Out] (2*d*Cos[a + b*x])/(3*b^2) + (d*Cos[a + b*x]^3)/(9*b^2) + (2*(c + d*x)*Sin[
a + b*x])/(3*b) + ((c + d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(3*b)
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cos^3(a+bx)\,dx &= \frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\cos^2(a+bx)\sin(a+bx)}{3b}+\frac{2}{3}\int (c+dx)\cos(a+bx)\,dx\\ &= \frac{d\cos^3(a+bx)}{9b^2}+\frac{2(c+dx)\sin(a+bx)}{3b}+\frac{(c+dx)\cos^2(a+bx)\sin(a+bx)}{3b}-\frac{(2d)\int \sin(\\ &= \frac{2d\cos(a+bx)}{3b^2}+\frac{d\cos^3(a+bx)}{9b^2}+\frac{2(c+dx)\sin(a+bx)}{3b}+\frac{(c+dx)\cos^2(a+bx)\sin(a+}{3b}\end{aligned}$$

Mathematica [A] time = 0.164027, size = 52, normalized size = 0.69

$$\frac{3b(c+dx)(9\sin(a+bx)+\sin(3(a+bx)))+27d\cos(a+bx)+d\cos(3(a+bx))}{36b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^3,x]
```

```
[Out] (27*d*Cos[a + b*x] + d*Cos[3*(a + b*x)] + 3*b*(c + d*x)*(9*Sin[a + b*x] + S
in[3*(a + b*x)]))/(36*b^2)
```

Maple [A] time = 0.027, size = 95, normalized size = 1.3

$$\frac{1}{b}\left(\frac{d}{b}\left(\frac{(bx+a)\left(2+(\cos(bx+a))^2\right)\sin(bx+a)}{3}+\frac{(\cos(bx+a))^3}{9}+\frac{2\cos(bx+a)}{3}\right)-\frac{da\left(2+(\cos(bx+a))^2\right)\sin(bx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b*d*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/9*cos(b*x+a)^3+2/3*co
s(b*x+a))-1/3*a*d/b*(2+cos(b*x+a)^2)*sin(b*x+a)+1/3*c*(2+cos(b*x+a)^2)*sin(
b*x+a))
```

Maxima [A] time = 0.996978, size = 139, normalized size = 1.85

$$-\frac{12\left(\sin(bx+a)^3-3\sin(bx+a)\right)c-\frac{12\left(\sin(bx+a)^3-3\sin(bx+a)\right)ad}{b}-\frac{(3(bx+a)\sin(3bx+3a)+27(bx+a)\sin(bx+a)+\cos(3bx+3a)+27\cos(bx+}{b}}{36b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/36*(12*(sin(b*x + a)^3 - 3*sin(b*x + a))*c - 12*(sin(b*x + a)^3 - 3*sin(
b*x + a))*a*d/b - (3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x + a)*sin(b*x + a)
 + cos(3*b*x + 3*a) + 27*cos(b*x + a))*d/b)/b
```

Fricas [A] time = 1.37993, size = 153, normalized size = 2.04

$$\frac{d\cos(bx+a)^3+6d\cos(bx+a)+3\left(2bdx+(bdx+bc)\cos(bx+a)^2+2bc\right)\sin(bx+a)}{9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/9*(d*cos(b*x + a)^3 + 6*d*cos(b*x + a) + 3*(2*b*d*x + (b*d*x + b*c)*cos(b
*x + a)^2 + 2*b*c)*sin(b*x + a))/b^2
```

Sympy [A] time = 1.18023, size = 126, normalized size = 1.68

$$\begin{cases}\frac{2c\sin^3(a+bx)}{3b}+\frac{c\sin(a+bx)\cos^2(a+bx)}{b}+\frac{2dx\sin^3(a+bx)}{3b}+\frac{dx\sin(a+bx)\cos^2(a+bx)}{b}+\frac{2d\sin^2(a+bx)\cos(a+bx)}{3b^2}+\frac{7d\cos^3(a+bx)}{9b^2} & \text{for } b\neq 0\\ \left(cx+\frac{dx^2}{2}\right)\cos^3(a) & \text{otherwis}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**3,x)
```

```
[Out] Piecewise((2*c*sin(a + b*x)**3/(3*b) + c*sin(a + b*x)*cos(a + b*x)**2/b + 2
*d*x*sin(a + b*x)**3/(3*b) + d*x*sin(a + b*x)*cos(a + b*x)**2/b + 2*d*sin(a
 + b*x)**2*cos(a + b*x)/(3*b**2) + 7*d*cos(a + b*x)**3/(9*b**2), Ne(b, 0)),
 ((c*x + d*x**2/2)*cos(a)**3, True))
```

Giac [A] time = 1.19975, size = 93, normalized size = 1.24

$$\frac{d\cos\left(3\,bx+3\,a\right)}{36\,b^2}+\frac{3\,d\cos\left(bx+a\right)}{4\,b^2}+\frac{\left(bdx+bc\right)\sin\left(3\,bx+3\,a\right)}{12\,b^2}+\frac{3\,\left(bdx+bc\right)\sin\left(bx+a\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/36*d*cos(3*b*x + 3*a)/b^2 + 3/4*d*cos(b*x + a)/b^2 + 1/12*(b*d*x + b*c)*s
in(3*b*x + 3*a)/b^2 + 3/4*(b*d*x + b*c)*sin(b*x + a)/b^2
```

3.20 $\int \frac{\cos^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=121

$$\frac{3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}-\frac{3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\sin\left(3a\right.}{}$$

```
[Out] (3*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d) + (Cos[3*a - (3*b*c)/
d]*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d) - (3*Sin[a - (b*c)/d]*SinIntegral[
(b*c)/d + b*x])/(4*d) - (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x
])/(4*d)
```

Rubi [A] time = 0.244037, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3312, 3303, 3299, 3302}

$$\frac{3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}-\frac{3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\sin\left(3a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/(c + d*x),x]
```

```
[Out] (3*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d) + (Cos[3*a - (3*b*c)/
d]*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d) - (3*Sin[a - (b*c)/d]*SinIntegral[
(b*c)/d + b*x])/(4*d) - (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x
])/(4*d)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)}{c+dx}\,dx = \int \left(\frac{3\cos(a+bx)}{4(c+dx)} + \frac{\cos(3a+3bx)}{4(c+dx)}\right)dx$$
$$= \frac{1}{4}\int \frac{\cos(3a+3bx)}{c+dx}\,dx + \frac{3}{4}\int \frac{\cos(a+bx)}{c+dx}\,dx$$
$$= \frac{1}{4}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx + \frac{1}{4}\left(3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx - \frac{1}{4}\sin\left(3a-\right.$$
$$= \frac{3\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{4d} + \frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{4d} - \frac{3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d} - \frac{\sin\left(\right.}{}$$

Mathematica [A] time = 0.262145, size = 103, normalized size = 0.85

$$\frac{3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)-3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)-\sin\left(3\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/(c + d*x),x]
```

```
[Out] (3*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] + Cos[3*a - (3*b*c)/d]*CosInte
gral[(3*b*(c + d*x))/d] - 3*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - Sin
[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d)
```

Maple [A] time = 0.03, size = 166, normalized size = 1.4

$$\frac{1}{b}\left(\frac{b}{12}\left(3\frac{1}{d}\text{Si}\left(3bx+3a+3\frac{-da+cb}{d}\right)\sin\left(3\frac{-da+cb}{d}\right)+3\frac{1}{d}\text{Ci}\left(3bx+3a+3\frac{-da+cb}{d}\right)\cos\left(3\frac{-da+cb}{d}\right)\right)+\frac{3b}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/(d*x+c),x)
```

```
[Out] 1/b*(1/12*b*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*
x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d)+3/4*b*(Si(b*x+a+(-a*d+b*c)/d)*
sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d))
```

Maxima [C] time = 1.34695, size = 373, normalized size = 3.08

$$-\frac{3b\left(E_1\left(\frac{ibc+i(bx+a)d-iad}{d}\right)+E_1\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b\left(E_1\left(\frac{3ibc+3i(bx+a)d-3iad}{d}\right)+E_1\left(-\frac{3ibc+3i(bx+a)d-3iad}{d}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*(3*b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integ
ral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b*(exp_
integral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(1,
-(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b*(3*I*e
```

```
xp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - 3*I*exp_integral_e(1,
 -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b*(I*exp_integr
al_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_integral_e(1, -(3*
I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d))/(b*d)
```

Fricas [A] time = 1.32774, size = 406, normalized size = 3.36

$$\frac{3\left(\text{Ci}\left(\frac{bdx+bc}{d}\right)+\text{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left(\text{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+\text{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)-2\,\sin\left(-\frac{3\,(bc-ad)}{d}\right)\text{Si}\left(\frac{3\,(}{}\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/8*(3*(cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*cos
(-(b*c - a*d)/d) + (cos_integral(3*(b*d*x + b*c)/d) + cos_integral(-3*(b*d*
x + b*c)/d))*cos(-3*(b*c - a*d)/d) - 2*sin(-3*(b*c - a*d)/d)*sin_integral(3
*(b*d*x + b*c)/d) - 6*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)**3/(c + d*x), x)
```

Giac [C] time = 2.0879, size = 8201, normalized size = 67.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/8*(real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan
(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*real_part(cos_integral(b*x + b*c/d))*tan
(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*real_part(cos_
integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a
)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*imag_part(cos_integral(b*x + b*c/
d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*imag_part
(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d) - 12*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*
tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_part(cos
_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1
/2*b*c/d)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*t
an(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 6*imag_part(cos_integral(b*x + b*c/d))*tan
```

```
(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*imag_part(cos_in
tegral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + 12*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c
/d)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2
*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integr
al(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*
c/d)^2*tan(1/2*b*c/d)^2 + real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*
a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*real_part(cos_integral(b*x + b*c/d))
*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*real_part(cos_integral(-b*x
 - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + real_part(cos_integ
ral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 12*real
_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d) + 12*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1
/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - real_part(cos_integral(3*b*x + 3*b*
c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*real_part(cos_integral
(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*real_part(cos
_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - real_
part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/
d)^2 + 4*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*t
an(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 4*real_part(cos_integral(-3*b*x - 3*b*c/d)
)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + real_part(cos_i
ntegral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 -
3*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/
2*b*c/d)^2 - 3*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/
2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(3*b*x + 3
*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*real_part(cos_i
ntegral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*re
al_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b
*c/d)^2 - real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*
c/d)^2*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/
2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 2*imag_part(cos_integral(-3*b*x - 3*b*
c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) - 4*sin_integral(3*(b*d*x +
b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) - 6*imag_part(cos_integral
(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 6*imag_part(cos_i
ntegral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 12*sin_in
tegral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 2*imag_p
art(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2
 - 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(
3/2*b*c/d)^2 + 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2 - 6*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/
2*a)^2*tan(1/2*b*c/d) + 6*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^
2*tan(1/2*a)^2*tan(1/2*b*c/d) - 12*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)
^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*imag_part(cos_integral(b*x + b*c/d))*tan
(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*imag_part(cos_integral(-b*x -
 b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*sin_integral((b*
d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*imag_part(co
s_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*i
mag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d) - 12*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d) + 6*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a
)*tan(1/2*b*c/d)^2 - 6*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*t
an(1/2*a)*tan(1/2*b*c/d)^2 + 12*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*
tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*ta
n(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(-3*b*x -
3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 4*sin_integral(3*(b*d*
x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 2*imag_part(cos_inte
gral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*ima
```

```
g_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*
b*c/d)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c/d)*ta
n(1/2*b*c/d)^2 + 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*ta
n(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))
*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 4*sin_integral(3*(b*d*x + b
*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integ
ral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*imag
_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b
*c/d)^2 + 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d)^2 + 6*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d)^2 - 6*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*
a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*sin_integral((b*d*x + b*c)/d)*tan
(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(3*b*x +
3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 - 3*real_part(cos_integral(b*x + b*c/d)
)*tan(3/2*a)^2*tan(1/2*a)^2 - 3*real_part(cos_integral(-b*x - b*c/d))*tan(3
/2*a)^2*tan(1/2*a)^2 - real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)
^2*tan(1/2*a)^2 + 4*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan
(1/2*a)^2*tan(3/2*b*c/d) + 4*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(
3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) + real_part(cos_integral(3*b*x + 3*b*c/d
))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 3*real_part(cos_integral(b*x + b*c/d))*t
an(3/2*a)^2*tan(3/2*b*c/d)^2 + 3*real_part(cos_integral(-b*x - b*c/d))*tan(
3/2*a)^2*tan(3/2*b*c/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3
/2*a)^2*tan(3/2*b*c/d)^2 - real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2
*a)^2*tan(3/2*b*c/d)^2 - 3*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^
2*tan(3/2*b*c/d)^2 - 3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*t
an(3/2*b*c/d)^2 - real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2 + 12*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1
/2*a)*tan(1/2*b*c/d) + 12*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^
2*tan(1/2*a)*tan(1/2*b*c/d) + 12*real_part(cos_integral(b*x + b*c/d))*tan(1
/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*real_part(cos_integral(-b*x - b*
c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - real_part(cos_integral(3
*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 3*real_part(cos_integral(b
*x + b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 3*real_part(cos_integral(-b*x
- b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(-3*b*x - 3
*b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + real_part(cos_integral(3*b*x + 3*b
*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*real_part(cos_integral(b*x + b*c/d
))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*real_part(cos_integral(-b*x - b*c/d))*
tan(1/2*a)^2*tan(1/2*b*c/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*t
an(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*real_part(cos_integral(3*b*x + 3*b*c/d))*t
an(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 4*real_part(cos_integral(-3*b*x
 - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - real_part(cos_int
egral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*real_part(cos
_integral(b*x + b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*real_part(cos
_integral(-b*x - b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_
integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*imag_part
(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a) + 6*imag_part(cos_integ
ral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a) - 12*sin_integral((b*d*x + b*c)/
d)*tan(3/2*a)^2*tan(1/2*a) - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan
(3/2*a)*tan(1/2*a)^2 + 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*
a)*tan(1/2*a)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2
 - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d) +
 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d) -
4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c/d) + 2*imag_part
(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d) - 2*imag_part(c
os_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d) + 4*sin_integral
(3*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d) + 2*imag_part(cos_integral(
3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2 - 2*imag_part(cos_integral(-3
*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2 + 4*sin_integral(3*(b*d*x + b*
c)/d)*tan(3/2*a)*tan(3/2*b*c/d)^2 - 6*imag_part(cos_integral(b*x + b*c/d))*
```

```
tan(1/2*a)*tan(3/2*b*c/d)^2 + 6*imag_part(cos_integral(-b*x - b*c/d))*tan(1
/2*a)*tan(3/2*b*c/d)^2 - 12*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(3/
2*b*c/d)^2 + 6*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*b*
c/d) - 6*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)
+ 12*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*b*c/d) - 6*imag_par
t(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*imag_part(cos_
integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 12*sin_integral((b*d*
x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*imag_part(cos_integral(b*x + b*
c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*imag_part(cos_integral(-b*x - b*c
/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*sin_integral((b*d*x + b*c)/d)*tan
(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*t
an(3/2*a)*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*ta
n(3/2*a)*tan(1/2*b*c/d)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*ta
n(1/2*b*c/d)^2 + 6*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*
b*c/d)^2 - 6*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d
)^2 + 12*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d)^2 + 2*imag
_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*im
ag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 4
*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - real_par
t(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2 + 3*real_part(cos_integral(b*
x + b*c/d))*tan(3/2*a)^2 + 3*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*
a)^2 - real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2 + real_part(c
os_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2 - 3*real_part(cos_integral(b*x +
 b*c/d))*tan(1/2*a)^2 - 3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^
2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2 + 4*real_part(co
s_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) + 4*real_part(cos_in
tegral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) - real_part(cos_integra
l(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)^2 + 3*real_part(cos_integral(b*x + b*c/d
))*tan(3/2*b*c/d)^2 + 3*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*c/d
)^2 - real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2 + 12*real_
part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + 12*real_part(co
s_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + real_part(cos_integra
l(3*b*x + 3*b*c/d))*tan(1/2*b*c/d)^2 - 3*real_part(cos_integral(b*x + b*c/d
))*tan(1/2*b*c/d)^2 - 3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d
)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*c/d)^2 - 2*imag_p
art(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a) + 2*imag_part(cos_integral(-3
*b*x - 3*b*c/d))*tan(3/2*a) - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)
- 6*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a) + 6*imag_part(cos_integ
ral(-b*x - b*c/d))*tan(1/2*a) - 12*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)
 + 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d) - 2*imag_part(
cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d) + 4*sin_integral(3*(b*d*x +
b*c)/d)*tan(3/2*b*c/d) + 6*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c
/d) - 6*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d) + 12*sin_integ
ral((b*d*x + b*c)/d)*tan(1/2*b*c/d) + real_part(cos_integral(3*b*x + 3*b*c/
d)) + 3*real_part(cos_integral(b*x + b*c/d)) + 3*real_part(cos_integral(-b*
x - b*c/d)) + real_part(cos_integral(-3*b*x - 3*b*c/d)))/(d*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(1/2*a)^2*
tan(3/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(3/2
*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*tan(1/2*a)^2*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(1/2*a)^2 + d*tan(3/2*a)^2*tan(3/2*b*c/
d)^2 + d*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(1/2*b*c/d)^2 +
d*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*t
an(3/2*a)^2 + d*tan(1/2*a)^2 + d*tan(3/2*b*c/d)^2 + d*tan(1/2*b*c/d)^2 + d)
```

3.21 $\int \frac{\cos^3(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=145

$$-\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{3b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{3b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{3b}{}$$

```
[Out] -(Cos[a + b*x]^3/(d*(c + d*x))) - (3*b*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3
*a - (3*b*c)/d])/(4*d^2) - (3*b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d]
)/(4*d^2) - (3*b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(4*d^2) - (3*
b*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(4*d^2)
```

Rubi [A] time = 0.225448, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3313, 3303, 3299, 3302}

$$-\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{3b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{3b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{3b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/(c + d*x)^2,x]
```



```
[Out] -(Cos[a + b*x]^3/(d*(c + d*x))) - (3*b*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3
*a - (3*b*c)/d])/(4*d^2) - (3*b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d]
)/(4*d^2) - (3*b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(4*d^2) - (3*
b*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(4*d^2)
```


Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)}{(c+dx)^2}\,dx = -\frac{\cos^3(a+bx)}{d(c+dx)} + \frac{(3b)\int\left(-\frac{\sin(a+bx)}{4(c+dx)} - \frac{\sin(3a+3bx)}{4(c+dx)}\right)dx}{d}$$
$$= -\frac{\cos^3(a+bx)}{d(c+dx)} - \frac{(3b)\int\frac{\sin(a+bx)}{c+dx}\,dx}{4d} - \frac{(3b)\int\frac{\sin(3a+3bx)}{c+dx}\,dx}{4d}$$
$$= -\frac{\cos^3(a+bx)}{d(c+dx)} - \frac{\left(3b\cos\left(3a-\frac{3bc}{d}\right)\right)\int\frac{\sin\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx}{4d} - \frac{\left(3b\cos\left(a-\frac{bc}{d}\right)\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{4d} - \frac{(3b\,\text{s}}{}$$
$$= -\frac{\cos^3(a+bx)}{d(c+dx)} - \frac{3b\text{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d^2} - \frac{3b\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{4d^2} - \frac{3b\cos\left(a-\frac{bc}{d}\right.}{4d}$$

Mathematica [A] time = 0.697888, size = 200, normalized size = 1.38

$$-\frac{3b(c+dx)\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)+3b(c+dx)\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+3bc\cos\left(a-\frac{bc}{d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] -(3*d*Cos[a + b*x] + d*Cos[3*(a + b*x)] + 3*b*(c + d*x)*CosIntegral[(3*b*(c
 + d*x))/d]*Sin[3*a - (3*b*c)/d] + 3*b*(c + d*x)*CosIntegral[b*(c/d + x)]*S
in[a - (b*c)/d] + 3*b*c*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 3*b*d*x
*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 3*b*c*Cos[3*a - (3*b*c)/d]*Sin
Integral[(3*b*(c + d*x))/d] + 3*b*d*x*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b
*(c + d*x))/d])/(4*d^2*(c + d*x))
```

Maple [A] time = 0.048, size = 242, normalized size = 1.7

$$\frac{1}{b}\left(\frac{b^2}{12}\left(-3\frac{\cos(3bx+3a)}{((bx+a)d-da+cb)d} - 3\frac{1}{d}\left(3\frac{1}{d}\text{Si}\left(3bx+3a+3\frac{-da+cb}{d}\right)\cos\left(3\frac{-da+cb}{d}\right) - 3\frac{1}{d}\text{Ci}\left(3bx+3a+3\frac{-}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] 1/b*(1/12*b^2*(-3*cos(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d-3*(3*Si(3*b*x+3*a+3*
(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-
a*d+b*c)/d)/d)/d)+3/4*b^2*(-cos(b*x+a)/((b*x+a)*d-d*a+c*b)/d-(Si(b*x+a+(-a*
d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d
))
```

Maxima [C] time = 1.61389, size = 410, normalized size = 2.83

$$-\frac{24576\,b^2\left(E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+8192\,b^2\left(E_2\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+E_2\left(-\frac{3i\,bc+3i}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/65536*(24576*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
 exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
 + 8192*b^2*(exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + e
xp_integral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a
*d)/d) - b^2*(24576*I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d)
- 24576*I*exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c
- a*d)/d) - b^2*(8192*I*exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*
a*d)/d) - 8192*I*exp_integral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d
))*sin(-3*(b*c - a*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Fricas [A] time = 1.43595, size = 571, normalized size = 3.94

$$-\frac{8\,d\cos\left(bx+a\right)^{3}+6\left(bdx+bc\right)\cos\left(-\frac{3\left(bc-ad\right)}{d}\right)\mathrm{Si}\left(\frac{3\left(bdx+bc\right)}{d}\right)+6\left(bdx+bc\right)\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)+3\left(\left(bdx+bc\right)\mathrm{Ci}\left(\right.\right.}{8\left(d^{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(8*d*cos(b*x + a)^3 + 6*(b*d*x + b*c)*cos(-3*(b*c - a*d)/d)*sin_integr
al(3*(b*d*x + b*c)/d) + 6*(b*d*x + b*c)*cos(-(b*c - a*d)/d)*sin_integral((b
*d*x + b*c)/d) + 3*((b*d*x + b*c)*cos_integral((b*d*x + b*c)/d) + (b*d*x +
b*c)*cos_integral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/d) + 3*((b*d*x + b*c)
*cos_integral(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-3*(b*d*x + b
*c)/d))*sin(-3*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^{3}\left(a+bx\right)}{\left(c+dx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)**3/(c + d*x)**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.22 $\int \frac{\cos^3(a+bx)}{(c+dx)^3} dx$

Optimal. Leaf size=184

$$-\frac{3b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}+\frac{3b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right.}{8d^3}$$

```
[Out] -Cos[a + b*x]^3/(2*d*(c + d*x)^2) - (3*b^2*Cos[a - (b*c)/d]*CosIntegral[(b*
c)/d + b*x])/(8*d^3) - (9*b^2*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/d +
3*b*x])/(8*d^3) + (3*b*Cos[a + b*x]^2*Sin[a + b*x])/(2*d^2*(c + d*x)) + (3*
b^2*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^3) + (9*b^2*Sin[3*a -
 (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^3)
```

Rubi [A] time = 0.344821, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3314, 3303, 3299, 3302, 3312}

$$-\frac{3b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}+\frac{3b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right.}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/(c + d*x)^3,x]
```



```
[Out] -Cos[a + b*x]^3/(2*d*(c + d*x)^2) - (3*b^2*Cos[a - (b*c)/d]*CosIntegral[(b*
c)/d + b*x])/(8*d^3) - (9*b^2*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/d +
3*b*x])/(8*d^3) + (3*b*Cos[a + b*x]^2*Sin[a + b*x])/(2*d^2*(c + d*x)) + (3*
b^2*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^3) + (9*b^2*Sin[3*a -
 (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^3)
```


Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{\cos^3(a+bx)}{(c+dx)^3}\,dx = -\frac{\cos^3(a+bx)}{2d(c+dx)^2}+\frac{3b\cos^2(a+bx)\sin(a+bx)}{2d^2(c+dx)}+\frac{\left(3b^2\right)\int\frac{\cos(a+bx)}{c+dx}\,dx}{d^2}-\frac{\left(9b^2\right)\int\frac{\cos^3(a+bx)}{c+dx}\,dx}{2d^2}$$

$$= -\frac{\cos^3(a+bx)}{2d(c+dx)^2}+\frac{3b\cos^2(a+bx)\sin(a+bx)}{2d^2(c+dx)}-\frac{\left(9b^2\right)\int\left(\frac{3\cos(a+bx)}{4(c+dx)}+\frac{\cos(3a+3bx)}{4(c+dx)}\right)dx}{2d^2}+\frac{\left(3b^2\cos\left(a-\right.\right.}{}$$

$$= -\frac{\cos^3(a+bx)}{2d(c+dx)^2}+\frac{3b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{Ci}\left(\frac{bc}{d}+bx\right)}{d^3}+\frac{3b\cos^2(a+bx)\sin(a+bx)}{2d^2(c+dx)}-\frac{3b^2\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\right.}{d^3}$$

$$= -\frac{\cos^3(a+bx)}{2d(c+dx)^2}+\frac{3b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{Ci}\left(\frac{bc}{d}+bx\right)}{d^3}+\frac{3b\cos^2(a+bx)\sin(a+bx)}{2d^2(c+dx)}-\frac{3b^2\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\right.}{d^3}$$

$$= -\frac{\cos^3(a+bx)}{2d(c+dx)^2}-\frac{3b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{Ci}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\operatorname{Ci}\left(\frac{3bc}{d}+3bx\right)}{8d^3}+\frac{3b\cos^2(a+b}{2d^2(c}$$

Mathematica [A] time = 0.851678, size = 221, normalized size = 1.2

$$\frac{-6b^2(c+dx)^2\left(\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+3\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)-\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/(c + d*x)^3,x]
```

```
[Out] (6*d*Cos[b*x]*(-(d*Cos[a]) + b*(c + d*x)*Sin[a]) + 2*d*Cos[3*b*x]*(-(d*Cos[
3*a]) + 3*b*(c + d*x)*Sin[3*a]) + 6*d*(b*(c + d*x)*Cos[a] + d*Sin[a])*Sin[b
*x] + 2*d*(3*b*(c + d*x)*Cos[3*a] + d*Sin[3*a])*Sin[3*b*x] - 6*b^2*(c + d*x
)^2*(Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] + 3*Cos[3*a - (3*b*c)/d]*Cos
Integral[(3*b*(c + d*x))/d] - Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - 3
*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d]))/(16*d^3*(c + d*x)^2)
```

Maple [A] time = 0.035, size = 311, normalized size = 1.7

$$\frac{1}{b}\left(\frac{b^3}{12}\left(-\frac{3\cos(3bx+3a)}{2\left((bx+a)d-da+cb\right)^2d}-\frac{3}{2d}\left(-3\frac{\sin(3bx+3a)}{\left((bx+a)d-da+cb\right)d}+3\frac{1}{d}\left(3\frac{1}{d}\operatorname{Si}\left(3bx+3a+3\frac{-da+cb}{d}\right)\sin\left(3\frac{-da}{}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/(d*x+c)^3,x)
```

```
[Out] 1/b*(1/12*b^3*(-3/2*cos(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)^2/d-3/2*(-3*sin(3*b*
x+3*a)/((b*x+a)*d-d*a+c*b)/d+3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+
b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d)/d)/d)+3/4*b
^3*(-1/2*cos(b*x+a)/((b*x+a)*d-d*a+c*b)^2/d-1/2*(-sin(b*x+a)/((b*x+a)*d-d*a
+c*b)/d+(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*
cos((-a*d+b*c)/d)/d)/d)/d))
```

Maxima [C] time = 1.9622, size = 458, normalized size = 2.49

$$-\frac{24576\,b^3\left(E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+8192\,b^3\left(E_3\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+E_3\left(-\frac{3i\,bc+3i}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/65536*(24576*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
 exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
 + 8192*b^3*(exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + e
xp_integral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a
*d)/d) - b^3*(24576*I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d)
- 24576*I*exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c
- a*d)/d) - b^3*(8192*I*exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*
a*d)/d) - 8192*I*exp_integral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d
))*sin(-3*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2
*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

Fricas [B] time = 1.61382, size = 861, normalized size = 4.68

$$-\frac{8\,d^2\cos\left(bx+a\right)^3-24\left(bd^2x+bcd\right)\cos\left(bx+a\right)^2\sin\left(bx+a\right)-18\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bdx}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(8*d^2*cos(b*x + a)^3 - 24*(b*d^2*x + b*c*d)*cos(b*x + a)^2*sin(b*x +
 a) - 18*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-3*(b*c - a*d)/d)*sin_in
tegral(3*(b*d*x + b*c)/d) - 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-(b
*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 3*((b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2)*cos_integral((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2)*cos_integral(-(b*d*x + b*c)/d))*cos(-(b*c - a*d)/d) + 9*((b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*cos_integral(-3*(b*d*x + b*c)/d))*cos(-3*(b*c - a*d)
/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^3(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] Integral(cos(a + b*x)**3/(c + d*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.23 $\int x^3 \cos^4(a + bx)\,dx$

Optimal. Leaf size=172

$$\frac{3x^2\cos^4(a+bx)}{16b^2}+\frac{9x^2\cos^2(a+bx)}{16b^2}-\frac{3\cos^4(a+bx)}{128b^4}-\frac{45\cos^2(a+bx)}{128b^4}-\frac{3x\sin(a+bx)\cos^3(a+bx)}{32b^3}-\frac{45x\sin(a+}{6}$$

```
[Out] (-45*x^2)/(128*b^2) + (3*x^4)/32 - (45*Cos[a + b*x]^2)/(128*b^4) + (9*x^2*C
os[a + b*x]^2)/(16*b^2) - (3*Cos[a + b*x]^4)/(128*b^4) + (3*x^2*Cos[a + b*x
]^4)/(16*b^2) - (45*x*Cos[a + b*x]*Sin[a + b*x])/(64*b^3) + (3*x^3*Cos[a +
b*x]*Sin[a + b*x])/(8*b) - (3*x*Cos[a + b*x]^3*Sin[a + b*x])/(32*b^3) + (x^
3*Cos[a + b*x]^3*Sin[a + b*x])/(4*b)
```

Rubi [A] time = 0.154006, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3311, 30, 3310}

$$\frac{3x^2\cos^4(a+bx)}{16b^2}+\frac{9x^2\cos^2(a+bx)}{16b^2}-\frac{3\cos^4(a+bx)}{128b^4}-\frac{45\cos^2(a+bx)}{128b^4}-\frac{3x\sin(a+bx)\cos^3(a+bx)}{32b^3}-\frac{45x\sin(a+}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[a + b*x]^4,x]
```

```
[Out] (-45*x^2)/(128*b^2) + (3*x^4)/32 - (45*Cos[a + b*x]^2)/(128*b^4) + (9*x^2*C
os[a + b*x]^2)/(16*b^2) - (3*Cos[a + b*x]^4)/(128*b^4) + (3*x^2*Cos[a + b*x
]^4)/(16*b^2) - (45*x*Cos[a + b*x]*Sin[a + b*x])/(64*b^3) + (3*x^3*Cos[a +
b*x]*Sin[a + b*x])/(8*b) - (3*x*Cos[a + b*x]^3*Sin[a + b*x])/(32*b^3) + (x^
3*Cos[a + b*x]^3*Sin[a + b*x])/(4*b)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int x^3\cos^4(a+bx)\,dx = \frac{3x^2\cos^4(a+bx)}{16b^2} + \frac{x^3\cos^3(a+bx)\sin(a+bx)}{4b} + \frac{3}{4}\int x^3\cos^2(a+bx)\,dx - \frac{3\int x\cos^4(a+bx)\,dx}{8b^2}$$
$$= \frac{9x^2\cos^2(a+bx)}{16b^2} - \frac{3\cos^4(a+bx)}{128b^4} + \frac{3x^2\cos^4(a+bx)}{16b^2} + \frac{3x^3\cos(a+bx)\sin(a+bx)}{8b} - \frac{3x\cos^3(a}{}$$
$$= \frac{3x^4}{32} - \frac{45\cos^2(a+bx)}{128b^4} + \frac{9x^2\cos^2(a+bx)}{16b^2} - \frac{3\cos^4(a+bx)}{128b^4} + \frac{3x^2\cos^4(a+bx)}{16b^2} - \frac{45x\cos(a+b}{64b}$$
$$= -\frac{45x^2}{128b^2} + \frac{3x^4}{32} - \frac{45\cos^2(a+bx)}{128b^4} + \frac{9x^2\cos^2(a+bx)}{16b^2} - \frac{3\cos^4(a+bx)}{128b^4} + \frac{3x^2\cos^4(a+bx)}{16b^2} - 45x$$

Mathematica [A] time = 0.414309, size = 100, normalized size = 0.58

$$\frac{4bx\left(32\left(2b^2x^2-3\right)\sin(2(a+bx))+\left(8b^2x^2-3\right)\sin(4(a+bx))+24b^3x^3\right)+192\left(2b^2x^2-1\right)\cos(2(a+bx))+3\left(8b^2x^2-\right.}{1024b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[a + b*x]^4,x]
```

```
[Out] (192*(-1 + 2*b^2*x^2)*Cos[2*(a + b*x)] + 3*(-1 + 8*b^2*x^2)*Cos[4*(a + b*x)
] + 4*b*x*(24*b^3*x^3 + 32*(-3 + 2*b^2*x^2)*Sin[2*(a + b*x)] + (-3 + 8*b^2*
x^2)*Sin[4*(a + b*x)]))/(1024*b^4)
```

Maple [B] time = 0.056, size = 440, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(b*x+a)^4,x)
```

```
[Out] 1/b^4*((b*x+a)^3*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*
a)+3/16*(b*x+a)^2*cos(b*x+a)^4-3/8*(b*x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a
))*sin(b*x+a)+3/8*b*x+3/8*a)+45/128*(b*x+a)^2-3/128*cos(b*x+a)^4-9/128*cos(
b*x+a)^2+9/16*(b*x+a)^2*cos(b*x+a)^2-9/8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)
+1/2*b*x+1/2*a)+9/32*sin(b*x+a)^2-9/32*(b*x+a)^4-3*a*((b*x+a)^2*(1/4*(cos(b
*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+1/8*(b*x+a)*cos(b*x+a)^4-
1/32*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)-15/64*b*x-15/64*a+3/8*(b*x+a)
*cos(b*x+a)^2-3/16*cos(b*x+a)*sin(b*x+a)-1/4*(b*x+a)^3)+3*a^2*((b*x+a)*(1/4
*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)-3/16*(b*x+a)^2+1/1
6*cos(b*x+a)^4+3/16*cos(b*x+a)^2)-a^3*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*si
n(b*x+a)+3/8*b*x+3/8*a))
```

Maxima [A] time = 1.05618, size = 409, normalized size = 2.38

$$96\,(bx+a)^4 - 32\,(12\,bx+12\,a+\sin(4\,bx+4\,a)+8\,\sin(2\,bx+2\,a))a^3 + 24\left(24\,(bx+a)^2+4\,(bx+a)\sin(4\,bx+4\,a)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/1024*(96*(b*x + a)^4 - 32*(12*b*x + 12*a + sin(4*b*x + 4*a) + 8*sin(2*b*x
 + 2*a))*a^3 + 24*(24*(b*x + a)^2 + 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x
+ a)*sin(2*b*x + 2*a) + cos(4*b*x + 4*a) + 16*cos(2*b*x + 2*a))*a^2 - 12*(3
2*(b*x + a)^3 + 4*(b*x + a)*cos(4*b*x + 4*a) + 64*(b*x + a)*cos(2*b*x + 2*a
) + (8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) + 32*(2*(b*x + a)^2 - 1)*sin(2*b*x
 + 2*a))*a + 3*(8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) + 192*(2*(b*x + a)^2 -
1)*cos(2*b*x + 2*a) + 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*b*x + 4*a) + 12
8*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b*x + 2*a))/b^4
```

Fricas [A] time = 1.41015, size = 271, normalized size = 1.58

$$\frac{12\,b^4x^4 + 3\left(8\,b^2x^2 - 1\right)\cos\left(bx + a\right)^4 - 45\,b^2x^2 + 9\left(8\,b^2x^2 - 5\right)\cos\left(bx + a\right)^2 + 2\left(2\left(8\,b^3x^3 - 3\,bx\right)\cos\left(bx + a\right)^3 + 3\left(\right.\right.}{128\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(12*b^4*x^4 + 3*(8*b^2*x^2 - 1)*cos(b*x + a)^4 - 45*b^2*x^2 + 9*(8*b^
2*x^2 - 5)*cos(b*x + a)^2 + 2*(2*(8*b^3*x^3 - 3*b*x)*cos(b*x + a)^3 + 3*(8*
b^3*x^3 - 15*b*x)*cos(b*x + a))*sin(b*x + a))/b^4
```

Sympy [A] time = 9.4835, size = 253, normalized size = 1.47

$$\begin{cases} \frac{3x^4\sin^4(a+bx)}{32} + \frac{3x^4\sin^2(a+bx)\cos^2(a+bx)}{16} + \frac{3x^4\cos^4(a+bx)}{32} + \frac{3x^3\sin^3(a+bx)\cos(a+bx)}{8b} + \frac{5x^3\sin(a+bx)\cos^3(a+bx)}{8b} - \frac{45x^2\sin^4(a+bx)}{128b^2} - \\ \frac{x^4\cos^4(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x**4*sin(a + b*x)**4/32 + 3*x**4*sin(a + b*x)**2*cos(a + b*x)*
*2/16 + 3*x**4*cos(a + b*x)**4/32 + 3*x**3*sin(a + b*x)**3*cos(a + b*x)/(8*
b) + 5*x**3*sin(a + b*x)*cos(a + b*x)**3/(8*b) - 45*x**2*sin(a + b*x)**4/(1
28*b**2) - 9*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(64*b**2) + 51*x**2*cos(a
 + b*x)**4/(128*b**2) - 45*x*sin(a + b*x)**3*cos(a + b*x)/(64*b**3) - 51*x*
sin(a + b*x)*cos(a + b*x)**3/(64*b**3) + 45*sin(a + b*x)**4/(256*b**4) - 51
*cos(a + b*x)**4/(256*b**4), Ne(b, 0)), (x**4*cos(a)**4/4, True))
```

Giac [A] time = 1.11344, size = 146, normalized size = 0.85

$$\frac{3}{32}\,x^4 + \frac{3\left(8\,b^2x^2 - 1\right)\cos\left(4\,bx + 4\,a\right)}{1024\,b^4} + \frac{3\left(2\,b^2x^2 - 1\right)\cos\left(2\,bx + 2\,a\right)}{16\,b^4} + \frac{\left(8\,b^3x^3 - 3\,bx\right)\sin\left(4\,bx + 4\,a\right)}{256\,b^4} + \frac{\left(2\,b^3x^3 - 3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/32*x^4 + 3/1024*(8*b^2*x^2 - 1)*cos(4*b*x + 4*a)/b^4 + 3/16*(2*b^2*x^2 -
1)*cos(2*b*x + 2*a)/b^4 + 1/256*(8*b^3*x^3 - 3*b*x)*sin(4*b*x + 4*a)/b^4 +
1/8*(2*b^3*x^3 - 3*b*x)*sin(2*b*x + 2*a)/b^4
```

3.24 $\int x^2 \cos^4(a+bx)\,dx$

Optimal. Leaf size=134

$$\frac{x\cos^4(a+bx)}{8b^2}+\frac{3x\cos^2(a+bx)}{8b^2}-\frac{\sin(a+bx)\cos^3(a+bx)}{32b^3}-\frac{15\sin(a+bx)\cos(a+bx)}{64b^3}+\frac{x^2\sin(a+bx)\cos^3(a+bx)}{4b}$$

```
[Out] (-15*x)/(64*b^2) + x^3/8 + (3*x*Cos[a + b*x]^2)/(8*b^2) + (x*Cos[a + b*x]^4
)/(8*b^2) - (15*Cos[a + b*x]*Sin[a + b*x])/(64*b^3) + (3*x^2*Cos[a + b*x]*S
in[a + b*x])/(8*b) - (Cos[a + b*x]^3*Sin[a + b*x])/(32*b^3) + (x^2*Cos[a +
b*x]^3*Sin[a + b*x])/(4*b)
```

Rubi [A] time = 0.108289, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3311, 30, 2635, 8}

$$\frac{x\cos^4(a+bx)}{8b^2}+\frac{3x\cos^2(a+bx)}{8b^2}-\frac{\sin(a+bx)\cos^3(a+bx)}{32b^3}-\frac{15\sin(a+bx)\cos(a+bx)}{64b^3}+\frac{x^2\sin(a+bx)\cos^3(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[a + b*x]^4,x]
```

```
[Out] (-15*x)/(64*b^2) + x^3/8 + (3*x*Cos[a + b*x]^2)/(8*b^2) + (x*Cos[a + b*x]^4
)/(8*b^2) - (15*Cos[a + b*x]*Sin[a + b*x])/(64*b^3) + (3*x^2*Cos[a + b*x]*S
in[a + b*x])/(8*b) - (Cos[a + b*x]^3*Sin[a + b*x])/(32*b^3) + (x^2*Cos[a +
b*x]^3*Sin[a + b*x])/(4*b)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int x^2\cos^4(a+bx)\,dx = \frac{x\cos^4(a+bx)}{8b^2} + \frac{x^2\cos^3(a+bx)\sin(a+bx)}{4b} + \frac{3}{4}\int x^2\cos^2(a+bx)\,dx - \frac{\int\cos^4(a+bx)\,dx}{8b^2}$$
$$= \frac{3x\cos^2(a+bx)}{8b^2} + \frac{x\cos^4(a+bx)}{8b^2} + \frac{3x^2\cos(a+bx)\sin(a+bx)}{8b} - \frac{\cos^3(a+bx)\sin(a+bx)}{32b^3} +$$
$$= \frac{x^3}{8} + \frac{3x\cos^2(a+bx)}{8b^2} + \frac{x\cos^4(a+bx)}{8b^2} - \frac{15\cos(a+bx)\sin(a+bx)}{64b^3} + \frac{3x^2\cos(a+bx)\sin(a-}{8b}$$
$$= -\frac{15x}{64b^2} + \frac{x^3}{8} + \frac{3x\cos^2(a+bx)}{8b^2} + \frac{x\cos^4(a+bx)}{8b^2} - \frac{15\cos(a+bx)\sin(a+bx)}{64b^3} + \frac{3x^2\cos(a+b}{8}$$

Mathematica [A] time = 0.176424, size = 92, normalized size = 0.69

$$\frac{64b^2x^2\sin(2(a+bx)) + 8b^2x^2\sin(4(a+bx)) - 32\sin(2(a+bx)) - \sin(4(a+bx)) + 64bx\cos(2(a+bx)) + 4bx\cos(4(a}{256b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[a + b*x]^4,x]
```

```
[Out] (32*b^3*x^3 + 64*b*x*Cos[2*(a + b*x)] + 4*b*x*Cos[4*(a + b*x)] - 32*Sin[2*(
a + b*x)] + 64*b^2*x^2*Sin[2*(a + b*x)] - Sin[4*(a + b*x)] + 8*b^2*x^2*Sin[
4*(a + b*x)])/(256*b^3)
```

Maple [B] time = 0.029, size = 241, normalized size = 1.8

$$\frac{1}{b^3}\left((bx+a)^2\left(\frac{\sin(bx+a)}{4}\left((\cos(bx+a))^3 + \frac{3\cos(bx+a)}{2}\right) + \frac{3bx}{8} + \frac{3a}{8}\right) + \frac{(bx+a)(\cos(bx+a))^4}{8} - \frac{\sin(bx+a)}{32}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(b*x+a)^4,x)
```

```
[Out] 1/b^3*((b*x+a)^2*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*
a)+1/8*(b*x+a)*cos(b*x+a)^4-1/32*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)-1
5/64*b*x-15/64*a+3/8*(b*x+a)*cos(b*x+a)^2-3/16*cos(b*x+a)*sin(b*x+a)-1/4*(b
*x+a)^3-2*a*((b*x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+
3/8*a)-3/16*(b*x+a)^2+1/16*cos(b*x+a)^4+3/16*cos(b*x+a)^2)+a^2*(1/4*(cos(b*
x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a))
```

Maxima [A] time = 1.06702, size = 254, normalized size = 1.9

$$\frac{32(bx+a)^3 + 8(12bx+12a+\sin(4bx+4a)+8\sin(2bx+2a))a^2 - 4\left(24(bx+a)^2 + 4(bx+a)\sin(4bx+4a) +\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/256*(32*(b*x + a)^3 + 8*(12*b*x + 12*a + sin(4*b*x + 4*a) + 8*sin(2*b*x +
 2*a))*a^2 - 4*(24*(b*x + a)^2 + 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a
)*sin(2*b*x + 2*a) + cos(4*b*x + 4*a) + 16*cos(2*b*x + 2*a))*a + 4*(b*x + a
```

```
)*cos(4*b*x + 4*a) + 64*(b*x + a)*cos(2*b*x + 2*a) + (8*(b*x + a)^2 - 1)*si
n(4*b*x + 4*a) + 32*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))/b^3
```

Fricas [A] time = 1.30974, size = 217, normalized size = 1.62

$$\frac{8\,b^3x^3 + 8\,bx\cos\left(bx+a\right)^4 + 24\,bx\cos\left(bx+a\right)^2 - 15\,bx + \left(2\left(8\,b^2x^2-1\right)\cos\left(bx+a\right)^3 + 3\left(8\,b^2x^2-5\right)\cos\left(bx+a\right)\right)\sin\left(\right.}{64\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/64*(8*b^3*x^3 + 8*b*x*cos(b*x + a)^4 + 24*b*x*cos(b*x + a)^2 - 15*b*x + (
2*(8*b^2*x^2 - 1)*cos(b*x + a)^3 + 3*(8*b^2*x^2 - 5)*cos(b*x + a))*sin(b*x
+ a))/b^3
```

Sympy [A] time = 6.46099, size = 209, normalized size = 1.56

$$\begin{cases} \frac{x^3\sin^4(a+bx)}{8} + \frac{x^3\sin^2(a+bx)\cos^2(a+bx)}{4} + \frac{x^3\cos^4(a+bx)}{8} + \frac{3x^2\sin^3(a+bx)\cos(a+bx)}{8b} + \frac{5x^2\sin(a+bx)\cos^3(a+bx)}{8b} - \frac{15x\sin^4(a+bx)}{64b^2} - \frac{3x\sin^2(\ldots}{} \\ \frac{x^3\cos^4(a)}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(b*x+a)**4,x)
```

```
[Out] Piecewise((x**3*sin(a + b*x)**4/8 + x**3*sin(a + b*x)**2*cos(a + b*x)**2/4
+ x**3*cos(a + b*x)**4/8 + 3*x**2*sin(a + b*x)**3*cos(a + b*x)/(8*b) + 5*x*
*2*sin(a + b*x)*cos(a + b*x)**3/(8*b) - 15*x*sin(a + b*x)**4/(64*b**2) - 3*
x*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**2) + 17*x*cos(a + b*x)**4/(64*b**2
) - 15*sin(a + b*x)**3*cos(a + b*x)/(64*b**3) - 17*sin(a + b*x)*cos(a + b*x
)**3/(64*b**3), Ne(b, 0)), (x**3*cos(a)**4/3, True))
```

Giac [A] time = 1.12872, size = 113, normalized size = 0.84

$$\frac{1}{8}\,x^3 + \frac{x\cos\left(4\,bx+4\,a\right)}{64\,b^2} + \frac{x\cos\left(2\,bx+2\,a\right)}{4\,b^2} + \frac{\left(8\,b^2x^2-1\right)\sin\left(4\,bx+4\,a\right)}{256\,b^3} + \frac{\left(2\,b^2x^2-1\right)\sin\left(2\,bx+2\,a\right)}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/8*x^3 + 1/64*x*cos(4*b*x + 4*a)/b^2 + 1/4*x*cos(2*b*x + 2*a)/b^2 + 1/256*
(8*b^2*x^2 - 1)*sin(4*b*x + 4*a)/b^3 + 1/8*(2*b^2*x^2 - 1)*sin(2*b*x + 2*a)
/b^3
```

3.25 $\int x\cos^4(a+bx)\,dx$

Optimal. Leaf size=80

$$\frac{\cos^4(a+bx)}{16b^2}+\frac{3\cos^2(a+bx)}{16b^2}+\frac{x\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3x\sin(a+bx)\cos(a+bx)}{8b}+\frac{3x^2}{16}$$

```
[Out] (3*x^2)/16 + (3*Cos[a + b*x]^2)/(16*b^2) + Cos[a + b*x]^4/(16*b^2) + (3*x*C
os[a + b*x]*Sin[a + b*x])/(8*b) + (x*Cos[a + b*x]^3*Sin[a + b*x])/(4*b)
```

Rubi [A] time = 0.0474994, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3310, 30}

$$\frac{\cos^4(a+bx)}{16b^2}+\frac{3\cos^2(a+bx)}{16b^2}+\frac{x\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3x\sin(a+bx)\cos(a+bx)}{8b}+\frac{3x^2}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]^4,x]
```

```
[Out] (3*x^2)/16 + (3*Cos[a + b*x]^2)/(16*b^2) + Cos[a + b*x]^4/(16*b^2) + (3*x*C
os[a + b*x]*Sin[a + b*x])/(8*b) + (x*Cos[a + b*x]^3*Sin[a + b*x])/(4*b)
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\cos^4(a+bx)\,dx&=\frac{\cos^4(a+bx)}{16b^2}+\frac{x\cos^3(a+bx)\sin(a+bx)}{4b}+\frac{3}{4}\int x\cos^2(a+bx)\,dx\\&=\frac{3\cos^2(a+bx)}{16b^2}+\frac{\cos^4(a+bx)}{16b^2}+\frac{3x\cos(a+bx)\sin(a+bx)}{8b}+\frac{x\cos^3(a+bx)\sin(a+bx)}{4b}+\frac{3}{\ldots}\\&=\frac{3x^2}{16}+\frac{3\cos^2(a+bx)}{16b^2}+\frac{\cos^4(a+bx)}{16b^2}+\frac{3x\cos(a+bx)\sin(a+bx)}{8b}+\frac{x\cos^3(a+bx)\sin(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.132604, size = 53, normalized size = 0.66

$$\frac{4bx(8\sin(2(a+bx))+\sin(4(a+bx))+6bx)+16\cos(2(a+bx))+\cos(4(a+bx))}{128b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]^4,x]
```

```
[Out] (16*Cos[2*(a + b*x)] + Cos[4*(a + b*x)] + 4*b*x*(6*b*x + 8*Sin[2*(a + b*x)]
 + Sin[4*(a + b*x)]))/(128*b^2)
```

Maple [A] time = 0.026, size = 110, normalized size = 1.4

$$\frac{1}{b^2}\left((bx+a)\left(\frac{\sin(bx+a)}{4}\left((\cos(bx+a))^3+\frac{3\cos(bx+a)}{2}\right)+\frac{3bx}{8}+\frac{3a}{8}\right)-\frac{3(bx+a)^2}{16}+\frac{(\cos(bx+a))^4}{16}+\frac{3(\cos(bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)^4,x)
```

```
[Out] 1/b^2*((b*x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)
-3/16*(b*x+a)^2+1/16*cos(b*x+a)^4+3/16*cos(b*x+a)^2-a*(1/4*(cos(b*x+a)^3+3/
2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a))
```

Maxima [A] time = 1.01717, size = 132, normalized size = 1.65

$$\frac{24(bx+a)^2-4(12bx+12a+\sin(4bx+4a)+8\sin(2bx+2a))a+4(bx+a)\sin(4bx+4a)+32(bx+a)\sin(2bx+}{128b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/128*(24*(b*x + a)^2 - 4*(12*b*x + 12*a + sin(4*b*x + 4*a) + 8*sin(2*b*x +
 2*a))*a + 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a) + c
os(4*b*x + 4*a) + 16*cos(2*b*x + 2*a))/b^2
```

Fricas [A] time = 1.36012, size = 161, normalized size = 2.01

$$\frac{3b^2x^2+\cos(bx+a)^4+3\cos(bx+a)^2+2\left(2bx\cos(bx+a)^3+3bx\cos(bx+a)\right)\sin(bx+a)}{16b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/16*(3*b^2*x^2 + cos(b*x + a)^4 + 3*cos(b*x + a)^2 + 2*(2*b*x*cos(b*x + a)
^3 + 3*b*x*cos(b*x + a))*sin(b*x + a))/b^2
```

Sympy [A] time = 3.41895, size = 138, normalized size = 1.72

$$\begin{cases}\frac{3x^2\sin^4(a+bx)}{16}+\frac{3x^2\sin^2(a+bx)\cos^2(a+bx)}{8}+\frac{3x^2\cos^4(a+bx)}{16}+\frac{3x\sin^3(a+bx)\cos(a+bx)}{8b}+\frac{5x\sin(a+bx)\cos^3(a+bx)}{8b}-\frac{3\sin^4(a+bx)}{32b^2}+\frac{5\cos^4(a+}{32b^2} \\ \frac{x^2\cos^4(a)}{2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x**2*sin(a + b*x)**4/16 + 3*x**2*sin(a + b*x)**2*cos(a + b*x)*
*2/8 + 3*x**2*cos(a + b*x)**4/16 + 3*x*sin(a + b*x)**3*cos(a + b*x)/(8*b) +
 5*x*sin(a + b*x)*cos(a + b*x)**3/(8*b) - 3*sin(a + b*x)**4/(32*b**2) + 5*c
os(a + b*x)**4/(32*b**2), Ne(b, 0)), (x**2*cos(a)**4/2, True))
```

Giac [A] time = 1.13597, size = 86, normalized size = 1.08

$$\frac{3}{16}\,x^2 + \frac{x\sin\left(4\,bx + 4\,a\right)}{32\,b} + \frac{x\sin\left(2\,bx + 2\,a\right)}{4\,b} + \frac{\cos\left(4\,bx + 4\,a\right)}{128\,b^2} + \frac{\cos\left(2\,bx + 2\,a\right)}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/16*x^2 + 1/32*x*sin(4*b*x + 4*a)/b + 1/4*x*sin(2*b*x + 2*a)/b + 1/128*cos
(4*b*x + 4*a)/b^2 + 1/8*cos(2*b*x + 2*a)/b^2
```

3.26 $\int \frac{\cos^4(a+bx)}{x} \, dx$

Optimal. Leaf size=59

$$\frac{1}{2}\cos(2a)\mathrm{CosIntegral}(2bx) + \frac{1}{8}\cos(4a)\mathrm{CosIntegral}(4bx) - \frac{1}{2}\sin(2a)\mathrm{Si}(2bx) - \frac{1}{8}\sin(4a)\mathrm{Si}(4bx) + \frac{3\log(x)}{8}$$

```
[Out] (Cos[2*a]*CosIntegral[2*b*x])/2 + (Cos[4*a]*CosIntegral[4*b*x])/8 + (3*Log[
x])/8 - (Sin[2*a]*SinIntegral[2*b*x])/2 - (Sin[4*a]*SinIntegral[4*b*x])/8
```

Rubi [A] time = 0.157571, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3312, 3303, 3299, 3302}

$$\frac{1}{2}\cos(2a)\mathrm{CosIntegral}(2bx) + \frac{1}{8}\cos(4a)\mathrm{CosIntegral}(4bx) - \frac{1}{2}\sin(2a)\mathrm{Si}(2bx) - \frac{1}{8}\sin(4a)\mathrm{Si}(4bx) + \frac{3\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4/x,x]
```

```
[Out] (Cos[2*a]*CosIntegral[2*b*x])/2 + (Cos[4*a]*CosIntegral[4*b*x])/8 + (3*Log[
x])/8 - (Sin[2*a]*SinIntegral[2*b*x])/2 - (Sin[4*a]*SinIntegral[4*b*x])/8
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos^4(a+bx)}{x}\,dx = \int \left(\frac{3}{8x}+\frac{\cos(2a+2bx)}{2x}+\frac{\cos(4a+4bx)}{8x}\right)dx$$
$$= \frac{3\log(x)}{8}+\frac{1}{8}\int \frac{\cos(4a+4bx)}{x}\,dx+\frac{1}{2}\int \frac{\cos(2a+2bx)}{x}\,dx$$
$$= \frac{3\log(x)}{8}+\frac{1}{2}\cos(2a)\int \frac{\cos(2bx)}{x}\,dx+\frac{1}{8}\cos(4a)\int \frac{\cos(4bx)}{x}\,dx-\frac{1}{2}\sin(2a)\int \frac{\sin(2bx)}{x}\,dx-\frac{1}{8}$$
$$= \frac{1}{2}\cos(2a)\text{Ci}(2bx)+\frac{1}{8}\cos(4a)\text{Ci}(4bx)+\frac{3\log(x)}{8}-\frac{1}{2}\sin(2a)\text{Si}(2bx)-\frac{1}{8}\sin(4a)\text{Si}(4bx)$$

Mathematica [A] time = 0.102006, size = 52, normalized size = 0.88

$$\frac{1}{8}(4\cos(2a)\text{CosIntegral}(2bx)+\cos(4a)\text{CosIntegral}(4bx)-4\sin(2a)\text{Si}(2bx)-\sin(4a)\text{Si}(4bx)+3\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4/x,x]
```

```
[Out] (4*Cos[2*a]*CosIntegral[2*b*x] + Cos[4*a]*CosIntegral[4*b*x] + 3*Log[x] - 4
*Sin[2*a]*SinIntegral[2*b*x] - Sin[4*a]*SinIntegral[4*b*x])/8
```

Maple [A] time = 0.03, size = 52, normalized size = 0.9

$$-\frac{\text{Si}(4bx)\sin(4a)}{8}+\frac{\text{Ci}(4bx)\cos(4a)}{8}-\frac{\text{Si}(2bx)\sin(2a)}{2}+\frac{\text{Ci}(2bx)\cos(2a)}{2}+\frac{3\ln(bx)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4/x,x)
```

```
[Out] -1/8*Si(4*b*x)*sin(4*a)+1/8*Ci(4*b*x)*cos(4*a)-1/2*Si(2*b*x)*sin(2*a)+1/2*C
i(2*b*x)*cos(2*a)+3/8*ln(b*x)
```

Maxima [C] time = 1.20682, size = 123, normalized size = 2.08

$$-\frac{1}{16}(E_1(4i\,bx)+E_1(-4i\,bx))\cos(4a)-\frac{1}{4}(E_1(2i\,bx)+E_1(-2i\,bx))\cos(2a)+\frac{1}{16}(i\,E_1(4i\,bx)-i\,E_1(-4i\,bx))\sin(4a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/x,x, algorithm="maxima")
```

```
[Out] -1/16*(exp_integral_e(1, 4*I*b*x) + exp_integral_e(1, -4*I*b*x))*cos(4*a) -
 1/4*(exp_integral_e(1, 2*I*b*x) + exp_integral_e(1, -2*I*b*x))*cos(2*a) +
1/16*(I*exp_integral_e(1, 4*I*b*x) - I*exp_integral_e(1, -4*I*b*x))*sin(4*a
) + 1/16*(4*I*exp_integral_e(1, 2*I*b*x) - 4*I*exp_integral_e(1, -2*I*b*x))
*sin(2*a) + 3/8*log(b*x)
```

Fricas [A] time = 1.47171, size = 274, normalized size = 4.64

$$\frac{1}{16}(\text{Ci}(4bx)+\text{Ci}(-4bx))\cos(4a)+\frac{1}{4}(\text{Ci}(2bx)+\text{Ci}(-2bx))\cos(2a)-\frac{1}{8}\sin(4a)\text{Si}(4bx)-\frac{1}{2}\sin(2a)\text{Si}(2bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/x,x, algorithm="fricas")
```

```
[Out] 1/16*(cos_integral(4*b*x) + cos_integral(-4*b*x))*cos(4*a) + 1/4*(cos_integ
ral(2*b*x) + cos_integral(-2*b*x))*cos(2*a) - 1/8*sin(4*a)*sin_integral(4*b
*x) - 1/2*sin(2*a)*sin_integral(2*b*x) + 3/8*log(x)
```

Sympy [A] time = 5.73744, size = 60, normalized size = 1.02

$$\frac{3\log(x)}{8} - \frac{\sin(2a)\operatorname{Si}(2bx)}{2} - \frac{\sin(4a)\operatorname{Si}(4bx)}{8} + \frac{\cos(2a)\operatorname{Ci}(2bx)}{2} + \frac{\cos(4a)\operatorname{Ci}(4bx)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4/x,x)
```

```
[Out] 3*log(x)/8 - sin(2*a)*Si(2*b*x)/2 - sin(4*a)*Si(4*b*x)/8 + cos(2*a)*Ci(2*b*
x)/2 + cos(4*a)*Ci(4*b*x)/8
```

Giac [C] time = 1.14128, size = 578, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/x,x, algorithm="giac")
```

```
[Out] 1/16*(6*log(abs(x))*tan(2*a)^2*tan(a)^2 - real_part(cos_integral(4*b*x))*ta
n(2*a)^2*tan(a)^2 - 4*real_part(cos_integral(2*b*x))*tan(2*a)^2*tan(a)^2 -
4*real_part(cos_integral(-2*b*x))*tan(2*a)^2*tan(a)^2 - real_part(cos_integ
ral(-4*b*x))*tan(2*a)^2*tan(a)^2 - 8*imag_part(cos_integral(2*b*x))*tan(2*a
)^2*tan(a) + 8*imag_part(cos_integral(-2*b*x))*tan(2*a)^2*tan(a) - 16*sin_i
ntegral(2*b*x)*tan(2*a)^2*tan(a) - 2*imag_part(cos_integral(4*b*x))*tan(2*a
)*tan(a)^2 + 2*imag_part(cos_integral(-4*b*x))*tan(2*a)*tan(a)^2 - 4*sin_in
tegral(4*b*x)*tan(2*a)*tan(a)^2 + 6*log(abs(x))*tan(2*a)^2 - real_part(cos_
integral(4*b*x))*tan(2*a)^2 + 4*real_part(cos_integral(2*b*x))*tan(2*a)^2 +
 4*real_part(cos_integral(-2*b*x))*tan(2*a)^2 - real_part(cos_integral(-4*b
*x))*tan(2*a)^2 + 6*log(abs(x))*tan(a)^2 + real_part(cos_integral(4*b*x))*t
an(a)^2 - 4*real_part(cos_integral(2*b*x))*tan(a)^2 - 4*real_part(cos_integ
ral(-2*b*x))*tan(a)^2 + real_part(cos_integral(-4*b*x))*tan(a)^2 - 2*imag_p
art(cos_integral(4*b*x))*tan(2*a) + 2*imag_part(cos_integral(-4*b*x))*tan(2
*a) - 4*sin_integral(4*b*x)*tan(2*a) - 8*imag_part(cos_integral(2*b*x))*tan
(a) + 8*imag_part(cos_integral(-2*b*x))*tan(a) - 16*sin_integral(2*b*x)*tan
(a) + 6*log(abs(x)) + real_part(cos_integral(4*b*x)) + 4*real_part(cos_inte
gral(2*b*x)) + 4*real_part(cos_integral(-2*b*x)) + real_part(cos_integral(-
4*b*x)))/(tan(2*a)^2*tan(a)^2 + tan(2*a)^2 + tan(a)^2 + 1)
```

3.27 $\int \frac{\cos^4(a+bx)}{x^2} \, dx$

Optimal. Leaf size=66

$$-b\sin(2a)\text{CosIntegral}(2bx) - \frac{1}{2}b\sin(4a)\text{CosIntegral}(4bx) - b\cos(2a)\text{Si}(2bx) - \frac{1}{2}b\cos(4a)\text{Si}(4bx) - \frac{\cos^4(a+bx)}{x}$$

```
[Out] -(Cos[a + b*x]^4/x) - b*CosIntegral[2*b*x]*Sin[2*a] - (b*CosIntegral[4*b*x]
*Sin[4*a])/2 - b*Cos[2*a]*SinIntegral[2*b*x] - (b*Cos[4*a]*SinIntegral[4*b*
x])/2
```

Rubi [A] time = 0.150996, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3313, 3303, 3299, 3302}

$$-b\sin(2a)\text{CosIntegral}(2bx) - \frac{1}{2}b\sin(4a)\text{CosIntegral}(4bx) - b\cos(2a)\text{Si}(2bx) - \frac{1}{2}b\cos(4a)\text{Si}(4bx) - \frac{\cos^4(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4/x^2,x]
```

```
[Out] -(Cos[a + b*x]^4/x) - b*CosIntegral[2*b*x]*Sin[2*a] - (b*CosIntegral[4*b*x]
*Sin[4*a])/2 - b*Cos[2*a]*SinIntegral[2*b*x] - (b*Cos[4*a]*SinIntegral[4*b*
x])/2
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(a+bx)}{x^2}\,dx &= -\frac{\cos^4(a+bx)}{x} + (4b)\int\left(-\frac{\sin(2a+2bx)}{4x} - \frac{\sin(4a+4bx)}{8x}\right)dx \\ &= -\frac{\cos^4(a+bx)}{x} - \frac{1}{2}b\int\frac{\sin(4a+4bx)}{x}\,dx - b\int\frac{\sin(2a+2bx)}{x}\,dx \\ &= -\frac{\cos^4(a+bx)}{x} - (b\cos(2a))\int\frac{\sin(2bx)}{x}\,dx - \frac{1}{2}(b\cos(4a))\int\frac{\sin(4bx)}{x}\,dx - (b\sin(2a))\int\frac{\cos(2bx)}{x} \\ &= -\frac{\cos^4(a+bx)}{x} - b\mathrm{Ci}(2bx)\sin(2a) - \frac{1}{2}b\mathrm{Ci}(4bx)\sin(4a) - b\cos(2a)\mathrm{Si}(2bx) - \frac{1}{2}b\cos(4a)\mathrm{Si}(4bx)\end{aligned}$$

Mathematica [A] time = 0.21995, size = 79, normalized size = 1.2

$$-\frac{8bx\sin(2a)\mathrm{CosIntegral}(2bx) + 4bx\sin(4a)\mathrm{CosIntegral}(4bx) + 8bx\cos(2a)\mathrm{Si}(2bx) + 4bx\cos(4a)\mathrm{Si}(4bx) + 4\cos(2(a}{8x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4/x^2,x]
```

```
[Out] -(3 + 4*Cos[2*(a + b*x)] + Cos[4*(a + b*x)] + 8*b*x*CosIntegral[2*b*x]*Sin[
2*a] + 4*b*x*CosIntegral[4*b*x]*Sin[4*a] + 8*b*x*Cos[2*a]*SinIntegral[2*b*x
] + 4*b*x*Cos[4*a]*SinIntegral[4*b*x])/(8*x)
```

Maple [A] time = 0.03, size = 90, normalized size = 1.4

$$b\left(-\frac{\cos(4bx+4a)}{8bx} - \frac{\mathrm{Si}(4bx)\cos(4a)}{2} - \frac{\mathrm{Ci}(4bx)\sin(4a)}{2} - \frac{\cos(2bx+2a)}{2bx} - \mathrm{Si}(2bx)\cos(2a) - \mathrm{Ci}(2bx)\sin(2a) - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4/x^2,x)
```

```
[Out] b*(-1/8*cos(4*b*x+4*a)/x/b-1/2*Si(4*b*x)*cos(4*a)-1/2*Ci(4*b*x)*sin(4*a)-1/
2*cos(2*b*x+2*a)/x/b-Si(2*b*x)*cos(2*a)-Ci(2*b*x)*sin(2*a)-3/8/x/b)
```

Maxima [C] time = 1.35022, size = 980, normalized size = 14.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/x^2,x, algorithm="maxima")
```

```
[Out] 1/1048576*(32768*((exp_integral_e(2, 4*I*b*x) + exp_integral_e(2, -4*I*b*x)
)*cos(2*a)^2 + (exp_integral_e(2, 4*I*b*x) + exp_integral_e(2, -4*I*b*x))*s
in(2*a)^2)*cos(4*a)^3 - ((32768*I*exp_integral_e(2, 4*I*b*x) - 32768*I*exp_
integral_e(2, -4*I*b*x))*cos(2*a)^2 + (32768*I*exp_integral_e(2, 4*I*b*x) -
 32768*I*exp_integral_e(2, -4*I*b*x))*sin(2*a)^2)*sin(4*a)^3 + (131072*(exp
_integral_e(2, 2*I*b*x) + exp_integral_e(2, -2*I*b*x))*cos(2*a)^3 - (131072
*I*exp_integral_e(2, 2*I*b*x) - 131072*I*exp_integral_e(2, -2*I*b*x))*sin(2
*a)^3 + 131072*((exp_integral_e(2, 2*I*b*x) + exp_integral_e(2, -2*I*b*x))*
cos(2*a) + 3)*sin(2*a)^2 + 131072*(exp_integral_e(2, 2*I*b*x) + exp_integra
```

```
l_e(2, -2*I*b*x))*cos(2*a) + 393216*cos(2*a)^2 - ((131072*I*exp_integral_e(
2, 2*I*b*x) - 131072*I*exp_integral_e(2, -2*I*b*x))*cos(2*a)^2 + 131072*I*e
xp_integral_e(2, 2*I*b*x) - 131072*I*exp_integral_e(2, -2*I*b*x))*sin(2*a))
*cos(4*a)^2 + (131072*(exp_integral_e(2, 2*I*b*x) + exp_integral_e(2, -2*I*
b*x))*cos(2*a)^3 - (131072*I*exp_integral_e(2, 2*I*b*x) - 131072*I*exp_inte
gral_e(2, -2*I*b*x))*sin(2*a)^3 + 131072*((exp_integral_e(2, 2*I*b*x) + exp
_integral_e(2, -2*I*b*x))*cos(2*a) + 3)*sin(2*a)^2 + 32768*((exp_integral_e
(2, 4*I*b*x) + exp_integral_e(2, -4*I*b*x))*cos(2*a)^2 + (exp_integral_e(2,
 4*I*b*x) + exp_integral_e(2, -4*I*b*x))*sin(2*a)^2)*cos(4*a) + 131072*(exp
_integral_e(2, 2*I*b*x) + exp_integral_e(2, -2*I*b*x))*cos(2*a) + 393216*co
s(2*a)^2 - ((131072*I*exp_integral_e(2, 2*I*b*x) - 131072*I*exp_integral_e(
2, -2*I*b*x))*cos(2*a)^2 + 131072*I*exp_integral_e(2, 2*I*b*x) - 131072*I*e
xp_integral_e(2, -2*I*b*x))*sin(2*a))*sin(4*a)^2 + 32768*((exp_integral_e(2
, 4*I*b*x) + exp_integral_e(2, -4*I*b*x))*cos(2*a)^2 + (exp_integral_e(2, 4
*I*b*x) + exp_integral_e(2, -4*I*b*x))*sin(2*a)^2)*cos(4*a) - (((32768*I*ex
p_integral_e(2, 4*I*b*x) - 32768*I*exp_integral_e(2, -4*I*b*x))*cos(2*a)^2
+ (32768*I*exp_integral_e(2, 4*I*b*x) - 32768*I*exp_integral_e(2, -4*I*b*x)
)*sin(2*a)^2)*cos(4*a)^2 + (32768*I*exp_integral_e(2, 4*I*b*x) - 32768*I*ex
p_integral_e(2, -4*I*b*x))*cos(2*a)^2 + (32768*I*exp_integral_e(2, 4*I*b*x)
 - 32768*I*exp_integral_e(2, -4*I*b*x))*sin(2*a)^2)*sin(4*a))*b/((a*cos(2*a
)^2 + a*sin(2*a)^2)*cos(4*a)^2 + (a*cos(2*a)^2 + a*sin(2*a)^2)*sin(4*a)^2 -
 ((cos(2*a)^2 + sin(2*a)^2)*cos(4*a)^2 + (cos(2*a)^2 + sin(2*a)^2)*sin(4*a)
^2)*(b*x + a))
```

Fricas [A] time = 1.44193, size = 312, normalized size = 4.73

$$-\frac{4\cos(bx+a)^4 + 2\,bx\cos(4\,a)\,\mathrm{Si}(4\,bx) + 4\,bx\cos(2\,a)\,\mathrm{Si}(2\,bx) + (bx\,\mathrm{Ci}(4\,bx) + bx\,\mathrm{Ci}(-4\,bx))\sin(4\,a) + 2\,(bx\,\mathrm{Ci}(}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*cos(b*x + a)^4 + 2*b*x*cos(4*a)*sin_integral(4*b*x) + 4*b*x*cos(2*a
)*sin_integral(2*b*x) + (b*x*cos_integral(4*b*x) + b*x*cos_integral(-4*b*x)
)*sin(4*a) + 2*(b*x*cos_integral(2*b*x) + b*x*cos_integral(-2*b*x))*sin(2*a
))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^4(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4/x**2,x)
```

```
[Out] Integral(cos(a + b*x)**4/x**2, x)
```

Giac [C] time = 1.25907, size = 4347, normalized size = 65.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*x*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*
tan(a)^2 + 2*b*x*imag_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan
(2*a)^2*tan(a)^2 - 2*b*x*imag_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(b
*x)^2*tan(2*a)^2*tan(a)^2 - b*x*imag_part(cos_integral(-4*b*x))*tan(2*b*x)^
2*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 2*b*x*sin_integral(4*b*x)*tan(2*b*x)^2*t
an(b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b*x*sin_integral(2*b*x)*tan(2*b*x)^2*tan(
b*x)^2*tan(2*a)^2*tan(a)^2 - 4*b*x*real_part(cos_integral(2*b*x))*tan(2*b*x
)^2*tan(b*x)^2*tan(2*a)^2*tan(a) - 4*b*x*real_part(cos_integral(-2*b*x))*ta
n(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a) - 2*b*x*real_part(cos_integral(4*b*
x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2 - 2*b*x*real_part(cos_integra
l(-4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2 + b*x*imag_part(cos_in
tegral(4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 - 2*b*x*imag_part(cos_int
egral(2*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 + 2*b*x*imag_part(cos_inte
gral(-2*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 - b*x*imag_part(cos_integr
al(-4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 + 2*b*x*sin_integral(4*b*x)*
tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 - 4*b*x*sin_integral(2*b*x)*tan(2*b*x)^2
*tan(b*x)^2*tan(2*a)^2 - b*x*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2*ta
n(b*x)^2*tan(a)^2 + 2*b*x*imag_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(b
*x)^2*tan(a)^2 - 2*b*x*imag_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(b*x
)^2*tan(a)^2 + b*x*imag_part(cos_integral(-4*b*x))*tan(2*b*x)^2*tan(b*x)^2*
tan(a)^2 - 2*b*x*sin_integral(4*b*x)*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 + 4*b
*x*sin_integral(2*b*x)*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 + b*x*imag_part(cos
_integral(4*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 + 2*b*x*imag_part(cos_in
tegral(2*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 - 2*b*x*imag_part(cos_integ
ral(-2*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 - b*x*imag_part(cos_integral(
-4*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 + 2*b*x*sin_integral(4*b*x)*tan(2
*b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b*x*sin_integral(2*b*x)*tan(2*b*x)^2*tan(2*
a)^2*tan(a)^2 + b*x*imag_part(cos_integral(4*b*x))*tan(b*x)^2*tan(2*a)^2*ta
n(a)^2 + 2*b*x*imag_part(cos_integral(2*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a)^
2 - 2*b*x*imag_part(cos_integral(-2*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a)^2 -
b*x*imag_part(cos_integral(-4*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 2*b*x*
sin_integral(4*b*x)*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b*x*sin_integral(2*b
*x)*tan(b*x)^2*tan(2*a)^2*tan(a)^2 - 2*b*x*real_part(cos_integral(4*b*x))*t
an(2*b*x)^2*tan(b*x)^2*tan(2*a) - 2*b*x*real_part(cos_integral(-4*b*x))*tan
(2*b*x)^2*tan(b*x)^2*tan(2*a) - 4*b*x*real_part(cos_integral(2*b*x))*tan(2*
b*x)^2*tan(b*x)^2*tan(a) - 4*b*x*real_part(cos_integral(-2*b*x))*tan(2*b*x)
^2*tan(b*x)^2*tan(a) - 4*b*x*real_part(cos_integral(2*b*x))*tan(2*b*x)^2*ta
n(2*a)^2*tan(a) - 4*b*x*real_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(2*
a)^2*tan(a) - 4*b*x*real_part(cos_integral(2*b*x))*tan(b*x)^2*tan(2*a)^2*ta
n(a) - 4*b*x*real_part(cos_integral(-2*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a) -
 2*b*x*real_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(2*a)*tan(a)^2 - 2*b*
x*real_part(cos_integral(-4*b*x))*tan(2*b*x)^2*tan(2*a)*tan(a)^2 - 2*b*x*re
al_part(cos_integral(4*b*x))*tan(b*x)^2*tan(2*a)*tan(a)^2 - 2*b*x*real_part
(cos_integral(-4*b*x))*tan(b*x)^2*tan(2*a)*tan(a)^2 - 4*tan(2*b*x)^2*tan(b*
x)^2*tan(2*a)^2*tan(a)^2 - b*x*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2*
tan(b*x)^2 - 2*b*x*imag_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(b*x)^2 +
 2*b*x*imag_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(b*x)^2 + b*x*imag_p
art(cos_integral(-4*b*x))*tan(2*b*x)^2*tan(b*x)^2 - 2*b*x*sin_integral(4*b*
x)*tan(2*b*x)^2*tan(b*x)^2 - 4*b*x*sin_integral(2*b*x)*tan(2*b*x)^2*tan(b*x
)^2 + b*x*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(2*a)^2 - 2*b*x*im
ag_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(2*a)^2 + 2*b*x*imag_part(cos_
integral(-2*b*x))*tan(2*b*x)^2*tan(2*a)^2 - b*x*imag_part(cos_integral(-4*b
*x))*tan(2*b*x)^2*tan(2*a)^2 + 2*b*x*sin_integral(4*b*x)*tan(2*b*x)^2*tan(2
*a)^2 - 4*b*x*sin_integral(2*b*x)*tan(2*b*x)^2*tan(2*a)^2 + b*x*imag_part(c
os_integral(4*b*x))*tan(b*x)^2*tan(2*a)^2 - 2*b*x*imag_part(cos_integral(2*
```

```
b*x))*tan(b*x)^2*tan(2*a)^2 + 2*b*x*imag_part(cos_integral(-2*b*x))*tan(b*x
)^2*tan(2*a)^2 - b*x*imag_part(cos_integral(-4*b*x))*tan(b*x)^2*tan(2*a)^2
+ 2*b*x*sin_integral(4*b*x)*tan(b*x)^2*tan(2*a)^2 - 4*b*x*sin_integral(2*b*
x)*tan(b*x)^2*tan(2*a)^2 - b*x*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2*
tan(a)^2 + 2*b*x*imag_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(a)^2 - 2*b
*x*imag_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(a)^2 + b*x*imag_part(co
s_integral(-4*b*x))*tan(2*b*x)^2*tan(a)^2 - 2*b*x*sin_integral(4*b*x)*tan(2
*b*x)^2*tan(a)^2 + 4*b*x*sin_integral(2*b*x)*tan(2*b*x)^2*tan(a)^2 - b*x*im
ag_part(cos_integral(4*b*x))*tan(b*x)^2*tan(a)^2 + 2*b*x*imag_part(cos_inte
gral(2*b*x))*tan(b*x)^2*tan(a)^2 - 2*b*x*imag_part(cos_integral(-2*b*x))*ta
n(b*x)^2*tan(a)^2 + b*x*imag_part(cos_integral(-4*b*x))*tan(b*x)^2*tan(a)^2
 - 2*b*x*sin_integral(4*b*x)*tan(b*x)^2*tan(a)^2 + 4*b*x*sin_integral(2*b*x
)*tan(b*x)^2*tan(a)^2 + b*x*imag_part(cos_integral(4*b*x))*tan(2*a)^2*tan(a
)^2 + 2*b*x*imag_part(cos_integral(2*b*x))*tan(2*a)^2*tan(a)^2 - 2*b*x*imag
_part(cos_integral(-2*b*x))*tan(2*a)^2*tan(a)^2 - b*x*imag_part(cos_integra
l(-4*b*x))*tan(2*a)^2*tan(a)^2 + 2*b*x*sin_integral(4*b*x)*tan(2*a)^2*tan(a
)^2 + 4*b*x*sin_integral(2*b*x)*tan(2*a)^2*tan(a)^2 - 2*b*x*real_part(cos_i
ntegral(4*b*x))*tan(2*b*x)^2*tan(2*a) - 2*b*x*real_part(cos_integral(-4*b*x
))*tan(2*b*x)^2*tan(2*a) - 2*b*x*real_part(cos_integral(4*b*x))*tan(b*x)^2*
tan(2*a) - 2*b*x*real_part(cos_integral(-4*b*x))*tan(b*x)^2*tan(2*a) - 4*b*
x*real_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(a) - 4*b*x*real_part(cos_
integral(-2*b*x))*tan(2*b*x)^2*tan(a) - 4*b*x*real_part(cos_integral(2*b*x)
)*tan(b*x)^2*tan(a) - 4*b*x*real_part(cos_integral(-2*b*x))*tan(b*x)^2*tan(
a) - 4*b*x*real_part(cos_integral(2*b*x))*tan(2*a)^2*tan(a) - 4*b*x*real_pa
rt(cos_integral(-2*b*x))*tan(2*a)^2*tan(a) + 8*tan(2*b*x)^2*tan(b*x)*tan(2*
a)^2*tan(a) - 3*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 - 2*b*x*real_part(cos_inte
gral(4*b*x))*tan(2*a)*tan(a)^2 - 2*b*x*real_part(cos_integral(-4*b*x))*tan(
2*a)*tan(a)^2 + 2*tan(2*b*x)*tan(b*x)^2*tan(2*a)*tan(a)^2 - 3*tan(b*x)^2*ta
n(2*a)^2*tan(a)^2 - b*x*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2 - 2*b*x
*imag_part(cos_integral(2*b*x))*tan(2*b*x)^2 + 2*b*x*imag_part(cos_integral
(-2*b*x))*tan(2*b*x)^2 + b*x*imag_part(cos_integral(-4*b*x))*tan(2*b*x)^2 -
 2*b*x*sin_integral(4*b*x)*tan(2*b*x)^2 - 4*b*x*sin_integral(2*b*x)*tan(2*b
*x)^2 - b*x*imag_part(cos_integral(4*b*x))*tan(b*x)^2 - 2*b*x*imag_part(cos
_integral(2*b*x))*tan(b*x)^2 + 2*b*x*imag_part(cos_integral(-2*b*x))*tan(b*
x)^2 + b*x*imag_part(cos_integral(-4*b*x))*tan(b*x)^2 - 2*b*x*sin_integral(
4*b*x)*tan(b*x)^2 - 4*b*x*sin_integral(2*b*x)*tan(b*x)^2 + b*x*imag_part(co
s_integral(4*b*x))*tan(2*a)^2 - 2*b*x*imag_part(cos_integral(2*b*x))*tan(2*
a)^2 + 2*b*x*imag_part(cos_integral(-2*b*x))*tan(2*a)^2 - b*x*imag_part(cos
_integral(-4*b*x))*tan(2*a)^2 + 2*b*x*sin_integral(4*b*x)*tan(2*a)^2 - 4*b*
x*sin_integral(2*b*x)*tan(2*a)^2 - b*x*imag_part(cos_integral(4*b*x))*tan(a
)^2 + 2*b*x*imag_part(cos_integral(2*b*x))*tan(a)^2 - 2*b*x*imag_part(cos_i
ntegral(-2*b*x))*tan(a)^2 + b*x*imag_part(cos_integral(-4*b*x))*tan(a)^2 -
2*b*x*sin_integral(4*b*x)*tan(a)^2 + 4*b*x*sin_integral(2*b*x)*tan(a)^2 + t
an(2*b*x)^2*tan(b*x)^2 - 2*b*x*real_part(cos_integral(4*b*x))*tan(2*a) - 2*
b*x*real_part(cos_integral(-4*b*x))*tan(2*a) + 2*tan(2*b*x)*tan(b*x)^2*tan(
2*a) - 4*tan(2*b*x)^2*tan(2*a)^2 + tan(b*x)^2*tan(2*a)^2 - 4*b*x*real_part(
cos_integral(2*b*x))*tan(a) - 4*b*x*real_part(cos_integral(-2*b*x))*tan(a)
+ 8*tan(2*b*x)^2*tan(b*x)*tan(a) + 8*tan(b*x)*tan(2*a)^2*tan(a) + tan(2*b*x
)^2*tan(a)^2 - 4*tan(b*x)^2*tan(a)^2 + 2*tan(2*b*x)*tan(2*a)*tan(a)^2 + tan
(2*a)^2*tan(a)^2 - b*x*imag_part(cos_integral(4*b*x)) - 2*b*x*imag_part(cos
_integral(2*b*x)) + 2*b*x*imag_part(cos_integral(-2*b*x)) + b*x*imag_part(c
os_integral(-4*b*x)) - 2*b*x*sin_integral(4*b*x) - 4*b*x*sin_integral(2*b*x
) - 3*tan(2*b*x)^2 + 2*tan(2*b*x)*tan(2*a) - 3*tan(2*a)^2 + 8*tan(b*x)*tan(
a) - 4)/(x*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + x*tan(2*b*x)^2*tan
(b*x)^2*tan(2*a)^2 + x*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 + x*tan(2*b*x)^2*ta
n(2*a)^2*tan(a)^2 + x*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + x*tan(2*b*x)^2*tan(b
*x)^2 + x*tan(2*b*x)^2*tan(2*a)^2 + x*tan(b*x)^2*tan(2*a)^2 + x*tan(2*b*x)^
2*tan(a)^2 + x*tan(b*x)^2*tan(a)^2 + x*tan(2*a)^2*tan(a)^2 + x*tan(2*b*x)^2
 + x*tan(b*x)^2 + x*tan(2*a)^2 + x*tan(a)^2 + x)
```

3.28 $\int \frac{\cos^4(a+bx)}{x^3} dx$

Optimal. Leaf size=90

$$-b^2\cos(2a)\text{CosIntegral}(2bx)-b^2\cos(4a)\text{CosIntegral}(4bx)+b^2\sin(2a)\text{Si}(2bx)+b^2\sin(4a)\text{Si}(4bx)-\frac{\cos^4(a+bx)}{2x^2}$$

```
[Out] -Cos[a + b*x]^4/(2*x^2) - b^2*Cos[2*a]*CosIntegral[2*b*x] - b^2*Cos[4*a]*Co
sIntegral[4*b*x] + (2*b*Cos[a + b*x]^3*Sin[a + b*x])/x + b^2*Sin[2*a]*SinIn
tegral[2*b*x] + b^2*Sin[4*a]*SinIntegral[4*b*x]
```

Rubi [A] time = 0.296895, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3314, 3312, 3303, 3299, 3302}

$$-b^2\cos(2a)\text{CosIntegral}(2bx)-b^2\cos(4a)\text{CosIntegral}(4bx)+b^2\sin(2a)\text{Si}(2bx)+b^2\sin(4a)\text{Si}(4bx)-\frac{\cos^4(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4/x^3,x]
```

```
[Out] -Cos[a + b*x]^4/(2*x^2) - b^2*Cos[2*a]*CosIntegral[2*b*x] - b^2*Cos[4*a]*Co
sIntegral[4*b*x] + (2*b*Cos[a + b*x]^3*Sin[a + b*x])/x + b^2*Sin[2*a]*SinIn
tegral[2*b*x] + b^2*Sin[4*a]*SinIntegral[4*b*x]
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos^4(a+bx)}{x^3}\,dx = -\frac{\cos^4(a+bx)}{2x^2} + \frac{2b\cos^3(a+bx)\sin(a+bx)}{x} + \left(6b^2\right)\int \frac{\cos^2(a+bx)}{x}\,dx - \left(8b^2\right)\int \frac{\cos^4(a+bx)}{x}\,d$$

$$= -\frac{\cos^4(a+bx)}{2x^2} + \frac{2b\cos^3(a+bx)\sin(a+bx)}{x} + \left(6b^2\right)\int \left(\frac{1}{2x} + \frac{\cos(2a+2bx)}{2x}\right)dx - \left(8b^2\right)\int \left(\frac{3}{8x}\right.$$

$$= -\frac{\cos^4(a+bx)}{2x^2} + \frac{2b\cos^3(a+bx)\sin(a+bx)}{x} - b^2\int \frac{\cos(4a+4bx)}{x}\,dx + \left(3b^2\right)\int \frac{\cos(2a+2bx)}{x}\,d$$

$$= -\frac{\cos^4(a+bx)}{2x^2} + \frac{2b\cos^3(a+bx)\sin(a+bx)}{x} + \left(3b^2\cos(2a)\right)\int \frac{\cos(2bx)}{x}\,dx - \left(4b^2\cos(2a)\right)\int \frac{\text{co}}{}$$

$$= -\frac{\cos^4(a+bx)}{2x^2} - b^2\cos(2a)\text{Ci}(2bx) - b^2\cos(4a)\text{Ci}(4bx) + \frac{2b\cos^3(a+bx)\sin(a+bx)}{x} + b^2\sin(2a)$$

Mathematica [A] time = 0.302518, size = 119, normalized size = 1.32

$$-\frac{16b^2x^2\cos(2a)\text{CosIntegral}(2bx) + 16b^2x^2\cos(4a)\text{CosIntegral}(4bx) - 16b^2x^2\sin(2a)\text{Si}(2bx) - 16b^2x^2\sin(4a)\text{Si}(4bx)}{16x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4/x^3,x]
```

```
[Out] -(3 + 4*Cos[2*(a + b*x)] + Cos[4*(a + b*x)] + 16*b^2*x^2*Cos[2*a]*CosIntegr
al[2*b*x] + 16*b^2*x^2*Cos[4*a]*CosIntegral[4*b*x] - 8*b*x*Sin[2*(a + b*x)]
 - 4*b*x*Sin[4*(a + b*x)] - 16*b^2*x^2*Sin[2*a]*SinIntegral[2*b*x] - 16*b^2
*x^2*Sin[4*a]*SinIntegral[4*b*x])/(16*x^2)
```

Maple [A] time = 0.033, size = 124, normalized size = 1.4

$$b^2\left(-\frac{\cos(4bx+4a)}{16x^2b^2} + \frac{\sin(4bx+4a)}{4bx} + \text{Si}(4bx)\sin(4a) - \text{Ci}(4bx)\cos(4a) - \frac{\cos(2bx+2a)}{4x^2b^2} + \frac{\sin(2bx+2a)}{2bx} + \text{Si}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4/x^3,x)
```

```
[Out] b^2*(-1/16*cos(4*b*x+4*a)/x^2/b^2+1/4*sin(4*b*x+4*a)/x/b+Si(4*b*x)*sin(4*a)
-Ci(4*b*x)*cos(4*a)-1/4*cos(2*b*x+2*a)/x^2/b^2+1/2*sin(2*b*x+2*a)/x/b+Si(2*
b*x)*sin(2*a)-Ci(2*b*x)*cos(2*a)-3/16/x^2/b^2)
```

Maxima [C] time = 1.38394, size = 1073, normalized size = 11.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/x^3,x, algorithm="maxima")
```

```
[Out] -1/2097152*(65536*((exp_integral_e(3, 4*I*b*x) + exp_integral_e(3, -4*I*b*x
))*cos(2*a)^2 + (exp_integral_e(3, 4*I*b*x) + exp_integral_e(3, -4*I*b*x))*
sin(2*a)^2)*cos(4*a)^3 - ((65536*I*exp_integral_e(3, 4*I*b*x) - 65536*I*exp
_integral_e(3, -4*I*b*x))*cos(2*a)^2 + (65536*I*exp_integral_e(3, 4*I*b*x)
- 65536*I*exp_integral_e(3, -4*I*b*x))*sin(2*a)^2)*sin(4*a)^3 + (262144*(ex
p_integral_e(3, 2*I*b*x) + exp_integral_e(3, -2*I*b*x))*cos(2*a)^3 - (26214
4*I*exp_integral_e(3, 2*I*b*x) - 262144*I*exp_integral_e(3, -2*I*b*x))*sin(
2*a)^3 + 131072*(2*(exp_integral_e(3, 2*I*b*x) + exp_integral_e(3, -2*I*b*x
))*cos(2*a) + 3)*sin(2*a)^2 + 262144*(exp_integral_e(3, 2*I*b*x) + exp_inte
gral_e(3, -2*I*b*x))*cos(2*a) + 393216*cos(2*a)^2 - ((262144*I*exp_integral
_e(3, 2*I*b*x) - 262144*I*exp_integral_e(3, -2*I*b*x))*cos(2*a)^2 + 262144*
I*exp_integral_e(3, 2*I*b*x) - 262144*I*exp_integral_e(3, -2*I*b*x))*sin(2*
a))*cos(4*a)^2 + (262144*(exp_integral_e(3, 2*I*b*x) + exp_integral_e(3, -2
*I*b*x))*cos(2*a)^3 - (262144*I*exp_integral_e(3, 2*I*b*x) - 262144*I*exp_i
ntegral_e(3, -2*I*b*x))*sin(2*a)^3 + 131072*(2*(exp_integral_e(3, 2*I*b*x)
+ exp_integral_e(3, -2*I*b*x))*cos(2*a) + 3)*sin(2*a)^2 + 65536*((exp_integ
ral_e(3, 4*I*b*x) + exp_integral_e(3, -4*I*b*x))*cos(2*a)^2 + (exp_integral
_e(3, 4*I*b*x) + exp_integral_e(3, -4*I*b*x))*sin(2*a)^2)*cos(4*a) + 262144
*(exp_integral_e(3, 2*I*b*x) + exp_integral_e(3, -2*I*b*x))*cos(2*a) + 3932
16*cos(2*a)^2 - ((262144*I*exp_integral_e(3, 2*I*b*x) - 262144*I*exp_integr
al_e(3, -2*I*b*x))*cos(2*a)^2 + 262144*I*exp_integral_e(3, 2*I*b*x) - 26214
4*I*exp_integral_e(3, -2*I*b*x))*sin(2*a))*sin(4*a)^2 + 65536*((exp_integra
l_e(3, 4*I*b*x) + exp_integral_e(3, -4*I*b*x))*cos(2*a)^2 + (exp_integral_e
(3, 4*I*b*x) + exp_integral_e(3, -4*I*b*x))*sin(2*a)^2)*cos(4*a) - (((65536
*I*exp_integral_e(3, 4*I*b*x) - 65536*I*exp_integral_e(3, -4*I*b*x))*cos(2*
a)^2 + (65536*I*exp_integral_e(3, 4*I*b*x) - 65536*I*exp_integral_e(3, -4*I
*b*x))*sin(2*a)^2)*cos(4*a)^2 + (65536*I*exp_integral_e(3, 4*I*b*x) - 65536
*I*exp_integral_e(3, -4*I*b*x))*cos(2*a)^2 + (65536*I*exp_integral_e(3, 4*I
*b*x) - 65536*I*exp_integral_e(3, -4*I*b*x))*sin(2*a)^2)*sin(4*a))*b^2/(((c
os(2*a)^2 + sin(2*a)^2)*cos(4*a)^2 + (cos(2*a)^2 + sin(2*a)^2)*sin(4*a)^2)*
(b*x + a)^2 + (a^2*cos(2*a)^2 + a^2*sin(2*a)^2)*cos(4*a)^2 + (a^2*cos(2*a)^
2 + a^2*sin(2*a)^2)*sin(4*a)^2 - 2*((a*cos(2*a)^2 + a*sin(2*a)^2)*cos(4*a)^
2 + (a*cos(2*a)^2 + a*sin(2*a)^2)*sin(4*a)^2)*(b*x + a))
```

Fricas [A] time = 1.44777, size = 389, normalized size = 4.32

$$\frac{4\,bx\cos\left(bx+a\right)^{3}\sin\left(bx+a\right)+2\,b^{2}x^{2}\sin\left(4\,a\right)\operatorname{Si}\left(4\,bx\right)+2\,b^{2}x^{2}\sin\left(2\,a\right)\operatorname{Si}\left(2\,bx\right)-\cos\left(bx+a\right)^{4}-\left(b^{2}x^{2}\operatorname{Ci}\left(4\,bx\right)+\right.}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*b*x*cos(b*x + a)^3*sin(b*x + a) + 2*b^2*x^2*sin(4*a)*sin_integral(4*
b*x) + 2*b^2*x^2*sin(2*a)*sin_integral(2*b*x) - cos(b*x + a)^4 - (b^2*x^2*c
os_integral(4*b*x) + b^2*x^2*cos_integral(-4*b*x))*cos(4*a) - (b^2*x^2*cos_
integral(2*b*x) + b^2*x^2*cos_integral(-2*b*x))*cos(2*a))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^{4}\left(a+bx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4/x**3,x)
```

```
[Out] Integral(cos(a + b*x)**4/x**3, x)
```

Giac [C] time = 1.26472, size = 5292, normalized size = 58.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*b^2*x^2*real_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2
*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(
b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(-2*b*x))*tan(
2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(
-4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 8*b^2*x^2*imag_part(
cos_integral(2*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a) - 8*b^2*x^2*
imag_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a) +
 16*b^2*x^2*sin_integral(2*b*x)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a) +
 8*b^2*x^2*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*
tan(a)^2 - 8*b^2*x^2*imag_part(cos_integral(-4*b*x))*tan(2*b*x)^2*tan(b*x)^
2*tan(2*a)*tan(a)^2 + 16*b^2*x^2*sin_integral(4*b*x)*tan(2*b*x)^2*tan(b*x)^
2*tan(2*a)*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(4*b*x))*tan(2*b*x)^2
*tan(b*x)^2*tan(2*a)^2 - 4*b^2*x^2*real_part(cos_integral(2*b*x))*tan(2*b*x
)^2*tan(b*x)^2*tan(2*a)^2 - 4*b^2*x^2*real_part(cos_integral(-2*b*x))*tan(2
*b*x)^2*tan(b*x)^2*tan(2*a)^2 + 4*b^2*x^2*real_part(cos_integral(-4*b*x))*t
an(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 - 4*b^2*x^2*real_part(cos_integral(4*b*x)
)*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(2*b*x
))*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(-2*b
*x))*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 - 4*b^2*x^2*real_part(cos_integral(-4
*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(
4*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral
(2*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integra
l(-2*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integ
ral(-4*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_int
egral(4*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_inte
gral(2*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integ
ral(-2*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integ
ral(-4*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 8*b^2*x^2*imag_part(cos_integ
ral(4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a) - 8*b^2*x^2*imag_part(cos_inte
gral(-4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a) + 16*b^2*x^2*sin_integral(4*
b*x)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a) + 8*b^2*x^2*imag_part(cos_integral(2*
b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(a) - 8*b^2*x^2*imag_part(cos_integral(-2*
b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(a) + 16*b^2*x^2*sin_integral(2*b*x)*tan(2
*b*x)^2*tan(b*x)^2*tan(a) + 8*b^2*x^2*imag_part(cos_integral(2*b*x))*tan(2*
b*x)^2*tan(2*a)^2*tan(a) - 8*b^2*x^2*imag_part(cos_integral(-2*b*x))*tan(2*
b*x)^2*tan(2*a)^2*tan(a) + 16*b^2*x^2*sin_integral(2*b*x)*tan(2*b*x)^2*tan(
2*a)^2*tan(a) + 8*b^2*x^2*imag_part(cos_integral(2*b*x))*tan(b*x)^2*tan(2*a
)^2*tan(a) - 8*b^2*x^2*imag_part(cos_integral(-2*b*x))*tan(b*x)^2*tan(2*a)^
2*tan(a) + 16*b^2*x^2*sin_integral(2*b*x)*tan(b*x)^2*tan(2*a)^2*tan(a) + 8*
b^2*x^2*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(2*a)*tan(a)^2 - 8*b
^2*x^2*imag_part(cos_integral(-4*b*x))*tan(2*b*x)^2*tan(2*a)*tan(a)^2 + 16*
b^2*x^2*sin_integral(4*b*x)*tan(2*b*x)^2*tan(2*a)*tan(a)^2 + 8*b^2*x^2*imag
_part(cos_integral(4*b*x))*tan(b*x)^2*tan(2*a)*tan(a)^2 - 8*b^2*x^2*imag_pa
rt(cos_integral(-4*b*x))*tan(b*x)^2*tan(2*a)*tan(a)^2 + 16*b^2*x^2*sin_inte
gral(4*b*x)*tan(b*x)^2*tan(2*a)*tan(a)^2 - 4*b^2*x^2*real_part(cos_integral
```

```
(4*b*x))*tan(2*b*x)^2*tan(b*x)^2 - 4*b^2*x^2*real_part(cos_integral(2*b*x))
*tan(2*b*x)^2*tan(b*x)^2 - 4*b^2*x^2*real_part(cos_integral(-2*b*x))*tan(2*
b*x)^2*tan(b*x)^2 - 4*b^2*x^2*real_part(cos_integral(-4*b*x))*tan(2*b*x)^2*
tan(b*x)^2 + 4*b^2*x^2*real_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(2*a)
^2 - 4*b^2*x^2*real_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(2*a)^2 - 4*b
^2*x^2*real_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(2*a)^2 + 4*b^2*x^2*
real_part(cos_integral(-4*b*x))*tan(2*b*x)^2*tan(2*a)^2 + 4*b^2*x^2*real_pa
rt(cos_integral(4*b*x))*tan(b*x)^2*tan(2*a)^2 - 4*b^2*x^2*real_part(cos_int
egral(2*b*x))*tan(b*x)^2*tan(2*a)^2 - 4*b^2*x^2*real_part(cos_integral(-2*b
*x))*tan(b*x)^2*tan(2*a)^2 + 4*b^2*x^2*real_part(cos_integral(-4*b*x))*tan(
b*x)^2*tan(2*a)^2 - 8*b*x*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a) - 4*b^2
*x^2*real_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(a)^2 + 4*b^2*x^2*real_
part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_i
ntegral(-2*b*x))*tan(2*b*x)^2*tan(a)^2 - 4*b^2*x^2*real_part(cos_integral(-
4*b*x))*tan(2*b*x)^2*tan(a)^2 - 4*b^2*x^2*real_part(cos_integral(4*b*x))*ta
n(b*x)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(2*b*x))*tan(b*x)^2*tan
(a)^2 + 4*b^2*x^2*real_part(cos_integral(-2*b*x))*tan(b*x)^2*tan(a)^2 - 4*b
^2*x^2*real_part(cos_integral(-4*b*x))*tan(b*x)^2*tan(a)^2 - 4*b*x*tan(2*b*
x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(4*b*x)
)*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(2*b*x))*tan(2*a)^2
*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(-2*b*x))*tan(2*a)^2*tan(a)^2 +
 4*b^2*x^2*real_part(cos_integral(-4*b*x))*tan(2*a)^2*tan(a)^2 - 8*b*x*tan(
2*b*x)^2*tan(b*x)*tan(2*a)^2*tan(a)^2 - 4*b*x*tan(2*b*x)*tan(b*x)^2*tan(2*a
)^2*tan(a)^2 + 8*b^2*x^2*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(2*
a) - 8*b^2*x^2*imag_part(cos_integral(-4*b*x))*tan(2*b*x)^2*tan(2*a) + 16*b
^2*x^2*sin_integral(4*b*x)*tan(2*b*x)^2*tan(2*a) + 8*b^2*x^2*imag_part(cos_
integral(4*b*x))*tan(b*x)^2*tan(2*a) - 8*b^2*x^2*imag_part(cos_integral(-4*
b*x))*tan(b*x)^2*tan(2*a) + 16*b^2*x^2*sin_integral(4*b*x)*tan(b*x)^2*tan(2
*a) + 8*b^2*x^2*imag_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(a) - 8*b^2*
x^2*imag_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(a) + 16*b^2*x^2*sin_in
tegral(2*b*x)*tan(2*b*x)^2*tan(a) + 8*b^2*x^2*imag_part(cos_integral(2*b*x)
)*tan(b*x)^2*tan(a) - 8*b^2*x^2*imag_part(cos_integral(-2*b*x))*tan(b*x)^2*
tan(a) + 16*b^2*x^2*sin_integral(2*b*x)*tan(b*x)^2*tan(a) + 8*b^2*x^2*imag_
part(cos_integral(2*b*x))*tan(2*a)^2*tan(a) - 8*b^2*x^2*imag_part(cos_integ
ral(-2*b*x))*tan(2*a)^2*tan(a) + 16*b^2*x^2*sin_integral(2*b*x)*tan(2*a)^2*
tan(a) + 8*b^2*x^2*imag_part(cos_integral(4*b*x))*tan(2*a)*tan(a)^2 - 8*b^2
*x^2*imag_part(cos_integral(-4*b*x))*tan(2*a)*tan(a)^2 + 16*b^2*x^2*sin_int
egral(4*b*x)*tan(2*a)*tan(a)^2 - 4*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a
)^2 - 4*b^2*x^2*real_part(cos_integral(4*b*x))*tan(2*b*x)^2 - 4*b^2*x^2*rea
l_part(cos_integral(2*b*x))*tan(2*b*x)^2 - 4*b^2*x^2*real_part(cos_integral
(-2*b*x))*tan(2*b*x)^2 - 4*b^2*x^2*real_part(cos_integral(-4*b*x))*tan(2*b*
x)^2 - 4*b^2*x^2*real_part(cos_integral(4*b*x))*tan(b*x)^2 - 4*b^2*x^2*real
_part(cos_integral(2*b*x))*tan(b*x)^2 - 4*b^2*x^2*real_part(cos_integral(-2
*b*x))*tan(b*x)^2 - 4*b^2*x^2*real_part(cos_integral(-4*b*x))*tan(b*x)^2 -
4*b*x*tan(2*b*x)^2*tan(b*x)^2*tan(2*a) + 4*b^2*x^2*real_part(cos_integral(4
*b*x))*tan(2*a)^2 - 4*b^2*x^2*real_part(cos_integral(2*b*x))*tan(2*a)^2 - 4
*b^2*x^2*real_part(cos_integral(-2*b*x))*tan(2*a)^2 + 4*b^2*x^2*real_part(c
os_integral(-4*b*x))*tan(2*a)^2 + 8*b*x*tan(2*b*x)^2*tan(b*x)*tan(2*a)^2 -
4*b*x*tan(2*b*x)*tan(b*x)^2*tan(2*a)^2 - 8*b*x*tan(2*b*x)^2*tan(b*x)^2*tan(
a) + 8*b*x*tan(2*b*x)^2*tan(2*a)^2*tan(a) - 8*b*x*tan(b*x)^2*tan(2*a)^2*tan
(a) - 4*b^2*x^2*real_part(cos_integral(4*b*x))*tan(a)^2 + 4*b^2*x^2*real_pa
rt(cos_integral(2*b*x))*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(-2*b*x)
)*tan(a)^2 - 4*b^2*x^2*real_part(cos_integral(-4*b*x))*tan(a)^2 - 8*b*x*tan
(2*b*x)^2*tan(b*x)*tan(a)^2 + 4*b*x*tan(2*b*x)*tan(b*x)^2*tan(a)^2 - 4*b*x*
tan(2*b*x)^2*tan(2*a)*tan(a)^2 + 4*b*x*tan(b*x)^2*tan(2*a)*tan(a)^2 - 4*b*x
*tan(2*b*x)*tan(2*a)^2*tan(a)^2 - 8*b*x*tan(b*x)*tan(2*a)^2*tan(a)^2 + 8*b^
2*x^2*imag_part(cos_integral(4*b*x))*tan(2*a) - 8*b^2*x^2*imag_part(cos_int
egral(-4*b*x))*tan(2*a) + 16*b^2*x^2*sin_integral(4*b*x)*tan(2*a) + 8*b^2*x
^2*imag_part(cos_integral(2*b*x))*tan(a) - 8*b^2*x^2*imag_part(cos_integral
```

```
(-2*b*x))*tan(a) + 16*b^2*x^2*sin_integral(2*b*x)*tan(a) + 8*tan(2*b*x)^2*t
an(b*x)*tan(2*a)^2*tan(a) - 3*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 + 2*tan(2*b*
x)*tan(b*x)^2*tan(2*a)*tan(a)^2 - 3*tan(b*x)^2*tan(2*a)^2*tan(a)^2 - 4*b^2*
x^2*real_part(cos_integral(4*b*x)) - 4*b^2*x^2*real_part(cos_integral(2*b*x
)) - 4*b^2*x^2*real_part(cos_integral(-2*b*x)) - 4*b^2*x^2*real_part(cos_in
tegral(-4*b*x)) + 8*b*x*tan(2*b*x)^2*tan(b*x) + 4*b*x*tan(2*b*x)*tan(b*x)^2
 - 4*b*x*tan(2*b*x)^2*tan(2*a) + 4*b*x*tan(b*x)^2*tan(2*a) - 4*b*x*tan(2*b*
x)*tan(2*a)^2 + 8*b*x*tan(b*x)*tan(2*a)^2 + 8*b*x*tan(2*b*x)^2*tan(a) - 8*b
*x*tan(b*x)^2*tan(a) + 8*b*x*tan(2*a)^2*tan(a) + 4*b*x*tan(2*b*x)*tan(a)^2
- 8*b*x*tan(b*x)*tan(a)^2 + 4*b*x*tan(2*a)*tan(a)^2 + tan(2*b*x)^2*tan(b*x)
^2 + 2*tan(2*b*x)*tan(b*x)^2*tan(2*a) - 4*tan(2*b*x)^2*tan(2*a)^2 + tan(b*x
)^2*tan(2*a)^2 + 8*tan(2*b*x)^2*tan(b*x)*tan(a) + 8*tan(b*x)*tan(2*a)^2*tan
(a) + tan(2*b*x)^2*tan(a)^2 - 4*tan(b*x)^2*tan(a)^2 + 2*tan(2*b*x)*tan(2*a)
*tan(a)^2 + tan(2*a)^2*tan(a)^2 + 4*b*x*tan(2*b*x) + 8*b*x*tan(b*x) + 4*b*x
*tan(2*a) + 8*b*x*tan(a) - 3*tan(2*b*x)^2 + 2*tan(2*b*x)*tan(2*a) - 3*tan(2
*a)^2 + 8*tan(b*x)*tan(a) - 4)/(x^2*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(
a)^2 + x^2*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 + x^2*tan(2*b*x)^2*tan(b*x)^2
*tan(a)^2 + x^2*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 + x^2*tan(b*x)^2*tan(2*a)^
2*tan(a)^2 + x^2*tan(2*b*x)^2*tan(b*x)^2 + x^2*tan(2*b*x)^2*tan(2*a)^2 + x^
2*tan(b*x)^2*tan(2*a)^2 + x^2*tan(2*b*x)^2*tan(a)^2 + x^2*tan(b*x)^2*tan(a)
^2 + x^2*tan(2*a)^2*tan(a)^2 + x^2*tan(2*b*x)^2 + x^2*tan(b*x)^2 + x^2*tan(
2*a)^2 + x^2*tan(a)^2 + x^2)
```

3.29 $\int (c+dx)^3 \sec(a+bx)\,dx$

Optimal. Leaf size=205

$$-\frac{6d^2(c+dx)\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-$$

```
[Out] ((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + ((3*I)*d*(c + d*x)^2*PolyL
og[2, (-I)*E^(I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(
a + b*x))])/b^2 - (6*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 +
(6*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*PolyLog[4,
 (-I)*E^(I*(a + b*x))])/b^4 + ((6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4
```

Rubi [A] time = 0.15739, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4181, 2531, 6609, 2282, 6589}

$$-\frac{6d^2(c+dx)\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x],x]
```


```
[Out] ((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + ((3*I)*d*(c + d*x)^2*PolyL
og[2, (-I)*E^(I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(
a + b*x))])/b^2 - (6*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 +
(6*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*PolyLog[4,
 (-I)*E^(I*(a + b*x))])/b^4 + ((6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4
```


Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3\sec(a+bx)\,dx = -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{(3d)\int (c+dx)^2\log\left(1-ie^{i(a+bx)}\right)dx}{b} + \frac{(3d)\int (c+dx)^2\log\left(1\right.}{b}$$
$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2} - \frac{3id(c+dx)^2\mathrm{Li}_2\left(ie^{i(a+bx)}\right)}{b^2} - \frac{(6}{}$$
$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2} - \frac{3id(c+dx)^2\mathrm{Li}_2\left(ie^{i(a+bx)}\right)}{b^2} - \frac{6c}{}$$
$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2} - \frac{3id(c+dx)^2\mathrm{Li}_2\left(ie^{i(a+bx)}\right)}{b^2} - \frac{6c}{}$$
$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2} - \frac{3id(c+dx)^2\mathrm{Li}_2\left(ie^{i(a+bx)}\right)}{b^2} - \frac{6c}{}$$

Mathematica [A] time = 0.188366, size = 196, normalized size = 0.96

$$-\frac{i\left(-3d\left(b^2(c+dx)^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)+2ibd(c+dx)\mathrm{Li}_3\left(-ie^{i(a+bx)}\right)-2d^2\mathrm{Li}_4\left(-ie^{i(a+bx)}\right)\right)+3d\left(b^2(c+dx)^2\mathrm{Li}_2\left(ie^{i(a+bx)}\right)+2\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x],x]
```

```
[Out] ((-I)*(2*b^3*(c + d*x)^3*ArcTan[E^(I*(a + b*x))] - 3*d*(b^2*(c + d*x)^2*Pol
yLog[2, (-I)*E^(I*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[3, (-I)*E^(I*(a
 + b*x))] - 2*d^2*PolyLog[4, (-I)*E^(I*(a + b*x))]) + 3*d*(b^2*(c + d*x)^2*
PolyLog[2, I*E^(I*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[3, I*E^(I*(a +
b*x))] - 2*d^2*PolyLog[4, I*E^(I*(a + b*x))])))/b^4
```

Maple [B] time = 0.397, size = 685, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a),x)
```

```
[Out] -1/b^4*d^3*a^3*ln(1+I*exp(I*(b*x+a)))+6/b^3*d^3*polylog(3,I*exp(I*(b*x+a)))
*x+6/b^3*c*d^2*polylog(3,I*exp(I*(b*x+a)))-6/b^3*d^3*polylog(3,-I*exp(I*(b*
x+a)))*x-2*I/b*c^3*arctan(exp(I*(b*x+a)))-6*I*d^3*polylog(4,-I*exp(I*(b*x+a
)))/b^4+1/b^4*d^3*a^3*ln(1-I*exp(I*(b*x+a)))-6/b^3*c*d^2*polylog(3,-I*exp(I
```

```
*(b*x+a)))-1/b*d^3*ln(1+I*exp(I*(b*x+a)))*x^3+1/b*d^3*ln(1-I*exp(I*(b*x+a))
)*x^3+3/b^3*a^2*c*d^2*ln(1+I*exp(I*(b*x+a)))-3/b*c*d^2*ln(1+I*exp(I*(b*x+a)
))*x^2+3*I/b^2*d^3*polylog(2,-I*exp(I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog(2,I
*exp(I*(b*x+a)))*x^2+3*I/b^2*c^2*d*polylog(2,-I*exp(I*(b*x+a)))+2*I/b^4*d^3
*a^3*arctan(exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,I*exp(I*(b*x+a)))+3/b*c
*d^2*ln(1-I*exp(I*(b*x+a)))*x^2-3/b*c^2*d*ln(1+I*exp(I*(b*x+a)))*x-3/b^2*c^
2*d*ln(1+I*exp(I*(b*x+a)))*a-3/b^3*a^2*c*d^2*ln(1-I*exp(I*(b*x+a)))+3/b*c^2
*d*ln(1-I*exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-I*exp(I*(b*x+a)))*a+6*I*d^3*po
lylog(4,I*exp(I*(b*x+a)))/b^4-6*I/b^3*c*d^2*a^2*arctan(exp(I*(b*x+a)))+6*I/
b^2*c^2*d*a*arctan(exp(I*(b*x+a)))+6*I/b^2*c*d^2*polylog(2,-I*exp(I*(b*x+a)
))*x-6*I/b^2*c*d^2*polylog(2,I*exp(I*(b*x+a)))*x
```

Maxima [B] time = 2.07532, size = 961, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*c^3*log(sec(b*x + a) + tan(b*x + a)) - 6*a*c^2*d*log(sec(b*x + a) +
tan(b*x + a))/b + 6*a^2*c*d^2*log(sec(b*x + a) + tan(b*x + a))/b^2 - 2*a^3*
d^3*log(sec(b*x + a) + tan(b*x + a))/b^3 + (12*I*d^3*polylog(4, I*e^(I*b*x
+ I*a)) - 12*I*d^3*polylog(4, -I*e^(I*b*x + I*a)) + (-2*I*(b*x + a)^3*d^3 +
 (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2
- 6*I*a^2*d^3)*(b*x + a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (-2*I*(
b*x + a)^3*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d +
 12*I*a*b*c*d^2 - 6*I*a^2*d^3)*(b*x + a))*arctan2(cos(b*x + a), -sin(b*x +
a) + 1) + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*
d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*dilog(I*e^(I*b*x + I*a)) + (6
*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b
*c*d^2 - 12*I*a*d^3)*(b*x + a))*dilog(-I*e^(I*b*x + I*a)) + ((b*x + a)^3*d^
3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)
*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - ((b
*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^
2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a
) + 1) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, I*e^(I*b*x + I*a))
 - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, -I*e^(I*b*x + I*a)))/b^3
)/b
```

Fricas [C] time = 1.81295, size = 2367, normalized size = 11.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, I*cos(b*x + a) + sin(b*x + a)) + 6*I*d^3*polylog(4,
 I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) + sin(
b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) + (-3*I*b^2*
d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(I*cos(b*x + a) + sin(b*x +
 a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(I*cos(b*x
 + a) - sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)
*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x
```

```
 + 3*I*b^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (b^3*c^3 - 3*a*b^
2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin(b*x + a) + I) -
 (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I*s
in(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1) -
(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*
d^2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(
-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) -
sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log
(-cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b*d^3*x + b*c*
d^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*poly
log(3, I*cos(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -I
*cos(b*x + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x
 + a) - sin(b*x + a)))/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^3 \sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*sec(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a), x)
```

3.30 $\int (c+dx)^2 \sec(a+bx)\,dx$

Optimal. Leaf size=137

$$\frac{2id(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-\frac{2d^2\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2d^2\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}-\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b}$$

```
[Out] ((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + ((2*I)*d*(c + d*x)*PolyLog
[2, (-I)*E^(I*(a + b*x))])/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, I*E^(I*(a +
b*x))])/b^2 - (2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 + (2*d^2*PolyLog
[3, I*E^(I*(a + b*x))])/b^3
```

Rubi [A] time = 0.0920892, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4181, 2531, 2282, 6589}

$$\frac{2id(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-\frac{2d^2\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2d^2\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}-\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + ((2*I)*d*(c + d*x)*PolyLog
[2, (-I)*E^(I*(a + b*x))])/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, I*E^(I*(a +
b*x))])/b^2 - (2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 + (2*d^2*PolyLog
[3, I*E^(I*(a + b*x))])/b^3
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \sec(a+bx)\,dx = -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{(2d)\int (c+dx)\log\left(1-ie^{i(a+bx)}\right)dx}{b} + \frac{(2d)\int (c+dx)\log\left(1+\right.}{b}$$
$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2} - \frac{2id(c+dx)\mathrm{Li}_2\left(ie^{i(a+bx)}\right)}{b^2} - \frac{(2id}{}$$
$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2} - \frac{2id(c+dx)\mathrm{Li}_2\left(ie^{i(a+bx)}\right)}{b^2} - \frac{(2d^2}{}$$
$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2} - \frac{2id(c+dx)\mathrm{Li}_2\left(ie^{i(a+bx)}\right)}{b^2} - \frac{2d^2\mathrm{L}}{}$$

Mathematica [A] time = 0.108983, size = 130, normalized size = 0.95

$$-\frac{2i\left(b^2(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)-d\left(b(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)+id\mathrm{Li}_3\left(-ie^{i(a+bx)}\right)\right)+d\left(b(c+dx)\mathrm{Li}_2\left(ie^{i(a+bx)}\right)+id\mathrm{Li}_3\left(ie^{i(}\right.\right.\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*(b^2*(c + d*x)^2*ArcTan[E^(I*(a + b*x))] - d*(b*(c + d*x)*PolyLog[2
, (-I)*E^(I*(a + b*x))] + I*d*PolyLog[3, (-I)*E^(I*(a + b*x))]) + d*(b*(c +
 d*x)*PolyLog[2, I*E^(I*(a + b*x))] + I*d*PolyLog[3, I*E^(I*(a + b*x))])))/
b^3
```

Maple [B] time = 0.366, size = 392, normalized size = 2.9

$$\frac{4\,icda\arctan\left(e^{i(bx+a)}\right)}{b^2} - \frac{2\,id^2\mathrm{polylog}\left(2, ie^{i(bx+a)}\right)x}{b^2} + \frac{2\,idc\mathrm{polylog}\left(2, -ie^{i(bx+a)}\right)}{b^2} - \frac{2\,idc\mathrm{polylog}\left(2, ie^{i(bx+a)}\right)}{b^2} + \frac{2\,id^2\mathrm{po}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a),x)
```

```
[Out] 4*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))-2*I/b^2*d^2*polylog(2,I*exp(I*(b*x+a))
)*x+2*I/b^2*c*d*polylog(2,-I*exp(I*(b*x+a)))-2*I/b^2*c*d*polylog(2,I*exp(I*
(b*x+a)))+2*I/b^2*d^2*polylog(2,-I*exp(I*(b*x+a)))*x-1/b*d^2*ln(1+I*exp(I*(
b*x+a)))*x^2+2*d^2*polylog(3,I*exp(I*(b*x+a)))/b^3-2*I/b^3*d^2*a^2*arctan(e
xp(I*(b*x+a)))+2/b*c*d*ln(1-I*exp(I*(b*x+a)))*x-2/b*c*d*ln(1+I*exp(I*(b*x+a
)))*x+1/b*d^2*ln(1-I*exp(I*(b*x+a)))*x^2-1/b^3*a^2*d^2*ln(1-I*exp(I*(b*x+a)
))+1/b^3*a^2*d^2*ln(1+I*exp(I*(b*x+a)))-2/b^2*c*d*ln(1+I*exp(I*(b*x+a)))*a-
2*I/b*c^2*arctan(exp(I*(b*x+a)))-2*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+2/b
^2*c*d*ln(1-I*exp(I*(b*x+a)))*a
```

Maxima [B] time = 1.85081, size = 535, normalized size = 3.91

$$2\,c^2\log\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right) - \frac{4\,acd\log(\sec(bx+a)+\tan(bx+a))}{b} + \frac{2\,a^2d^2\log(\sec(bx+a)+\tan(bx+a))}{b^2} + \frac{4\,d^2\mathrm{Li}_3(i\,e^{(i\,bx+i\,a)})-4\,d^2\mathrm{Li}_3(-i\,e^{(i}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*c^2*log(sec(b*x + a) + tan(b*x + a)) - 4*a*c*d*log(sec(b*x + a) + ta
n(b*x + a))/b + 2*a^2*d^2*log(sec(b*x + a) + tan(b*x + a))/b^2 + (4*d^2*pol
ylog(3, I*e^(I*b*x + I*a)) - 4*d^2*polylog(3, -I*e^(I*b*x + I*a)) + (-2*I*(
b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a))*arctan2(cos(b*x + a),
sin(b*x + a) + 1) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x +
 a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) + (-4*I*b*c*d - 4*I*(b*x + a)
*d^2 + 4*I*a*d^2)*dilog(I*e^(I*b*x + I*a)) + (4*I*b*c*d + 4*I*(b*x + a)*d^2
 - 4*I*a*d^2)*dilog(-I*e^(I*b*x + I*a)) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d
^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) -
((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*
x + a)^2 - 2*sin(b*x + a) + 1))/b^2)/b
```

Fricas [C] time = 1.70654, size = 1534, normalized size = 11.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*polylog(3, I*
cos(b*x + a) - sin(b*x + a)) + 2*d^2*polylog(3, -I*cos(b*x + a) + sin(b*x +
 a)) - 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - (-2*I*b*d^2*x - 2
*I*b*c*d)*dilog(I*cos(b*x + a) + sin(b*x + a)) - (-2*I*b*d^2*x - 2*I*b*c*d)
*dilog(I*cos(b*x + a) - sin(b*x + a)) - (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-I*
cos(b*x + a) + sin(b*x + a)) - (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-I*cos(b*x +
 a) - sin(b*x + a)) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) + I*
sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) - I*si
n(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*c
os(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d -
a^2*d^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*
x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2
*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x + a
) + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a)
 + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a)
+ I))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \sec (a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*sec(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \sec(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a), x)
```

3.31 $\int (c+dx)\sec(a+bx)\,dx$

Optimal. Leaf size=75

$$\frac{id\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{id\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}$$

```
[Out] ((-2*I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b + (I*d*PolyLog[2, (-I)*E^(I*(a
 + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2
```

Rubi [A] time = 0.0415003, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4181, 2279, 2391}

$$\frac{id\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{id\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b + (I*d*PolyLog[2, (-I)*E^(I*(a
 + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\sec(a+bx)\,dx &= -\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{d\int \log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{d\int \log\left(1+ie^{i(a+bx)}\right)dx}{b}\\ &= -\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{(id)\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{i(a+bx)}\right)}{b^2}-\frac{(id)\,\text{Subst}\left(\int \frac{\log(1+i}{x}\right.}{b^2}\\ &= -\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{id\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{id\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.0061443, size = 87, normalized size = 1.16

$$\frac{id\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{id\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}+\frac{c\tanh^{-1}(\sin(a+bx))}{b}-\frac{2idx\tan^{-1}\left(e^{ia+ibx}\right)}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*d*x*ArcTan[E^(I*a + I*b*x)])/b + (c*ArcTanh[Sin[a + b*x]])/b + (I*d
*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))]
)/b^2
```

Maple [B] time = 0.368, size = 167, normalized size = 2.2

$$\frac{-2\,ic\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b}-\frac{d\ln\left(1+i\mathrm{e}^{i(bx+a)}\right)x}{b}-\frac{d\ln\left(1+i\mathrm{e}^{i(bx+a)}\right)a}{b^2}+\frac{d\ln\left(1-i\mathrm{e}^{i(bx+a)}\right)x}{b}+\frac{d\ln\left(1-i\mathrm{e}^{i(bx+a)}\right)a}{b^2}+\frac{id\,\mathrm{dilo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a),x)
```

```
[Out] -2*I/b*c*arctan(exp(I*(b*x+a)))-1/b*d*ln(1+I*exp(I*(b*x+a)))*x-1/b^2*d*ln(1
+I*exp(I*(b*x+a)))*a+1/b*d*ln(1-I*exp(I*(b*x+a)))*x+1/b^2*d*ln(1-I*exp(I*(b
*x+a)))*a+I/b^2*d*dilog(1+I*exp(I*(b*x+a)))-I/b^2*d*dilog(1-I*exp(I*(b*x+a)
))+2*I/b^2*d*a*arctan(exp(I*(b*x+a)))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.56131, size = 837, normalized size = 11.16

$$\frac{-i\,d\text{Li}_2\left(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)-i\,d\text{Li}_2\left(i\,\cos\left(bx+a\right)-\sin\left(bx+a\right)\right)+i\,d\text{Li}_2\left(-i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)+i\,d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) - I*d*dilog(I*cos(b*x + a) -
 sin(b*x + a)) + I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(-I*c
os(b*x + a) - sin(b*x + a)) + (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x + a)
 + I) - (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) + (b*d*x + a*d)*
log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*x + a*d)*log(I*cos(b*x + a) -
 sin(b*x + a) + 1) + (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)
```

```
- (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d)*log(-
cos(b*x + a) + I*sin(b*x + a) + I) - (b*c - a*d)*log(-cos(b*x + a) - I*sin(
b*x + a) + I))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \sec (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)*sec(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \sec (bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a), x)
```

3.32 $\int \frac{\sec(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\sec(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable[Sec[a + b*x]/(c + d*x), x]
```

Rubi [A] time = 0.0224226, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[a + b*x]/(c + d*x),x]
```

```
[Out] Defer[Int][Sec[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\sec(a+bx)}{c+dx}\,dx = \int \frac{\sec(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 4.46836, size = 0, normalized size = 0.

$$\int \frac{\sec(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[a + b*x]/(c + d*x),x]
```

```
[Out] Integrate[Sec[a + b*x]/(c + d*x), x]
```

Maple [A] time = 0.243, size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)/(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(bx+a)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(sec(a + b*x)/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)/(d*x + c), x)
```

3.33 $\int (c+dx)^3 \sec^2(a+bx)\,dx$

Optimal. Leaf size=114

$$-\frac{3id^2(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2}+\frac{3d^3\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^4}+\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{i(c+dx)^3}{b}$$

```
[Out] ((-I)*(c + d*x)^3)/b + (3*d*(c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b^2 -
 ((3*I)*d^2*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^3 + (3*d^3*PolyLo
g[3, -E^((2*I)*(a + b*x))])/(2*b^4) + ((c + d*x)^3*Tan[a + b*x])/b
```

Rubi [A] time = 0.215341, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4184, 3719, 2190, 2531, 2282, 6589}

$$-\frac{3id^2(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2}+\frac{3d^3\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^4}+\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{i(c+dx)^3}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^3)/b + (3*d*(c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b^2 -
 ((3*I)*d^2*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^3 + (3*d^3*PolyLo
g[3, -E^((2*I)*(a + b*x))])/(2*b^4) + ((c + d*x)^3*Tan[a + b*x])/b
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3\sec^2(a+bx)\,dx = \frac{(c+dx)^3\tan(a+bx)}{b} - \frac{(3d)\int (c+dx)^2\tan(a+bx)\,dx}{b}$$
$$= -\frac{i(c+dx)^3}{b} + \frac{(c+dx)^3\tan(a+bx)}{b} + \frac{(6id)\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}\,dx}{b}$$
$$= -\frac{i(c+dx)^3}{b} + \frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} + \frac{(c+dx)^3\tan(a+bx)}{b} - \frac{\left(6d^2\right)\int (c+dx)}{}$$
$$= -\frac{i(c+dx)^3}{b} + \frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\,\mathrm{t}}{}$$
$$= -\frac{i(c+dx)^3}{b} + \frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\,\mathrm{t}}{}$$
$$= -\frac{i(c+dx)^3}{b} + \frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3} + \frac{3d^3\mathrm{Li}_3\left(-\right.}{2b}$$

Mathematica [A] time = 0.486614, size = 109, normalized size = 0.96

$$\frac{2b^2(c+dx)^2\left(b(c+dx)\tan(a+bx)+3d\log\left(1+e^{2i(a+bx)}\right)-ib(c+dx)\right)-6ibd^2(c+dx)\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)+3d^3\mathrm{Li}_3\left(-e^{2i}\right.}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]^2,x]
```

```
[Out] ((-6*I)*b*d^2*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))] + 3*d^3*PolyLog[3,
 -E^((2*I)*(a + b*x))] + 2*b^2*(c + d*x)^2*((-I)*b*(c + d*x) + 3*d*Log[1 +
E^((2*I)*(a + b*x))] + b*(c + d*x)*Tan[a + b*x]))/(2*b^4)
```

Maple [B] time = 0.432, size = 316, normalized size = 2.8

$$\frac{-3\,id^2c\mathrm{polylog}\left(2,-\mathrm{e}^{2\,i(bx+a)}\right)}{b^3}+3\,\frac{c^2d\ln\left(\mathrm{e}^{2\,i(bx+a)}+1\right)}{b^2}-6\,\frac{c^2d\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^2}-6\,\frac{d^3a^2\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^4}+\frac{2\,i\left(d^3x^3+3\,cd^2x^2\right.}{b\left(\mathrm{e}^{2\,i(bx+a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)^2,x)
```

```
[Out] -3*I*d^2/b^3*c*polylog(2,-exp(2*I*(b*x+a)))+3*d/b^2*c^2*ln(exp(2*I*(b*x+a))
+1)-6*d/b^2*c^2*ln(exp(I*(b*x+a)))-6*d^3/b^4*a^2*ln(exp(I*(b*x+a)))+2*I*(d^
```

```
3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(exp(2*I*(b*x+a))+1)-12*I*d^2/b^2*c*a*x+
6*d^2/b^2*c*ln(exp(2*I*(b*x+a))+1)*x+3*d^3/b^2*ln(exp(2*I*(b*x+a))+1)*x^2-2
*I*d^3/b*x^3+3/2*d^3*polylog(3,-exp(2*I*(b*x+a)))/b^4+12*d^2/b^3*c*a*ln(exp
(I*(b*x+a)))+6*I*d^3/b^3*a^2*x-3*I*d^3/b^3*polylog(2,-exp(2*I*(b*x+a)))*x+4
*I*d^3/b^4*a^3-6*I*d^2/b*c*x^2-6*I*d^2/b^3*c*a^2
```

Maxima [B] time = 2.733, size = 1426, normalized size = 12.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*c^3*tan(b*x + a) - 6*a*c^2*d*tan(b*x + a)/b + 6*a^2*c*d^2*tan(b*x +
a)/b^2 - 2*a^3*d^3*tan(b*x + a)/b^3 + 3*((cos(2*b*x + 2*a)^2 + sin(2*b*x +
2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^
2 + 2*cos(2*b*x + 2*a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*c^2*d/((cos(2*b
*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b) - 6*((cos(2*b
*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x +
2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*(b*x + a)*sin(2*b
*x + 2*a))*a*c*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
+ 2*a) + 1)*b^2) + 3*((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*
x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2
*a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^3/((cos(2*b*x + 2*a)^2 + sin
(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b^3) + 2*((6*(b*x + a)^2*d^3 + 12
*(b*c*d^2 - a*d^3)*(b*x + a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*
x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^
3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a)
+ 1) - 4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2)*cos(2*b*x + 2*
a) - (6*b*c*d^2 + 6*(b*x + a)*d^3 - 6*a*d^3 + 6*(b*c*d^2 + (b*x + a)*d^3 -
a*d^3)*cos(2*b*x + 2*a) - (-6*I*b*c*d^2 - 6*I*(b*x + a)*d^3 + 6*I*a*d^3)*si
n(2*b*x + 2*a))*dilog(-e^(2*I*b*x + 2*I*a)) + (-3*I*(b*x + a)^2*d^3 + (-6*I
*b*c*d^2 + 6*I*a*d^3)*(b*x + a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6
*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a
*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a
)^2 + 2*cos(2*b*x + 2*a) + 1) + (-3*I*d^3*cos(2*b*x + 2*a) + 3*d^3*sin(2*b*
x + 2*a) - 3*I*d^3)*polylog(3, -e^(2*I*b*x + 2*I*a)) + (-4*I*(b*x + a)^3*d^
3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^2)*sin(2*b*x + 2*a))/(-2*I*b^3*c
os(2*b*x + 2*a) + 2*b^3*sin(2*b*x + 2*a) - 2*I*b^3))/b
```

Fricas [C] time = 1.78137, size = 2010, normalized size = 17.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*d^3*cos(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 6*d^3*c
os(b*x + a)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*d^3*cos(b*x + a)*
polylog(3, -I*cos(b*x + a) + sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3,
-I*cos(b*x + a) - sin(b*x + a)) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b*x + a)*
dilog(I*cos(b*x + a) + sin(b*x + a)) + (-6*I*b*d^3*x - 6*I*b*c*d^2)*cos(b*x
 + a)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-6*I*b*d^3*x - 6*I*b*c*d^2)*c
```

```
os(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (6*I*b*d^3*x + 6*I*b*c*
d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 3*(b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) +
3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) - I*sin
(b*x + a) + I) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*co
s(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*
c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x
+ a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x
 + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^
2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a
) + 1) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x +
a) + I*sin(b*x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x +
a)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x
^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(b*x + a))/(b^4*cos(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \sec^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*sec(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \sec(bx + a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)^2, x)
```

3.34 $\int (c+dx)^2 \sec^2(a+bx)\,dx$

Optimal. Leaf size=82

$$\frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2}-\frac{id^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{i(c+dx)^2}{b}$$

```
[Out] ((-I)*(c + d*x)^2)/b + (2*d*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b^2 - (
I*d^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^3 + ((c + d*x)^2*Tan[a + b*x])/b
```

Rubi [A] time = 0.134178, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4184, 3719, 2190, 2279, 2391}

$$\frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2}-\frac{id^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{i(c+dx)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^2)/b + (2*d*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b^2 - (
I*d^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^3 + ((c + d*x)^2*Tan[a + b*x])/b
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)^2\sec^2(a+bx)\,dx = \frac{(c+dx)^2\tan(a+bx)}{b} - \frac{(2d)\int (c+dx)\tan(a+bx)\,dx}{b}$$
$$= -\frac{i(c+dx)^2}{b} + \frac{(c+dx)^2\tan(a+bx)}{b} + \frac{(4id)\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}\,dx}{b}$$
$$= -\frac{i(c+dx)^2}{b} + \frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\tan(a+bx)}{b} - \frac{\left(2d^2\right)\int \log\left(1+e\right.}{b^2}$$
$$= -\frac{i(c+dx)^2}{b} + \frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\tan(a+bx)}{b} + \frac{\left(id^2\right)\text{Subst}\left(\int \frac{\text{lo}}{}\right.}{}$$
$$= -\frac{i(c+dx)^2}{b} + \frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{id^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)^2\tan(a+bx}{b}$$

Mathematica [A] time = 0.242623, size = 75, normalized size = 0.91

$$\frac{b(c+dx)\left(b(c+dx)\tan(a+bx)+2d\log\left(1+e^{2i(a+bx)}\right)-ib(c+dx)\right)-id^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]^2,x]
```

```
[Out] ((-I)*d^2*PolyLog[2, -E^((2*I)*(a + b*x))] + b*(c + d*x)*((-I)*b*(c + d*x)
+ 2*d*Log[1 + E^((2*I)*(a + b*x))] + b*(c + d*x)*Tan[a + b*x]))/b^3
```

Maple [B] time = 0.374, size = 170, normalized size = 2.1

$$\frac{2i\left(d^2x^2+2cdx+c^2\right)}{b\left(e^{2i(bx+a)}+1\right)} - 4\frac{cd\ln\left(e^{i(bx+a)}\right)}{b^2} + 2\frac{cd\ln\left(e^{2i(bx+a)}+1\right)}{b^2} - \frac{2id^2x^2}{b} - \frac{4id^2ax}{b^2} - \frac{2id^2a^2}{b^3} + 2\frac{d^2\ln\left(e^{2i(bx+a)}+\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)^2,x)
```

```
[Out] 2*I*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*x+a))+1)-4*d/b^2*c*ln(exp(I*(b*x+a)
))+2*d/b^2*c*ln(exp(2*I*(b*x+a))+1)-2*I*d^2/b*x^2-4*I*d^2/b^2*a*x-2*I*d^2/b
^3*a^2+2*d^2/b^2*ln(exp(2*I*(b*x+a))+1)*x-I*d^2*polylog(2,-exp(2*I*(b*x+a))
)/b^3+4*d^2/b^3*a*ln(exp(I*(b*x+a)))
```

Maxima [B] time = 2.51338, size = 437, normalized size = 5.33

$$\frac{2b^2c^2+\left(2bd^2x+2bcd+2\left(bd^2x+bcd\right)\cos(2bx+2a)+\left(2ibd^2x+2ibcd\right)\sin(2bx+2a)\right)\arctan(\sin(2bx+2a)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (2*b^2*c^2 + (2*b*d^2*x + 2*b*c*d + 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a) +
(2*I*b*d^2*x + 2*I*b*c*d)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2
*b*x + 2*a) + 1) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x)*cos(2*b*x + 2*a) - (d^2*co
s(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) + d^2)*dilog(-e^(2*I*b*x + 2*I*a))
+ (-I*b*d^2*x - I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(2*b*x + 2*a) + (b*d^2*
x + b*c*d)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 +
2*cos(2*b*x + 2*a) + 1) + (-2*I*b^2*d^2*x^2 - 4*I*b^2*c*d*x)*sin(2*b*x + 2*
a))/(-I*b^3*cos(2*b*x + 2*a) + b^3*sin(2*b*x + 2*a) - I*b^3)
```

Fricas [B] time = 1.73393, size = 1187, normalized size = 14.48

$$i\,d^2\cos(bx+a)\,\mathrm{Li}_2(i\cos(bx+a)+\sin(bx+a)) - i\,d^2\cos(bx+a)\,\mathrm{Li}_2(i\cos(bx+a)-\sin(bx+a)) - i\,d^2\cos(bx+a)\,\mathrm{I}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) - I*d^2*cos(b*x +
a)*dilog(I*cos(b*x + a) - sin(b*x + a)) - I*d^2*cos(b*x + a)*dilog(-I*cos(b
*x + a) + sin(b*x + a)) + I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*
x + a)) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) +
I) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + I) +
(b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*
d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b*d^2
*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d^2*x
 + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c*d - a
*d^2)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*c*d - a*d^2
)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2)*sin(b*x + a))/(b^3*cos(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^2 \sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*sec(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \sec(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)^2, x)
```

3.35 $\int (c+dx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=28

$$\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}$$

```
[Out] (d*Log[Cos[a + b*x]])/b^2 + ((c + d*x)*Tan[a + b*x])/b
```

Rubi [A] time = 0.0273841, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4184, 3475}

$$\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]^2,x]
```

```
[Out] (d*Log[Cos[a + b*x]])/b^2 + ((c + d*x)*Tan[a + b*x])/b
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\sec^2(a+bx)\,dx &= \frac{(c+dx)\tan(a+bx)}{b}-\frac{d\int \tan(a+bx)\,dx}{b}\\ &= \frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.012524, size = 36, normalized size = 1.29

$$\frac{d\log(\cos(a+bx))}{b^2}+\frac{c\tan(a+bx)}{b}+\frac{dx\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]^2,x]
```

```
[Out] (d*Log[Cos[a + b*x]])/b^2 + (c*Tan[a + b*x])/b + (d*x*Tan[a + b*x])/b
```

Maple [A] time = 0.026, size = 37, normalized size = 1.3

$$\frac{d\tan(bx+a)x}{b}+\frac{d\ln(\cos(bx+a))}{b^2}+\frac{c\tan(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)^2,x)
```

```
[Out] 1/b*d*tan(b*x+a)*x+d*ln(cos(b*x+a))/b^2+1/b*c*tan(b*x+a)
```

Maxima [B] time = 2.11492, size = 215, normalized size = 7.68

$$\frac{2\,c\tan(bx+a)-\frac{2\,ad\tan(bx+a)}{b}+\frac{\left(\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)+1\right)\log\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)+1\right)+4\,(bx+}{\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)+1\right)b}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*c*tan(b*x + a) - 2*a*d*tan(b*x + a)/b + ((cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x
+ 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*d/((cos(
2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b))/b
```

Fricas [A] time = 1.40773, size = 115, normalized size = 4.11

$$\frac{d\cos(bx+a)\log(-\cos(bx+a))+(bdx+bc)\sin(bx+a)}{b^2\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (d*cos(b*x + a)*log(-cos(b*x + a)) + (b*d*x + b*c)*sin(b*x + a))/(b^2*cos(b
*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(c+dx)\sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*sec(a + b*x)**2, x)
```

Giac [B] time = 1.59505, size = 1970, normalized size = 70.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*b*d*x*tan(1/2*b*x)^2*tan(1/2*a) + 4*b*d*x*tan(1/2*b*x)*tan(1/2*a)^2
 - d*log(4*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^4
 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*
b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*t
an(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)
^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a
)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*ta
n(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*ta
n(1/2*a) - 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 4*b*c*tan(1/2
*b*x)^2*tan(1/2*a) + 4*b*c*tan(1/2*b*x)*tan(1/2*a)^2 - 4*b*d*x*tan(1/2*b*x)
 + d*log(4*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^4
 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*
b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*t
an(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)
^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a
)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*ta
n(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*ta
n(1/2*a) - 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2 - 4*b*d*x*tan(1/2*a) + 4*d*l
og(4*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*t
an(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8
 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2
*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan
(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 -
2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*
a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*
a) - 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a) + d*log(4*(tan(1/2*a)^4 +
 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/
2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*
tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3
 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2
*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8
*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x
)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2
+ 1))*tan(1/2*a)^2 - 4*b*c*tan(1/2*b*x) - 4*b*c*tan(1/2*a) - d*log(4*(tan(1
/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)
^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/
2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan
(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 3
6*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b
*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*ta
n(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(
1/2*a)^2 + 1)))/(b^2*tan(1/2*b*x)^2*tan(1/2*a)^2 - b^2*tan(1/2*b*x)^2 - 4*b
^2*tan(1/2*b*x)*tan(1/2*a) - b^2*tan(1/2*a)^2 + b^2)
```

3.36 $\int \frac{\sec^2(a+bx)}{c+dx} \, dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\sec^2(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable[Sec[a + b*x]^2/(c + d*x), x]
```

Rubi [A] time = 0.0390762, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^2(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[a + b*x]^2/(c + d*x),x]
```

```
[Out] Defer[Int][Sec[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\sec^2(a+bx)}{c+dx} \, dx = \int \frac{\sec^2(a+bx)}{c+dx} \, dx$$

Mathematica [A] time = 5.59674, size = 0, normalized size = 0.

$$\int \frac{\sec^2(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[a + b*x]^2/(c + d*x),x]
```

```
[Out] Integrate[Sec[a + b*x]^2/(c + d*x), x]
```

Maple [A] time = 0.332, size = 0, normalized size = 0.

$$\int \frac{(\sec(bx+a))^2}{dx+c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)^2/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(bx+a\right)^{2}}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^2/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{2}\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sec(a + b*x)**2/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(bx+a\right)^{2}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^2/(d*x + c), x)
```

3.37 $\int (c+dx)^3 \sec^3(a+bx)\,dx$

Optimal. Leaf size=337

$$-\frac{3d^2(c+dx)\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{3d^2(c+dx)\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{2b^2}-3$$

```
[Out] ((-6*I)*d^2*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^3 - (I*(c + d*x)^3*ArcTan[
E^(I*(a + b*x))])/b + ((3*I)*d^3*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^4 + ((
(3*I)/2)*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((3*I)*d^3*P
olyLog[2, I*E^(I*(a + b*x))])/b^4 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, I*E
^(I*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b
^3 + (3*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - ((3*I)*d^3*PolyL
og[4, (-I)*E^(I*(a + b*x))])/b^4 + ((3*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))]
)/b^4 - (3*d*(c + d*x)^2*Sec[a + b*x])/(2*b^2) + ((c + d*x)^3*Sec[a + b*x]*
Tan[a + b*x])/(2*b)
```

Rubi [A] time = 0.268809, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4186, 4181, 2279, 2391, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{3d^2(c+dx)\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{2b^2}-3$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]^3,x]
```

```
[Out] ((-6*I)*d^2*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^3 - (I*(c + d*x)^3*ArcTan[
E^(I*(a + b*x))])/b + ((3*I)*d^3*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^4 + ((
(3*I)/2)*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((3*I)*d^3*P
olyLog[2, I*E^(I*(a + b*x))])/b^4 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, I*E
^(I*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b
^3 + (3*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - ((3*I)*d^3*PolyL
og[4, (-I)*E^(I*(a + b*x))])/b^4 + ((3*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))]
)/b^4 - (3*d*(c + d*x)^2*Sec[a + b*x])/(2*b^2) + ((c + d*x)^3*Sec[a + b*x]*
Tan[a + b*x])/(2*b)
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3\sec^3(a+bx)\,dx = -\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\sec(a+bx)\tan(a+bx)}{2b}+\frac{1}{2}\int (c+dx)^3\sec(a+b$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3}-\frac{i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3}-\frac{i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3id(c+dx)^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{2b^2}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3}-\frac{i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3id^3\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^4}+\frac{3id(}{}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3}-\frac{i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3id^3\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^4}+\frac{3id(}{}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3}-\frac{i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3id^3\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^4}+\frac{3id(}{}$$

Mathematica [A] time = 2.93546, size = 311, normalized size = 0.92

$$\frac{3id\left(b^2(c+dx)^2\text{Li}_2\left(-ie^{i(a+bx)}\right)+2ibd(c+dx)\text{Li}_3\left(-ie^{i(a+bx)}\right)-2d^2\text{Li}_4\left(-ie^{i(a+bx)}\right)\right)-3id\left(b^2(c+dx)^2\text{Li}_2\left(ie^{i(a+bx)}\right)+2ibd\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]^3,x]
```

```
[Out] ((-2*I)*b^3*(c + d*x)^3*ArcTan[E^(I*(a + b*x))] - (6*I)*d^2*(2*b*(c + d*x)*
ArcTan[E^(I*(a + b*x))] - d*PolyLog[2, (-I)*E^(I*(a + b*x))] + d*PolyLog[2,
 I*E^(I*(a + b*x))]) + (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b
*x))] + (2*I)*b*d*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))] - 2*d^2*PolyLo
g[4, (-I)*E^(I*(a + b*x))]) - (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, I*E^(I*(a
 + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))] - 2*d^2*PolyL
og[4, I*E^(I*(a + b*x))]) - 3*b^2*d*(c + d*x)^2*Sec[a + b*x] + b^3*(c + d*x
)^3*Sec[a + b*x]*Tan[a + b*x])/(2*b^4)
```

Maple [B] time = 0.458, size = 1127, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)^3,x)
```

```
[Out] I/b^4*d^3*a^3*arctan(exp(I*(b*x+a)))+3/2*I/b^2*c^2*d*polylog(2,-I*exp(I*(b*
x+a)))-3/2*I/b^2*c^2*d*polylog(2,I*exp(I*(b*x+a)))-3/2*I/b^2*d^3*polylog(2,
I*exp(I*(b*x+a)))*x^2+3/2*I/b^2*d^3*polylog(2,-I*exp(I*(b*x+a)))*x^2+6*I/b^
4*d^3*a*arctan(exp(I*(b*x+a)))-6*I/b^3*c*d^2*arctan(exp(I*(b*x+a)))-1/2/b^4
*d^3*a^3*ln(1+I*exp(I*(b*x+a)))+3/b^3*d^3*polylog(3,I*exp(I*(b*x+a)))*x+3/b
^3*c*d^2*polylog(3,I*exp(I*(b*x+a)))-3/b^3*d^3*polylog(3,-I*exp(I*(b*x+a)))
*x+3/b^3*d^3*ln(1-I*exp(I*(b*x+a)))*x+3/b^4*d^3*ln(1-I*exp(I*(b*x+a)))*a-3/
b^3*d^3*ln(1+I*exp(I*(b*x+a)))*x-I/b*c^3*arctan(exp(I*(b*x+a)))-I/b^2/(exp(
2*I*(b*x+a))+1)^2*(d^3*x^3*b*exp(3*I*(b*x+a))+3*c*d^2*x^2*b*exp(3*I*(b*x+a)
)+3*c^2*d*x*b*exp(3*I*(b*x+a))-d^3*x^3*b*exp(I*(b*x+a))+c^3*b*exp(3*I*(b*x+
a))-3*c*d^2*x^2*b*exp(I*(b*x+a))-3*I*d^3*x^2*exp(3*I*(b*x+a))-3*c^2*d*x*b*e
xp(I*(b*x+a))-6*I*c*d^2*x*exp(3*I*(b*x+a))-c^3*b*exp(I*(b*x+a))-3*I*c^2*d*e
xp(3*I*(b*x+a))-3*I*d^3*x^2*exp(I*(b*x+a))-6*I*c*d^2*x*exp(I*(b*x+a))-3*I*c
^2*d*exp(I*(b*x+a)))+3*I/b^2*c*d^2*polylog(2,-I*exp(I*(b*x+a)))*x-3*I/b^2*c
*d^2*polylog(2,I*exp(I*(b*x+a)))*x-3*I/b^3*c*d^2*a^2*arctan(exp(I*(b*x+a)))
+3*I/b^2*c^2*d*a*arctan(exp(I*(b*x+a)))-3/b^4*d^3*ln(1+I*exp(I*(b*x+a)))*a+
1/2/b^4*d^3*a^3*ln(1-I*exp(I*(b*x+a)))-3/b^3*c*d^2*polylog(3,-I*exp(I*(b*x+
a)))-1/2/b*d^3*ln(1+I*exp(I*(b*x+a)))*x^3+1/2/b*d^3*ln(1-I*exp(I*(b*x+a)))*
x^3-3*I*d^3*polylog(2,I*exp(I*(b*x+a)))/b^4-3*I*d^3*polylog(4,-I*exp(I*(b*x
+a)))/b^4+3*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4+3/2/b^3*a^2*c*d^2*ln(1+I*
exp(I*(b*x+a)))-3/2/b*c*d^2*ln(1+I*exp(I*(b*x+a)))*x^2+3/2/b*c*d^2*ln(1-I*e
xp(I*(b*x+a)))*x^2-3/2/b*c^2*d*ln(1+I*exp(I*(b*x+a)))*x-3/2/b^2*c^2*d*ln(1+
I*exp(I*(b*x+a)))*a-3/2/b^3*a^2*c*d^2*ln(1-I*exp(I*(b*x+a)))+3/2/b*c^2*d*ln
(1-I*exp(I*(b*x+a)))*x+3/2/b^2*c^2*d*ln(1-I*exp(I*(b*x+a)))*a+3*I*d^3*polyl
og(2,-I*exp(I*(b*x+a)))/b^4
```

Maxima [B] time = 7.55126, size = 5168, normalized size = 15.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(c^3*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + lo
g(sin(b*x + a) - 1)) - 3*a*c^2*d*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log
(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*sin(b*x + a)
/(sin(b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/b^2
- a^3*d^3*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + lo
g(sin(b*x + a) - 1))/b^3 + 4*((2*(b*x + a)^3*d^3 + 12*b*c*d^2 - 12*a*d^3 +
6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^
3)*(b*x + a) + 2*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^
3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))*cos
(4*b*x + 4*a) + 4*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d
^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))*co
s(2*b*x + 2*a) - (-2*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (-6*I*
b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I
*a^2 - 12*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (-4*I*(b*x + a)^3*d^3 - 24*
I*b*c*d^2 + 24*I*a*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^2 + (-12*I*
b^2*c^2*d + 24*I*a*b*c*d^2 + (-12*I*a^2 - 24*I)*d^3)*(b*x + a))*sin(2*b*x +
 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (2*(b*x + a)^3*d^3 + 12*b*
c*d^2 - 12*a*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c
*d^2 + (a^2 + 2)*d^3)*(b*x + a) + 2*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3
+ 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*
d^3)*(b*x + a))*cos(4*b*x + 4*a) + 4*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3
 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)
*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-2*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 +
12*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*
I*a*b*c*d^2 + (-6*I*a^2 - 12*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (-4*I*(b
*x + a)^3*d^3 - 24*I*b*c*d^2 + 24*I*a*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b
*x + a)^2 + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 + (-12*I*a^2 - 24*I)*d^3)*(b*
x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) + (4*(b*
x + a)^3*d^3 - 12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*a^2*d^3 + (12*b*c*d^2
 - (12*a + 12*I)*d^3)*(b*x + a)^2 + (12*b^2*c^2*d - (24*a + 24*I)*b*c*d^2 +
 12*(a^2 + 2*I*a)*d^3)*(b*x + a))*cos(3*b*x + 3*a) - (4*(b*x + a)^3*d^3 + 1
2*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^3 + (12*b*c*d^2 - (12*a - 12*I)
*d^3)*(b*x + a)^2 + (12*b^2*c^2*d - (24*a - 24*I)*b*c*d^2 + 12*(a^2 - 2*I*a
)*d^3)*(b*x + a))*cos(b*x + a) + (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^
2*d^3 + 6*(a^2 + 2)*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2
*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a
))*cos(4*b*x + 4*a) + 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2
+ 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-6*I*b^2*c^2*
d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 + (-6*I*a^2 - 12*I)*d^3 + (-12*I*b
*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (-12*I*b^2*c^2*d + 24*I*
a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 + (-12*I*a^2 - 24*I)*d^3 + (-24*I*b*c*d^2
+ 24*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - (6*b^
2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*(a^2 + 2)*d^3 + 12*(b*c*d^2
- a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 +
2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 12*(b^2*c^2*d -
2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x +
a))*cos(2*b*x + 2*a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^
3 + (6*I*a^2 + 12*I)*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*sin(4*b*x
 + 4*a) + (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*(b*x + a)^2*d^3 + (12*I*a
^2 + 24*I)*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*d
ilog(-I*e^(I*b*x + I*a)) - (-I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 +
(-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 +
(-3*I*a^2 - 6*I)*d^3)*(b*x + a) + (-I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a
*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c
*d^2 + (-3*I*a^2 - 6*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^
```

```
3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2
+ (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 - 12*I)*d^3)*(b*x + a))*cos(
2*b*x + 2*a) + ((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^3)
*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))*sin(4
*b*x + 4*a) + 2*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^3
)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))*sin(
2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (
I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*(b*
x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + (3*I*a^2 + 6*I)*d^3)*(b*x + a)
+ (I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*
(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + (3*I*a^2 + 6*I)*d^3)*(b*x +
a))*cos(4*b*x + 4*a) + (2*I*(b*x + a)^3*d^3 + 12*I*b*c*d^2 - 12*I*a*d^3 + (
6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + (6
*I*a^2 + 12*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^3*d^3 + 6*b*c*
d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^
2 + (a^2 + 2)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*((b*x + a)^3*d^3 + 6*b*c
*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d
^2 + (a^2 + 2)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b
*x + a)^2 - 2*sin(b*x + a) + 1) - (12*d^3*cos(4*b*x + 4*a) + 24*d^3*cos(2*b
*x + 2*a) + 12*I*d^3*sin(4*b*x + 4*a) + 24*I*d^3*sin(2*b*x + 2*a) + 12*d^3)
*polylog(4, I*e^(I*b*x + I*a)) + (12*d^3*cos(4*b*x + 4*a) + 24*d^3*cos(2*b*
x + 2*a) + 12*I*d^3*sin(4*b*x + 4*a) + 24*I*d^3*sin(2*b*x + 2*a) + 12*d^3)*
polylog(4, -I*e^(I*b*x + I*a)) - (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I
*a*d^3 + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*cos(4*b*x + 4*a)
 + (-24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*cos(2*b*x + 2*a) + 12*
(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) + 24*(b*c*d^2 + (b*x + a
)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, I*e^(I*b*x + I*a)) - (12*I*b*c*
d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3 + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3
- 12*I*a*d^3)*cos(4*b*x + 4*a) + (24*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 24*I*
a*d^3)*cos(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x +
4*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, -I
*e^(I*b*x + I*a)) - (-4*I*(b*x + a)^3*d^3 - 12*b^2*c^2*d + 24*a*b*c*d^2 - 1
2*a^2*d^3 - 12*(I*b*c*d^2 + (-I*a + 1)*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d
- 24*(-I*a + 1)*b*c*d^2 + (-12*I*a^2 + 24*a)*d^3)*(b*x + a))*sin(3*b*x + 3*
a) - (4*I*(b*x + a)^3*d^3 - 12*b^2*c^2*d + 24*a*b*c*d^2 - 12*a^2*d^3 - 12*(
-I*b*c*d^2 + (I*a + 1)*d^3)*(b*x + a)^2 + (12*I*b^2*c^2*d - 24*(I*a + 1)*b*
c*d^2 + (12*I*a^2 + 24*a)*d^3)*(b*x + a))*sin(b*x + a))/(-4*I*b^3*cos(4*b*x
 + 4*a) - 8*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin(4*b*x + 4*a) + 8*b^3*sin(2*b
*x + 2*a) - 4*I*b^3))/b
```

Fricas [C] time = 2.48923, size = 3213, normalized size = 9.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(6*I*d^3*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) + sin(b*x + a)) + 6*I
*d^3*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*cos
(b*x + a)^2*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) - 6*I*d^3*cos(b*x +
a)^2*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b
^2*c*d^2*x - 3*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) +
 sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 6*I*
d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + (3*I*b^2*d^3*x^2
 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 6*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b
*x + a) + sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*
```

```
d + 6*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (b^3*c^
3 - 3*a*b^2*c^2*d + 3*(a^2 + 2)*b*c*d^2 - (a^3 + 6*a)*d^3)*cos(b*x + a)^2*l
og(cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 +
 2)*b*c*d^2 - (a^3 + 6*a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x
+ a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
+ (a^3 + 6*a)*d^3 + 3*(b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^2*log(I*cos(b*x
 + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d
- 3*a^2*b*c*d^2 + (a^3 + 6*a)*d^3 + 3*(b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)
^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
+ 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 6*a)*d^3 + 3*(b^3*c^2*d + 2*b*d^3)
*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 +
 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 6*a)*d^3 + 3*(b^3
*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1)
 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 2)*b*c*d^2 - (a^3 + 6*a)*d^3)*cos(b*
x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d
 + 3*(a^2 + 2)*b*c*d^2 - (a^3 + 6*a)*d^3)*cos(b*x + a)^2*log(-cos(b*x + a)
- I*sin(b*x + a) + I) - 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, I*c
os(b*x + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(
3, I*cos(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*po
lylog(3, -I*cos(b*x + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*cos(b*x +
a)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 6*(b^2*d^3*x^2 + 2*b^2*c*
d^2*x + b^2*c^2*d)*cos(b*x + a) + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*
c^2*d*x + b^3*c^3)*sin(b*x + a))/(b^4*cos(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \sec^3(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**3*sec(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \sec(bx + a)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)^3, x)
```

3.38 $\int (c+dx)^2 \sec^3(a+bx)\,dx$

Optimal. Leaf size=193

$$\frac{id(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-\frac{d(c+dx)\sec(a+bx)}{b^2}-\frac{d^2\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{d^2\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}+\frac{d^2\text{t}}{}$$

```
[Out] ((-I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (d^2*ArcTanh[Sin[a + b*x]])/
b^3 + (I*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*(c + d*x)
*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (d^2*PolyLog[3, (-I)*E^(I*(a + b*x))]
)/b^3 + (d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - (d*(c + d*x)*Sec[a + b*x]
)/b^2 + ((c + d*x)^2*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rubi [A] time = 0.142798, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4186, 3770, 4181, 2531, 2282, 6589}

$$\frac{id(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-\frac{d(c+dx)\sec(a+bx)}{b^2}-\frac{d^2\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{d^2\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}+\frac{d^2\text{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]^3,x]
```


```
[Out] ((-I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (d^2*ArcTanh[Sin[a + b*x]])/
b^3 + (I*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*(c + d*x)
*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (d^2*PolyLog[3, (-I)*E^(I*(a + b*x))]
)/b^3 + (d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - (d*(c + d*x)*Sec[a + b*x]
)/b^2 + ((c + d*x)^2*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```


Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2\sec^3(a+bx)\,dx = -\frac{d(c+dx)\sec(a+bx)}{b^2}+\frac{(c+dx)^2\sec(a+bx)\tan(a+bx)}{2b}+\frac{1}{2}\int (c+dx)^2\sec(a+bx)$$

$$= -\frac{i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3}-\frac{d(c+dx)\sec(a+bx)}{b^2}+\frac{(c+dx}{}$$

$$= -\frac{i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3}+\frac{id(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{id(c}{}$$

$$= -\frac{i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3}+\frac{id(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{id(c}{}$$

$$= -\frac{i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3}+\frac{id(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{id(c}{}$$

Mathematica [A] time = 1.23401, size = 184, normalized size = 0.95

$$\frac{-2ib^2(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)+b^2(c+dx)^2\tan(a+bx)\sec(a+bx)+2ibd(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)-2ibd(c+dx)\mathrm{Li}_2\left(\right.}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]^3,x]
```

```
[Out] ((-2*I)*b^2*(c + d*x)^2*ArcTan[E^(I*(a + b*x))] + 2*d^2*ArcTanh[Sin[a + b*x
]] + (2*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(c +
d*x)*PolyLog[2, I*E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))]
 + 2*d^2*PolyLog[3, I*E^(I*(a + b*x))] - 2*b*d*(c + d*x)*Sec[a + b*x] + b^2
*(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x])/(2*b^3)
```

Maple [B] time = 0.411, size = 584, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)^3,x)
```

```
[Out] 2*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))-1/b^2*c*d*ln(1+I*exp(I*(b*x+a)))*a-1/2
/b^3*a^2*d^2*ln(1-I*exp(I*(b*x+a)))+1/2/b^3*a^2*d^2*ln(1+I*exp(I*(b*x+a)))-
2*I/b^3*d^2*arctan(exp(I*(b*x+a)))-I/b*c^2*arctan(exp(I*(b*x+a)))-I/b^2/(ex
p(2*I*(b*x+a))+1)^2*(d^2*x^2*b*exp(3*I*(b*x+a))+2*c*d*x*b*exp(3*I*(b*x+a))+
c^2*b*exp(3*I*(b*x+a))-d^2*x^2*b*exp(I*(b*x+a))-2*c*d*x*b*exp(I*(b*x+a))-2*
I*d^2*x*exp(3*I*(b*x+a))-c^2*b*exp(I*(b*x+a))-2*I*c*d*exp(3*I*(b*x+a))-2*I*
d^2*x*exp(I*(b*x+a))-2*I*c*d*exp(I*(b*x+a)))+I/b^2*d^2*polylog(2,-I*exp(I*(
b*x+a)))*x-I/b^2*c*d*polylog(2,I*exp(I*(b*x+a)))+d^2*polylog(3,I*exp(I*(b*x
+a)))/b^3-I/b^2*d^2*polylog(2,I*exp(I*(b*x+a)))*x+I/b^2*c*d*polylog(2,-I*ex
p(I*(b*x+a)))-I/b^3*d^2*a^2*arctan(exp(I*(b*x+a)))-1/b*c*d*ln(1+I*exp(I*(b*
x+a)))*x-1/2/b*d^2*ln(1+I*exp(I*(b*x+a)))*x^2+1/2/b*d^2*ln(1-I*exp(I*(b*x+a
)))*x^2+1/b^2*c*d*ln(1-I*exp(I*(b*x+a)))*a-d^2*polylog(3,-I*exp(I*(b*x+a)))
/b^3+1/b*c*d*ln(1-I*exp(I*(b*x+a)))*x
```

Maxima [B] time = 2.99682, size = 2556, normalized size = 13.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(c^2*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + lo
g(sin(b*x + a) - 1)) - 2*a*c*d*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(s
in(b*x + a) + 1) + log(sin(b*x + a) - 1))/b + a^2*d^2*(2*sin(b*x + a)/(sin(
b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/b^2 + 4*((
2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + 4*d^2 + 2*((b*x + a)^2*d^
2 + 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) + 4*((b*x + a)^2*
d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*a) - (-2*I*(b*x +
a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a) - 4*I*d^2)*sin(4*b*x + 4*a) -
 (-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) - 8*I*d^2)*sin(
2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (2*(b*x + a)^2*d^2
+ 4*(b*c*d - a*d^2)*(b*x + a) + 4*d^2 + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d
^2)*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a
*d^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*a) - (-2*I*(b*x + a)^2*d^2 + (-4*I*b
*c*d + 4*I*a*d^2)*(b*x + a) - 4*I*d^2)*sin(4*b*x + 4*a) - (-4*I*(b*x + a)^2
*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) - 8*I*d^2)*sin(2*b*x + 2*a))*arct
an2(cos(b*x + a), -sin(b*x + a) + 1) + (4*(b*x + a)^2*d^2 - 8*I*b*c*d + 8*I
*a*d^2 + (8*b*c*d - (8*a + 8*I)*d^2)*(b*x + a))*cos(3*b*x + 3*a) - (4*(b*x
+ a)^2*d^2 + 8*I*b*c*d - 8*I*a*d^2 + (8*b*c*d - (8*a - 8*I)*d^2)*(b*x + a))
*cos(b*x + a) + (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)
*d^2 - a*d^2)*cos(4*b*x + 4*a) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*
x + 2*a) - (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*sin(4*b*x + 4*a) -
(-8*I*b*c*d - 8*I*(b*x + a)*d^2 + 8*I*a*d^2)*sin(2*b*x + 2*a))*dilog(I*e^(I
*b*x + I*a)) - (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*
d^2 - a*d^2)*cos(4*b*x + 4*a) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x
 + 2*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*sin(4*b*x + 4*a) + (8
*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-I*e^(I*b
*x + I*a)) - (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) - 2*I
*d^2 + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) - 2*I*d^2)*
cos(4*b*x + 4*a) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x +
a) - 4*I*d^2)*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
+ a) + 2*d^2)*sin(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*
x + a) + 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*s
in(b*x + a) + 1) - (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a) +
```

```
 2*I*d^2 + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a) + 2*I*d^2
)*cos(4*b*x + 4*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x +
a) + 4*I*d^2)*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
+ a) + 2*d^2)*sin(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*
x + a) + 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*s
in(b*x + a) + 1) - (-4*I*d^2*cos(4*b*x + 4*a) - 8*I*d^2*cos(2*b*x + 2*a) +
4*d^2*sin(4*b*x + 4*a) + 8*d^2*sin(2*b*x + 2*a) - 4*I*d^2)*polylog(3, I*e^(
I*b*x + I*a)) - (4*I*d^2*cos(4*b*x + 4*a) + 8*I*d^2*cos(2*b*x + 2*a) - 4*d^
2*sin(4*b*x + 4*a) - 8*d^2*sin(2*b*x + 2*a) + 4*I*d^2)*polylog(3, -I*e^(I*b
*x + I*a)) - (-4*I*(b*x + a)^2*d^2 - 8*b*c*d + 8*a*d^2 - 8*(I*b*c*d + (-I*a
 + 1)*d^2)*(b*x + a))*sin(3*b*x + 3*a) - (4*I*(b*x + a)^2*d^2 - 8*b*c*d + 8
*a*d^2 - 8*(-I*b*c*d + (I*a + 1)*d^2)*(b*x + a))*sin(b*x + a))/(-4*I*b^2*co
s(4*b*x + 4*a) - 8*I*b^2*cos(2*b*x + 2*a) + 4*b^2*sin(4*b*x + 4*a) + 8*b^2*
sin(2*b*x + 2*a) - 4*I*b^2))/b
```

Fricas [C] time = 2.03719, size = 2034, normalized size = 10.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*d^
2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*cos(b*x
+ a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*cos(b*x + a)^2*po
lylog(3, -I*cos(b*x + a) - sin(b*x + a)) - (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b
*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) - (-2*I*b*d^2*x - 2*I*b*c*d)
*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - (2*I*b*d^2*x + 2*I*b
*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) - (2*I*b*d^2*x +
 2*I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^2*c^2
 - 2*a*b*c*d + (a^2 + 2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x +
 a) + I) + (b^2*c^2 - 2*a*b*c*d + (a^2 + 2)*d^2)*cos(b*x + a)^2*log(cos(b*x
 + a) - I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*
d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 +
 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin
(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x
+ a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x
 + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) +
 1) - (b^2*c^2 - 2*a*b*c*d + (a^2 + 2)*d^2)*cos(b*x + a)^2*log(-cos(b*x + a
) + I*sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + (a^2 + 2)*d^2)*cos(b*x + a
)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 4*(b*d^2*x + b*c*d)*cos(b*x +
 a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(b*x + a))/(b^3*cos(b*x +
a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \sec^3 (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**2*sec(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \sec(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)^3, x)
```

3.39 $\int (c+dx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=117

$$\frac{id\text{Li}_2\left(-ie^{i(a+bx)}\right)}{2b^2}-\frac{id\text{Li}_2\left(ie^{i(a+bx)}\right)}{2b^2}-\frac{d\sec(a+bx)}{2b^2}-\frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{(c+dx)\tan(a+bx)\sec(a+bx)}{2b}$$

```
[Out] ((-I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b + ((I/2)*d*PolyLog[2, (-I)*E^(I*
(a + b*x))])/b^2 - ((I/2)*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (d*Sec[a +
 b*x])/(2*b^2) + ((c + d*x)*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rubi [A] time = 0.0688011, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4185, 4181, 2279, 2391}

$$\frac{id\text{Li}_2\left(-ie^{i(a+bx)}\right)}{2b^2}-\frac{id\text{Li}_2\left(ie^{i(a+bx)}\right)}{2b^2}-\frac{d\sec(a+bx)}{2b^2}-\frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{(c+dx)\tan(a+bx)\sec(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]^3,x]
```


```
[Out] ((-I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b + ((I/2)*d*PolyLog[2, (-I)*E^(I*
(a + b*x))])/b^2 - ((I/2)*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (d*Sec[a +
 b*x])/(2*b^2) + ((c + d*x)*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```


Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)\sec^3(a+bx)\,dx = -\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\sec(a+bx)\tan(a+bx)}{2b}+\frac{1}{2}\int (c+dx)\sec(a+bx)\,dx$$
$$= -\frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\sec(a+bx)\tan(a+bx)}{2b}-\frac{d\int\log\left(1-\right.}{2}$$
$$= -\frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\sec(a+bx)\tan(a+bx)}{2b}+\frac{(id)\,\text{Subst}}{}$$
$$= -\frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{id\text{Li}_2\left(-ie^{i(a+bx)}\right)}{2b^2}-\frac{id\text{Li}_2\left(ie^{i(a+bx)}\right)}{2b^2}-\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)}{}$$

Mathematica [B] time = 3.79828, size = 389, normalized size = 3.32

$$\frac{d\left(i\left(\text{Li}_2\left(-e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)-\text{Li}_2\left(e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)\right)+\left(-a-bx+\frac{\pi}{2}\right)\left(\log\left(1-e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)-\log\left(1+e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)\right)-\left(\frac{\pi}{2}-a\right)\log\right.}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]^3,x]
```

```
[Out] (c*ArcTanh[Sin[a + b*x]])/(2*b) + (d*((-a + Pi/2 - b*x)*(Log[1 - E^(I*(-a +
 Pi/2 - b*x))] - Log[1 + E^(I*(-a + Pi/2 - b*x))]) - (-a + Pi/2)*Log[Tan[(-
a + Pi/2 - b*x)/2]] + I*(PolyLog[2, -E^(I*(-a + Pi/2 - b*x))] - PolyLog[2,
E^(I*(-a + Pi/2 - b*x))])))/(2*b^2) + (d*x)/(4*b*(Cos[a/2 + (b*x)/2] - Sin[
a/2 + (b*x)/2])^2) - (d*Sin[(b*x)/2])/(2*b^2*(Cos[a/2] - Sin[a/2])*(Cos[a/2
 + (b*x)/2] - Sin[a/2 + (b*x)/2])) - (d*x)/(4*b*(Cos[a/2 + (b*x)/2] + Sin[a
/2 + (b*x)/2])^2) + (d*Sin[(b*x)/2])/(2*b^2*(Cos[a/2] + Sin[a/2])*(Cos[a/2
+ (b*x)/2] + Sin[a/2 + (b*x)/2])) + (c*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Maple [B] time = 0.197, size = 267, normalized size = 2.3

$$\frac{-i\left(dxbe^{3\,i(bx+a)}+cbe^{3\,i(bx+a)}-dxbe^{i(bx+a)}-cbe^{i(bx+a)}-ide^{3\,i(bx+a)}-ide^{i(bx+a)}\right)}{b^2\left(e^{2\,i(bx+a)}+1\right)^2}-\frac{ic\arctan\left(e^{i(bx+a)}\right)}{b}-\frac{d\ln\left(1+ie^{i(bx+a)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)^3,x)
```

```
[Out] -I/b^2/(exp(2*I*(b*x+a))+1)^2*(d*x*b*exp(3*I*(b*x+a))+c*b*exp(3*I*(b*x+a))-
d*x*b*exp(I*(b*x+a))-c*b*exp(I*(b*x+a))-I*d*exp(3*I*(b*x+a))-I*d*exp(I*(b*x
+a)))-I/b*c*arctan(exp(I*(b*x+a)))-1/2/b*d*ln(1+I*exp(I*(b*x+a)))*x-1/2/b^2
*d*ln(1+I*exp(I*(b*x+a)))*a+1/2/b*d*ln(1-I*exp(I*(b*x+a)))*x+1/2/b^2*d*ln(1
-I*exp(I*(b*x+a)))*a+1/2*I/b^2*d*dilog(1+I*exp(I*(b*x+a)))-1/2*I/b^2*d*dilo
g(1-I*exp(I*(b*x+a)))+I/b^2*d*a*arctan(exp(I*(b*x+a)))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.78553, size = 1170, normalized size = 10.

$$\frac{-i\,d\cos\left(bx+a\right)^{2}\operatorname{Li}_{2}\left(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)-i\,d\cos\left(bx+a\right)^{2}\operatorname{Li}_{2}\left(i\,\cos\left(bx+a\right)-\sin\left(bx+a\right)\right)+i\,d\cos\left(bx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(-I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) - I*d*cos(b*x
 + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + I*d*cos(b*x + a)^2*dilog(-I*
cos(b*x + a) + sin(b*x + a)) + I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - s
in(b*x + a)) + (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a)
 + I) - (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I) +
 (b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d
*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b*d*x +
a*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*x + a*d)
*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d)*cos(b
*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) - (b*c - a*d)*cos(b*x + a
)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 2*d*cos(b*x + a) + 2*(b*d*x +
 b*c)*sin(b*x + a))/(b^2*cos(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)\sec^{3}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*sec(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\sec\left(bx+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a)^3, x)
```

3.40 $\int \frac{\sec^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\sec^2(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable[Sec[a + b*x]^2/(c + d*x), x]
```

Rubi [A] time = 0.0382456, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[a + b*x]^2/(c + d*x),x]
```

```
[Out] Defer[Int][Sec[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\sec^2(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 1.44811, size = 0, normalized size = 0.

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[a + b*x]^2/(c + d*x),x]
```

```
[Out] Integrate[Sec[a + b*x]^2/(c + d*x), x]
```

Maple [A] time = 0.018, size = 0, normalized size = 0.

$$\int \frac{(\sec(bx+a))^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)^2/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(bx+a)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^2/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sec(a + b*x)**2/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^2/(d*x + c), x)
```

3.41 $\int (c+dx)^{5/2}\cos(a+bx)\,dx$

Optimal. Leaf size=194

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{7/2}}-\frac{15d^2\sqrt{c+dx}\sin(a+bx)}{4b^3}+5d$$

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(2*b^2) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[a -
 (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4*b^(7/2))
 + (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[
d]]*Sin[a - (b*c)/d])/(4*b^(7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(4*
b^3) + ((c + d*x)^(5/2)*Sin[a + b*x])/b
```

Rubi [A] time = 0.422734, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{7/2}}-\frac{15d^2\sqrt{c+dx}\sin(a+bx)}{4b^3}+5d$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x],x]
```

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(2*b^2) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[a -
 (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4*b^(7/2))
 + (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[
d]]*Sin[a - (b*c)/d])/(4*b^(7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(4*
b^3) + ((c + d*x)^(5/2)*Sin[a + b*x])/b
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\,dx = \frac{(c+dx)^{5/2}\sin(a+bx)}{b} - \frac{(5d)\int (c+dx)^{3/2}\sin(a+bx)\,dx}{2b}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{2b^2} + \frac{(c+dx)^{5/2}\sin(a+bx)}{b} - \frac{\left(15d^2\right)\int \sqrt{c+dx}\cos(a+bx)\,dx}{4b^2}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{2b^2} - \frac{15d^2\sqrt{c+dx}\sin(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\sin(a+bx)}{b} + \frac{\left(15d^3 \right.}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{2b^2} - \frac{15d^2\sqrt{c+dx}\sin(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\sin(a+bx)}{b} + \frac{\left(15d^3 \right.}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{2b^2} - \frac{15d^2\sqrt{c+dx}\sin(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\sin(a+bx)}{b} + \frac{\left(15d^2 \right.}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{2b^2} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{7/2}} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}C\left(\frac{\sqrt{b}\sqrt{}}{}\right.}{4}$$

Mathematica [C] time = 0.056424, size = 124, normalized size = 0.64

$$-\frac{d^3e^{-\frac{i(ad+bc)}{d}}\left(e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\text{Gamma}\left(\frac{7}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\text{Gamma}\left(\frac{7}{2},\frac{ib(c+dx)}{d}\right)\right)}{2b^4\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x],x]
```

```
[Out] -(d^3*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[7/2, ((-I)*b*(c + d*x))
/d] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[7/2, (I*b*(c + d*x))/
d]))/(2*b^4*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x])
```

Maple [A] time = 0.031, size = 232, normalized size = 1.2

$$2\frac{1}{d}\left(1/2\frac{d(dx+c)^{5/2}}{b}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)-5/2\frac{d}{b}\left(-1/2\frac{d(dx+c)^{3/2}}{b}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)+3/2\frac{d}{b}\left(1/2\frac{d\sqrt{}}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a),x)
```

```
[Out] 2/d*(1/2/b*d*(d*x+c)^(5/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-5/2/b*d*(-1/2/b*d
*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2
)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin((
a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 2.24648, size = 886, normalized size = 4.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/32*(80*(d*x + c)^(3/2)*b*d^2*sqrt(abs(b)/abs(d))*cos(((d*x + c)*b - b*c +
 a*d)/d) + ((15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) + 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))))*d^3*cos(-(b*c - a*d)/d) + (15*sqrt(pi)*cos(1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(-1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sin(-(b*c - a
*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + ((-15*I*sqrt(pi)*cos(1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*cos(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*cos(-(b*c - a*d)/d
) + (15*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + 15*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqr
t(d^2))) - 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d
/sqrt(d^2))))*d^3*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + 8*
(4*(d*x + c)^(5/2)*b^2*d*sqrt(abs(b)/abs(d)) - 15*sqrt(d*x + c)*d^3*sqrt(ab
s(b)/abs(d)))*sin(((d*x + c)*b - b*c + a*d)/d))/(b^3*d*sqrt(abs(b)/abs(d)))
```

Fricas [A] time = 1.48212, size = 466, normalized size = 2.4

$$\frac{15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+2\sqrt{dx+c}\left(10\left(b^2d^2\right.\right.}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(
2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel
_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 2*sqrt(d*x
 + c)*(10*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) + (4*b^3*d^2*x^2 + 8*b^3*c*d*x
 + 4*b^3*c^2 - 15*b*d^2)*sin(b*x + a)))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.26431, size = 1374, normalized size = 7.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*(4*(-I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*
x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)*b) + 2*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c +
 I*a*d)/d)/b - 2*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b
)*c^2 - d^2*((sqrt(2)*sqrt(pi)*(4*I*b^2*c^2*d - 12*b*c*d^2 - 15*I*d^3)*d*er
f(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b
*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(-4*I*(d*x + c
)^(5/2)*b^2*d + 8*I*(d*x + c)^(3/2)*b^2*c*d - 4*I*sqrt(d*x + c)*b^2*c^2*d -
 10*(d*x + c)^(3/2)*b*d^2 + 12*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d
^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^2 + (sqrt(2)*sqrt(pi)*(-4
*I*b^2*c^2*d - 12*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(4*I*(d*x + c)^(5/2)*b^2*d - 8*I*(d*x + c)^(
3/2)*b^2*c*d + 4*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 12*
sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((I*(d*x + c)*b - I*b*c +
 I*a*d)/d)/b^3)/d^2) - 4*(sqrt(2)*sqrt(pi)*(-2*I*b*c*d + 3*d^2)*d*erf(-1/2*
sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*
a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + sqrt(2)*sqrt(pi)*(2*I*b
*c*d + 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^
2) - 2*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)
*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2 - 2*(-2*I*(d*x + c)^(3/2)*b
*d + 2*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*
b*c - I*a*d)/d)/b^2)*c)/d
```

3.42 $\int (c+dx)^{3/2}\cos(a+bx)\,dx$

Optimal. Leaf size=169

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}}+\frac{3d\sqrt{c+dx}\cos(a+bx)}{2b^2}+\frac{(c+ \ldots}{}$$

```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(2*b^2) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b
*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2*b^(5/2)) +
(3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*
Sin[a - (b*c)/d])/(2*b^(5/2)) + ((c + d*x)^(3/2)*Sin[a + b*x])/b
```

Rubi [A] time = 0.23601, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}}+\frac{3d\sqrt{c+dx}\cos(a+bx)}{2b^2}+\frac{(c+ \ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x],x]
```



```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(2*b^2) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b
*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2*b^(5/2)) +
(3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*
Sin[a - (b*c)/d])/(2*b^(5/2)) + ((c + d*x)^(3/2)*Sin[a + b*x])/b
```


Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\,dx=\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{(3d)\int \sqrt{c+dx}\sin(a+bx)\,dx}{2b}$$

$$=\frac{3d\sqrt{c+dx}\cos(a+bx)}{2b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{\left(3d^2\right)\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx}{4b^2}$$

$$=\frac{3d\sqrt{c+dx}\cos(a+bx)}{2b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{\left(3d^2\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{4b^2}+$$

$$=\frac{3d\sqrt{c+dx}\cos(a+bx)}{2b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{\left(3d\cos\left(a-\frac{bc}{d}\right)\right)\text{Subst}\left(\int\cos\left(\frac{bx^2}{d}\right.\right.}{2b^2}$$

$$=\frac{3d\sqrt{c+dx}\cos(a+bx)}{2b^2}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}}+\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+}}{\sqrt{d}}\right.}{2b^{5/2}}$$

Mathematica [C] time = 0.101229, size = 122, normalized size = 0.72

$$\frac{d\sqrt{c+dx}e^{-\frac{i(ad+bc)}{d}}\left(\frac{e^{2ia}\text{Gamma}\left(\frac{5}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}+\frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{5}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x],x]
```

```
[Out] (d*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[5/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)
*b*(c + d*x))/d] + (E^(((2*I)*b*c)/d)*Gamma[5/2, (I*b*(c + d*x))/d])/Sqrt[(
I*b*(c + d*x))/d]))/(2*b^2*E^((I*(b*c + a*d))/d))
```

Maple [A] time = 0.03, size = 189, normalized size = 1.1

$$2\frac{1}{d}\left(1/2\frac{d(dx+c)^{3/2}}{b}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)-3/2\frac{d}{b}\left(-1/2\frac{d\sqrt{dx+c}}{b}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)+1/4\frac{d\sqrt{2}\sqrt{\pi}}{b}\left(\text{co}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a),x)
```

```
[Out] 2/d*(1/2/b*d*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d
*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
```

```
)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d))))
```

Maxima [C] time = 2.22993, size = 855, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/16*(16*(d*x + c)^(3/2)*b*d*sqrt(abs(b)/abs(d))*sin(((d*x + c)*b - b*c + a
*d)/d) + 24*sqrt(d*x + c)*d^2*sqrt(abs(b)/abs(d))*cos(((d*x + c)*b - b*c +
a*d)/d) - ((3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/s
qrt(d^2))) - 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d
/sqrt(d^2))) + 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))))*d^2*cos(-(b*c - a*d)/d) - (3*I*sqrt(pi)*cos(1/4*pi + 1/2*a
rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*cos(-1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(-1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sin(-(b*c - a*d)/d))*e
rf(sqrt(d*x + c)*sqrt(I*b/d)) - ((3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*cos(-(b*c - a*d)/d) - (-3*I*sqrt
(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sq
rt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*s
qrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*s
qrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2
*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)))/(b^2*d*sqrt(abs(b)/a
bs(d)))
```

Fricas [A] time = 1.47785, size = 394, normalized size = 2.33

$$-\frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-2\left(3\,bd\cos\left(bx+a\right)\right.}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(
2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_
sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 2*(3*b*d*co
s(b*x + a) + 2*(b^2*d*x + b^2*c)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^{\frac{3}{2}}\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(3/2)*cos(a + b*x), x)
```

Giac [C] time = 1.20883, size = 767, normalized size = 4.54

$$-\frac{2\left(-\frac{i\sqrt{2}\sqrt{\pi}d^{2}\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)b}+\frac{i\sqrt{2}\sqrt{\pi}d^{2}\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)}{2\,d}\right)e^{\left(\frac{-i\,bc+i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)b}+\frac{2i\sqrt{dx+c}de^{\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{d}\right)}}{b}-2i\sqrt{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*(-I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)*b) + 2*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c +
I*a*d)/d)/b - 2*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)
*c - sqrt(2)*sqrt(pi)*(-2*I*b*c*d + 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqr
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b^2) - sqrt(2)*sqrt(pi)*(2*I*b*c*d + 3*d^2)*d*erf(-
1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*
c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)
^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)
*b - I*b*c + I*a*d)/d)/b^2 + 2*(-2*I*(d*x + c)^(3/2)*b*d + 2*I*sqrt(d*x + c
)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/
d
```

3.43 $\int \sqrt{c+dx}\cos(a+bx)\,dx$

Optimal. Leaf size=142

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}}+\frac{\sqrt{c+dx}\sin(a+bx)}{b}$$

```
[Out] -((Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]])/b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/P
i]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/b^(3/2) + (Sqrt[c + d*x]*Sin[a
 + b*x])/b
```

Rubi [A] time = 0.171514, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}}+\frac{\sqrt{c+dx}\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x],x]
```


```
[Out] -((Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]])/b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/P
i]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/b^(3/2) + (Sqrt[c + d*x]*Sin[a
 + b*x])/b
```


Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{c+dx}\cos(a+bx)\,dx = \frac{\sqrt{c+dx}\sin(a+bx)}{b} - \frac{d\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{2b}$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{b} - \frac{\left(d\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{2b} - \frac{\left(d\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{2b}$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{b} - \frac{\cos\left(a-\frac{bc}{d}\right)\mathrm{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)dx, x, \sqrt{c+dx}\right)}{b} - \frac{\sin\left(a-\frac{bc}{d}\right)\mathrm{Su}}{}$$

$$= -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}} - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{b^{3/2}} + \frac{\sqrt{c+dx}\sin}{b}$$

Mathematica [C] time = 0.0917543, size = 124, normalized size = 0.87

$$-\frac{i\sqrt{c+dx}e^{-\frac{i(ad+bc)}{d}}\left(\frac{e^{2ia}\mathrm{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}}\mathrm{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x],x]
```

```
[Out] ((-I/2)*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[
((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/S
qrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d))
```

Maple [A] time = 0.029, size = 144, normalized size = 1.

$$2\frac{1}{d}\left(1/2\frac{d\sqrt{dx+c}}{b}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right) - 1/4\frac{d\sqrt{2}\sqrt{\pi}}{b}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\sqrt{dx+cb}}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right) + \sin\left(\frac{da-}{d}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a),x)
```

```
[Out] 2/d*(1/2/b*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*P
i^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)
*(d*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*
(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 2.05253, size = 776, normalized size = 5.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(8*sqrt(d*x + c)*d*sqrt(abs(b)/abs(d))*sin(((d*x + c)*b - b*c + a*d)/d)
 + ((-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) - sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) + sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
))*d*cos(-(b*c - a*d)/d) - (sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))))*d*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/
d)) + ((I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) + sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))))*d*cos(-(b*c - a*d)/d) - (sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2
*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2
*arctan2(0, d/sqrt(d^2))))*d*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I
*b/d)))/(b*d*sqrt(abs(b)/abs(d)))
```

Fricas [A] time = 1.41488, size = 328, normalized size = 2.31

$$-\frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-2\sqrt{dx+c}\,b\sin\left(bx+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*s
qrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 2*sqrt(d*x + c)*b*s
in(b*x + a))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c+dx}\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x)*cos(a + b*x), x)
```

Giac [C] time = 1.16625, size = 335, normalized size = 2.36

$$-\frac{-\frac{i\sqrt{2}\sqrt{\pi}d^{2}\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^{2}d^{2}}}+1\right)}{2d}\right)e^{\left(\frac{ibc-iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^{2}d^{2}}}+1\right)b}+\frac{i\sqrt{2}\sqrt{\pi}d^{2}\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^{2}d^{2}}}+1\right)}{2d}\right)e^{\left(\frac{-ibc+iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^{2}d^{2}}}+1\right)b}+\frac{2i\sqrt{dx+c}de^{\left(\frac{i(dx+c)b-ibc+iad}{d}\right)}}{b}-2i\sqrt{dx+}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(-I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)*b) + 2*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a
*d)/d)/b - 2*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)/d
```

3.44 $\int \frac{\cos(a+bx)}{\sqrt{c+dx}} dx$

Optimal. Leaf size=118

$$\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] (Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sq
rt[d]])/(Sqrt[b]*Sqrt[d]) - (Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
 + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(Sqrt[b]*Sqrt[d])
```

Rubi [A] time = 0.133968, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sq
rt[d]])/(Sqrt[b]*Sqrt[d]) - (Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
 + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(Sqrt[b]*Sqrt[d])
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx = \cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx - \sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx$$
$$= \frac{\left(2\cos\left(a-\frac{bc}{d}\right)\right)\text{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)dx, x, \sqrt{c+dx}\right)}{d} - \frac{\left(2\sin\left(a-\frac{bc}{d}\right)\right)\text{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}} - \frac{\sqrt{2\pi}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{\sqrt{b}\sqrt{d}}$$

Mathematica [C] time = 0.0604899, size = 124, normalized size = 1.05

$$\frac{ie^{-\frac{i(ad+bc)}{d}}\left(e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\text{Gamma}\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right)-e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\text{Gamma}\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)\right)}{2b\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] ((I/2)*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/2, ((-I)*b*(c + d*
x))/d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[1/2, (I*b*(c + d*
x))/d]))/(b*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x])
```

Maple [A] time = 0.031, size = 100, normalized size = 0.9

$$\frac{\sqrt{2}\sqrt{\pi}}{d}\left(\cos\left(\frac{da-cb}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}b}{\sqrt{\pi}d}\sqrt{dx+c}\frac{1}{\sqrt{\frac{b}{d}}}\right)-\sin\left(\frac{da-cb}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}b}{\sqrt{\pi}d}\sqrt{dx+c}\frac{1}{\sqrt{\frac{b}{d}}}\right)\right)\frac{1}{\sqrt{\frac{b}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2
)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)
/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))
```

Maxima [C] time = 2.12306, size = 714, normalized size = 6.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)))*cos(-(b*c - a*d)/d) - (I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2
*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d))
 + ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
 + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
- I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
)*cos(-(b*c - a*d)/d) - (-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d))
)/(d*sqrt(abs(b)/abs(d)))
```

Fricas [A] time = 1.34967, size = 269, normalized size = 2.28

$$\frac{\sqrt{2}\pi\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\pi\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*pi*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x
 + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(
d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(a+bx\right)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(cos(a + b*x)/sqrt(c + d*x), x)
```

Giac [C] time = 1.14548, size = 224, normalized size = 1.9

$$-\frac{\frac{\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{ibc-iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}+\frac{\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{-ibc+iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
 1)) + sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^
2) + 1)))/d
```

3.45 $\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=139

$$-\frac{2\sqrt{2\pi}\sqrt{b}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sqrt{2\pi}\sqrt{b}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}$$

```
[Out] (-2*Cos[a + b*x])/(d*Sqrt[c + d*x]) - (2*Sqrt[b]*Sqrt[2*Pi]*Cos[a - (b*c)/d
]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/d^(3/2) - (2*Sqrt[b
]*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (
b*c)/d])/d^(3/2)
```

Rubi [A] time = 0.18567, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3297, 3306, 3305, 3351, 3304, 3352}

$$-\frac{2\sqrt{2\pi}\sqrt{b}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sqrt{2\pi}\sqrt{b}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(d*Sqrt[c + d*x]) - (2*Sqrt[b]*Sqrt[2*Pi]*Cos[a - (b*c)/d
]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/d^(3/2) - (2*Sqrt[b
]*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (
b*c)/d])/d^(3/2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx &= -\frac{2\cos(a+bx)}{d\sqrt{c+dx}} - \frac{(2b)\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{d}\\ &= -\frac{2\cos(a+bx)}{d\sqrt{c+dx}} - \frac{\left(2b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{d} - \frac{\left(2b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{d}\\ &= -\frac{2\cos(a+bx)}{d\sqrt{c+dx}} - \frac{\left(4b\cos\left(a-\frac{bc}{d}\right)\right)\text{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)dx, x, \sqrt{c+dx}\right)}{d^2} - \frac{\left(4b\sin\left(a-\frac{bc}{d}\right)\right)\text{Subst}}{}\\ &= -\frac{2\cos(a+bx)}{d\sqrt{c+dx}} - \frac{2\sqrt{b}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\sqrt{b}\sqrt{2\pi}C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{d^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.330311, size = 147, normalized size = 1.06

$$\frac{e^{-ia}\left(e^{2ia-\frac{ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)+e^{-ibx}\left(e^{\frac{ib(c+dx)}{d}}\sqrt{\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right)-e^{2i(a+bx)}-1\right)\right)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (E^((2*I)*a - (I*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/2, ((-I)*b*(c +
 d*x))/d] + (-1 - E^((2*I)*(a + b*x)) + E^((I*b*(c + d*x))/d)*Sqrt[(I*b*(c
+ d*x))/d]*Gamma[1/2, (I*b*(c + d*x))/d])/E^(I*b*x))/(d*E^(I*a)*Sqrt[c + d*
x])
```

Maple [A] time = 0.03, size = 140, normalized size = 1.

$$2\frac{1}{d}\left(-\frac{1}{\sqrt{dx+c}}\cos\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)-\frac{\sqrt{2}b\sqrt{\pi}}{d}\left(\cos\left(\frac{da-cb}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}\sqrt{dx+c}b}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right)+\sin\left(\frac{da-cb}{d}\right)\text{Fr}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^(3/2),x)
```

```
[Out] 2/d*(-1/(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)-b/d*2^(1/2)*Pi^(1/2)/(
b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^
```

```
(1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(
1/2)*b/d)))
```

Maxima [C] time = 1.56839, size = 632, normalized size = 4.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(((gamma(-1/2, I*(d*x + c)*b/d) + gamma(-1/2, -I*(d*x + c)*b/d))*cos(1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + (gamma(-1/2, I*(
d*x + c)*b/d) + gamma(-1/2, -I*(d*x + c)*b/d))*cos(-1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))) + (I*gamma(-1/2, I*(d*x + c)*b/d) - I*ga
mma(-1/2, -I*(d*x + c)*b/d))*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))) + (-I*gamma(-1/2, I*(d*x + c)*b/d) + I*gamma(-1/2, -I*(d*x
+ c)*b/d))*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*
cos(-(b*c - a*d)/d) + ((-I*gamma(-1/2, I*(d*x + c)*b/d) + I*gamma(-1/2, -I*
(d*x + c)*b/d))*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) + (-I*gamma(-1/2, I*(d*x + c)*b/d) + I*gamma(-1/2, -I*(d*x + c)*b/d))*co
s(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + (gamma(-1/2,
 I*(d*x + c)*b/d) + gamma(-1/2, -I*(d*x + c)*b/d))*sin(1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - (gamma(-1/2, I*(d*x + c)*b/d) + gam
ma(-1/2, -I*(d*x + c)*b/d))*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))))*sin(-(b*c - a*d)/d))*sqrt((d*x + c)*abs(b)/abs(d))/(sqrt(d
*x + c)*d)
```

Fricas [A] time = 1.42787, size = 363, normalized size = 2.61

$$-\frac{2\left(\sqrt{2}(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+\sqrt{2}(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+\sqrt{dx\cdots}\right.}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(2)*(pi*d*x + pi*c)*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(
sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(2)*(pi*d*x + pi*c)*sqrt(b/(pi*
d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) +
 sqrt(d*x + c)*cos(b*x + a))/(d^2*x + c*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(d*x + c)^(3/2), x)
```

3.46 $\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=168

$$-\frac{4\sqrt{2\pi}b^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{4\sqrt{2\pi}b^{3/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{4b\sin(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}$$

```
[Out] (-2*Cos[a + b*x])/(3*d*(c + d*x)^(3/2)) - (4*b^(3/2)*Sqrt[2*Pi]*Cos[a - (b*
c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) + (
4*b^(3/2)*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[a - (b*c)/d])/(3*d^(5/2)) + (4*b*Sin[a + b*x])/(3*d^2*Sqrt[c + d*x])
```

Rubi [A] time = 0.262334, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3297, 3306, 3305, 3351, 3304, 3352}

$$-\frac{4\sqrt{2\pi}b^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{4\sqrt{2\pi}b^{3/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{4b\sin(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(3*d*(c + d*x)^(3/2)) - (4*b^(3/2)*Sqrt[2*Pi]*Cos[a - (b*
c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) + (
4*b^(3/2)*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[a - (b*c)/d])/(3*d^(5/2)) + (4*b*Sin[a + b*x])/(3*d^2*Sqrt[c + d*x])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx &= -\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}} - \frac{(2b)\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}\,dx}{3d}\\ &= -\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}} + \frac{4b\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{\left(4b^2\right)\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx}{3d^2}\\ &= -\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}} + \frac{4b\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{\left(4b^2\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{3d^2} + \frac{\left(4b^2\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}\right.}{\sqrt{c}}}{3d^2}\\ &= -\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}} + \frac{4b\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{\left(8b^2\cos\left(a-\frac{bc}{d}\right)\right)\mathrm{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)dx, x, \sqrt{c+dx}\right)}{3d^3} + \frac{\left(8b^2\right.}{}\\ &= -\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}} - \frac{4b^{3/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{4b^{3/2}\sqrt{2\pi}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{3d^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.302547, size = 190, normalized size = 1.13

$$\frac{e^{-ia}\left(e^{-ibx}\left(-4de^{\frac{ib(c+dx)}{d}}\left(\frac{ib(c+dx)}{d}\right)^{3/2}\mathrm{Gamma}\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right)+4ib(c+dx)-2d\right)-2ie^{2ia-\frac{ibc}{d}}\left(e^{\frac{ib(c+dx)}{d}}(2b(c+dx)-id)-2id\right.\right.}{6d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] ((-2*I)*E^((2*I)*a - (I*b*c)/d)*(E^((I*b*(c + d*x))/d)*((-I)*d + 2*b*(c + d
*x)) - (2*I)*d*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-I)*b*(c + d*x))/d
]) + (-2*d + (4*I)*b*(c + d*x) - 4*d*E^((I*b*(c + d*x))/d)*((I*b*(c + d*x))
/d)^(3/2)*Gamma[1/2, (I*b*(c + d*x))/d])/E^(I*b*x))/(6*d^2*E^(I*a)*(c + d*x
)^(3/2))
```

Maple [A] time = 0.03, size = 180, normalized size = 1.1

$$2\,\frac{1}{d}\left(-1/3\,\frac{1}{(dx+c)^{3/2}}\cos\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)-2/3\,\frac{b}{d}\left(-\frac{1}{\sqrt{dx+c}}\sin\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)+\frac{\sqrt{2}b\sqrt{\pi}}{d}\left(\cos\left(\frac{da-}{d}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^(5/2),x)
```

```
[Out] 2/d*(-1/3/(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)-2/3*b/d*(-1/(d*x+c)^
(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)+b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(
(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin((
a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

Maxima [C] time = 1.5105, size = 632, normalized size = 3.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(((gamma(-3/2, I*(d*x + c)*b/d) + gamma(-3/2, -I*(d*x + c)*b/d))*cos(3
/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) + (gamma(-3/2, I*(
d*x + c)*b/d) + gamma(-3/2, -I*(d*x + c)*b/d))*cos(-3/4*pi + 3/2*arctan2(0,
 b) + 3/2*arctan2(0, d/sqrt(d^2))) + (I*gamma(-3/2, I*(d*x + c)*b/d) - I*ga
mma(-3/2, -I*(d*x + c)*b/d))*sin(3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0
, d/sqrt(d^2))) + (-I*gamma(-3/2, I*(d*x + c)*b/d) + I*gamma(-3/2, -I*(d*x
+ c)*b/d))*sin(-3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))))*
cos(-(b*c - a*d)/d) + ((-I*gamma(-3/2, I*(d*x + c)*b/d) + I*gamma(-3/2, -I*
(d*x + c)*b/d))*cos(3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2)
)) + (-I*gamma(-3/2, I*(d*x + c)*b/d) + I*gamma(-3/2, -I*(d*x + c)*b/d))*co
s(-3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) + (gamma(-3/2,
 I*(d*x + c)*b/d) + gamma(-3/2, -I*(d*x + c)*b/d))*sin(3/4*pi + 3/2*arctan2
(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) - (gamma(-3/2, I*(d*x + c)*b/d) + gam
ma(-3/2, -I*(d*x + c)*b/d))*sin(-3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0
, d/sqrt(d^2))))*sin(-(b*c - a*d)/d))*((d*x + c)*abs(b)/abs(d))^(3/2)/((d*x
 + c)^(3/2)*d)
```

Fricas [A] time = 1.47589, size = 510, normalized size = 3.04

$$-\frac{2\left(2\sqrt{2}\left(\pi bd^2x^2+2\pi bcdx+\pi bc^2\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-2\sqrt{2}\left(\pi bd^2x^2+2\pi bcdx+\pi bc^2\right)\sqrt{\frac{b}{\pi d}}\mathrm{S}\left(\sqrt{\cdots}\right.}{3\left(d^4x^2+2cd^3x+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*sqrt(2)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos
(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 2*sqrt
(2)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_sin(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + sqrt(d*x + c)*(d*c
os(b*x + a) - 2*(b*d*x + b*c)*sin(b*x + a)))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)}{(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(d*x + c)^(5/2), x)
```

3.47 $\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal. Leaf size=193

$$\frac{8\sqrt{2\pi}b^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{8\sqrt{2\pi}b^{5/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{8b^2\cos(a+bx)}{15d^3\sqrt{c+dx}}+\frac{4b\sin(a+b}{15d^2(c+dx)}$$

```
[Out] (-2*Cos[a + b*x])/(5*d*(c + d*x)^(5/2)) + (8*b^2*Cos[a + b*x])/(15*d^3*Sqrt
[c + d*x]) + (8*b^(5/2)*Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(15*d^(7/2)) + (8*b^(5/2)*Sqrt[2*Pi]*Fresnel
C[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(15*d^(7/2)
) + (4*b*Sin[a + b*x])/(15*d^2*(c + d*x)^(3/2))
```

Rubi [A] time = 0.296318, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3297, 3306, 3305, 3351, 3304, 3352}

$$\frac{8\sqrt{2\pi}b^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{8\sqrt{2\pi}b^{5/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{8b^2\cos(a+bx)}{15d^3\sqrt{c+dx}}+\frac{4b\sin(a+b}{15d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x)^(7/2),x]
```


```
[Out] (-2*Cos[a + b*x])/(5*d*(c + d*x)^(5/2)) + (8*b^2*Cos[a + b*x])/(15*d^3*Sqrt
[c + d*x]) + (8*b^(5/2)*Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(15*d^(7/2)) + (8*b^(5/2)*Sqrt[2*Pi]*Fresnel
C[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(15*d^(7/2)
) + (4*b*Sin[a + b*x])/(15*d^2*(c + d*x)^(3/2))
```


Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}} - \frac{(2b)\int \frac{\sin(a+bx)}{(c+dx)^{5/2}}\,dx}{5d}$$
$$= -\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}} + \frac{4b\sin(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{\left(4b^2\right)\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx}{15d^2}$$
$$= -\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}} + \frac{8b^2\cos(a+bx)}{15d^3\sqrt{c+dx}} + \frac{4b\sin(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{\left(8b^3\right)\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{15d^3}$$
$$= -\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}} + \frac{8b^2\cos(a+bx)}{15d^3\sqrt{c+dx}} + \frac{4b\sin(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{\left(8b^3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{15d^3} + \frac{\left(8b^3 \text{s}\right.}{}$$
$$= -\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}} + \frac{8b^2\cos(a+bx)}{15d^3\sqrt{c+dx}} + \frac{4b\sin(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{\left(16b^3\cos\left(a-\frac{bc}{d}\right)\right)\text{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)dx\right.}{15d^4}$$
$$= -\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}} + \frac{8b^2\cos(a+bx)}{15d^3\sqrt{c+dx}} + \frac{8b^{5/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{8b^{5/2}\sqrt{2\pi}C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\ldots}{\sqrt{d}}\right.}{15d}$$

Mathematica [C] time = 0.388496, size = 228, normalized size = 1.18

$$\frac{e^{-ia}\left(2e^{2ia}\left(2be^{-\frac{ibc}{d}}(c+dx)\left(e^{\frac{ib(c+dx)}{d}}(2b(c+dx)-id)-2id\left(-\frac{ib(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)\right)-3d^2e^{ibx}\right)+e^{-ibx}\left(8d^2e\right.\ldots}{30d^3(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(7/2),x]
```

```
[Out] (2*E^((2*I)*a)*(-3*d^2*E^(I*b*x) + (2*b*(c + d*x)*(E^((I*b*(c + d*x))/d)*((
-I)*d + 2*b*(c + d*x)) - (2*I)*d*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((
-I)*b*(c + d*x))/d]))/E^((I*b*c)/d)) + (-6*d^2 + (4*I)*b*d*(c + d*x) + 8*b^
2*(c + d*x)^2 + 8*d^2*E^((I*b*(c + d*x))/d)*((I*b*(c + d*x))/d)^(5/2)*Gamma
[1/2, (I*b*(c + d*x))/d])/E^(I*b*x))/(30*d^3*E^(I*a)*(c + d*x)^(5/2))
```

Maple [A] time = 0.03, size = 220, normalized size = 1.1

$$2\frac{1}{d}\left(-1/5\frac{1}{(dx+c)^{5/2}}\cos\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)-2/5\frac{b}{d}\left(-1/3\frac{1}{(dx+c)^{3/2}}\sin\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)+2/3\frac{b}{d}\left(-\frac{1}{\sqrt{dx+c}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^(7/2),x)
```

```
[Out] 2/d*(-1/5/(d*x+c)^(5/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)-2/5*b/d*(-1/3/(d*x+c
)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)+2/3*b/d*(-1/(d*x+c)^(1/2)*cos(1/d*(d
*x+c)*b+(a*d-b*c)/d)-b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*Fre
snelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*Fres
nelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 1.55995, size = 632, normalized size = 3.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/4*(((gamma(-5/2, I*(d*x + c)*b/d) + gamma(-5/2, -I*(d*x + c)*b/d))*cos(5
/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) + (gamma(-5/2, I*(
d*x + c)*b/d) + gamma(-5/2, -I*(d*x + c)*b/d))*cos(-5/4*pi + 5/2*arctan2(0,
 b) + 5/2*arctan2(0, d/sqrt(d^2))) + (I*gamma(-5/2, I*(d*x + c)*b/d) - I*ga
mma(-5/2, -I*(d*x + c)*b/d))*sin(5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0
, d/sqrt(d^2))) + (-I*gamma(-5/2, I*(d*x + c)*b/d) + I*gamma(-5/2, -I*(d*x
+ c)*b/d))*sin(-5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))))*
cos(-(b*c - a*d)/d) + ((-I*gamma(-5/2, I*(d*x + c)*b/d) + I*gamma(-5/2, -I*
(d*x + c)*b/d))*cos(5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2)
)) + (-I*gamma(-5/2, I*(d*x + c)*b/d) + I*gamma(-5/2, -I*(d*x + c)*b/d))*co
s(-5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) + (gamma(-5/2,
 I*(d*x + c)*b/d) + gamma(-5/2, -I*(d*x + c)*b/d))*sin(5/4*pi + 5/2*arctan2
(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) - (gamma(-5/2, I*(d*x + c)*b/d) + gam
ma(-5/2, -I*(d*x + c)*b/d))*sin(-5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0
, d/sqrt(d^2))))*sin(-(b*c - a*d)/d))*((d*x + c)*abs(b)/abs(d))^(5/2)/((d*x
 + c)^(5/2)*d)
```

Fricas [A] time = 1.58695, size = 680, normalized size = 3.52

$$\frac{2\left(4\sqrt{2}\left(\pi b^2d^3x^3+3\pi b^2cd^2x^2+3\pi b^2c^2dx+\pi b^2c^3\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+4\sqrt{2}\left(\pi b^2d^3x^3+3\pi b^2cd^2x\right.\right.}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(4*sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + p
i*b^2*c^3)*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x
+ c)*sqrt(b/(pi*d))) + 4*sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*p
```

```
i*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c
)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + sqrt(d*x + c)*((4*b^2*d^2*x^2 + 8*b
^2*c*d*x + 4*b^2*c^2 - 3*d^2)*cos(b*x + a) + 2*(b*d^2*x + b*c*d)*sin(b*x +
a)))/(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)}{(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(d*x + c)^(7/2), x)
```

3.48 $\int (c+dx)^{5/2}\cos^2(a+bx)\,dx$

Optimal. Leaf size=231

$$\frac{15\sqrt{\pi}d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{7/2}}+\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}}-\frac{15d^2\sqrt{c+dx}\sin(2a+2bx)}{64b^3}+5$$

```
[Out] (-5*d*(c + d*x)^(3/2))/(16*b^2) + (c + d*x)^(7/2)/(7*d) + (5*d*(c + d*x)^(3
/2)*Cos[a + b*x]^2)/(8*b^2) + (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fre
snelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(128*b^(7/2)) + (15*d^
(5/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2
*a - (2*b*c)/d])/(128*b^(7/2)) + ((c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]
)/(2*b) - (15*d^2*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(64*b^3)
```

Rubi [A] time = 0.434774, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3311, 32, 3312, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\pi}d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{7/2}}+\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}}-\frac{15d^2\sqrt{c+dx}\sin(2a+2bx)}{64b^3}+5$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2,x]
```

```
[Out] (-5*d*(c + d*x)^(3/2))/(16*b^2) + (c + d*x)^(7/2)/(7*d) + (5*d*(c + d*x)^(3
/2)*Cos[a + b*x]^2)/(8*b^2) + (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fre
snelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(128*b^(7/2)) + (15*d^
(5/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2
*a - (2*b*c)/d])/(128*b^(7/2)) + ((c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]
)/(2*b) - (15*d^2*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(64*b^3)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\,dx = \frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} + \frac{1}{2}\int (c+dx)^{5/2}\,dx -$$

$$= \frac{(c+dx)^{7/2}}{7d} + \frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} - \frac{(15d^2}{}$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} + \frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{5/2}\cos(a+bx)\sin}{2b}$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} + \frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{5/2}\cos(a+bx)\sin}{2b}$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} + \frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{5/2}\cos(a+bx)\sin}{2b}$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} + \frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{5/2}\cos(a+bx)\sin}{2b}$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} + \frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2} + \frac{15d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S}{128b^{7/2}}$$

Mathematica [A] time = 1.89746, size = 194, normalized size = 0.84

$$\frac{\sqrt{\frac{b}{d}}\left(2\sqrt{\frac{b}{d}}\sqrt{c+dx}\left(7d\sin(2(a+bx))\left(16b^2(c+dx)^2-15d^2\right)+140bd^2(c+dx)\cos(2(a+bx))+64b^3(c+dx)^3\right)+105\sqrt{}\right.}{896b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(105*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqr
t[c + d*x])/Sqrt[Pi]] + 105*d^3*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x
])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] + 2*Sqrt[b/d]*Sqrt[c + d*x]*(64*b^3*(c +
d*x)^3 + 140*b*d^2*(c + d*x)*Cos[2*(a + b*x)] + 7*d*(-15*d^2 + 16*b^2*(c +
d*x)^2)*Sin[2*(a + b*x)])))/(896*b^4)
```

Maple [A] time = 0.038, size = 242, normalized size = 1.1

$$2\frac{1}{d}\left(1/14\,(dx+c)^{7/2}+1/8\,\frac{d\,(dx+c)^{5/2}}{b}\sin\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{da-cb}{d}\right)-5/8\,\frac{d}{b}\left(-1/4\,\frac{d\,(dx+c)^{3/2}}{b}\cos\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{da}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2,x)
```

```
[Out] 2/d*(1/14*(d*x+c)^(7/2)+1/8/b*d*(d*x+c)^(5/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)
/d)-5/8/b*d*(-1/4/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/4/b*
d*(1/4/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/
(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/
d)))))
```

Maxima [C] time = 2.20208, size = 937, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/7168*sqrt(2)*(512*sqrt(2)*(d*x + c)^(7/2)*b^3*sqrt(abs(b)/abs(d)) + 1120*
sqrt(2)*(d*x + c)^(3/2)*b*d^2*sqrt(abs(b)/abs(d))*cos(2*((d*x + c)*b - b*c
+ a*d)/d) + ((105*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))) + 105*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) + 105*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))) - 105*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2
*arctan2(0, d/sqrt(d^2))))*d^3*cos(-2*(b*c - a*d)/d) + (105*sqrt(pi)*cos(1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 105*sqrt(pi)*cos(
-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 105*I*sqrt(pi)
*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 105*I*sqrt
(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*si
n(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + ((-105*I*sqrt(pi)*c
os(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 105*I*sqrt(p
i)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 105*sqr
t(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 105*s
qrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3
*cos(-2*(b*c - a*d)/d) + (105*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2
*arctan2(0, d/sqrt(d^2))) + 105*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) + 105*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b
```

```
) + 1/2*arctan2(0, d/sqrt(d^2))) - 105*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sin(-2*(b*c - a*d)/d))*erf(sqrt(
d*x + c)*sqrt(-2*I*b/d)) + 56*(16*sqrt(2)*(d*x + c)^(5/2)*b^2*d*sqrt(abs(b)
/abs(d)) - 15*sqrt(2)*sqrt(d*x + c)*d^3*sqrt(abs(b)/abs(d)))*sin(2*((d*x +
c)*b - b*c + a*d)/d))/(b^3*d*sqrt(abs(b)/abs(d)))
```

Fricas [A] time = 1.85298, size = 613, normalized size = 2.65

$$105\,\pi d^4\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+105\,\pi d^4\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+4\left(32\,b^4d^3x^3+96\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/896*(105*pi*d^4*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d
*x + c)*sqrt(b/(pi*d))) + 105*pi*d^4*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x
+ c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 4*(32*b^4*d^3*x^3 + 96*b^4*c*d
^2*x^2 + 32*b^4*c^3 - 70*b^2*c*d^2 + 140*(b^2*d^3*x + b^2*c*d^2)*cos(b*x +
a)^2 + 7*(16*b^3*d^3*x^2 + 32*b^3*c*d^2*x + 16*b^3*c^2*d - 15*b*d^3)*cos(b*
x + a)*sin(b*x + a) + 2*(48*b^4*c^2*d - 35*b^2*d^3)*x)*sqrt(d*x + c))/(b^4*
d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.37971, size = 1419, normalized size = 6.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/26880*(560*(-3*I*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
 1)*b) + 3*I*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*
b) - 16*(d*x + c)^(3/2) + 6*I*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c
 + 2*I*a*d)/d)/b - 6*I*sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I
*a*d)/d)/b)*c^2 - d^2*(256*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*
(d*x + c)^(3/2)*c^2)/d^2 + 105*(sqrt(pi)*(16*I*b^2*c^2*d - 24*b*c*d^2 - 15*
I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*
b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(-16*I*(d*x
```

```
 + c)^(5/2)*b^2*d + 32*I*(d*x + c)^(3/2)*b^2*c*d - 16*I*sqrt(d*x + c)*b^2*c
^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 24*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x
+ c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^3)/d^2 + 105*(sqrt
(pi)*(-16*I*b^2*c^2*d - 24*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(16*I*(d*x + c)^(5/2)*b^2*d - 32*I*(d*x + c
)^(3/2)*b^2*c*d + 16*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^(3/2)*b*d^2 +
 24*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((2*I*(d*x + c)*b - 2
*I*b*c + 2*I*a*d)/d)/b^3)/d^2) - 56*(192*(d*x + c)^(5/2) - 320*(d*x + c)^(3
/2)*c + 15*sqrt(pi)*(-4*I*b*c*d + 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b^2) + 15*sqrt(pi)*(4*I*b*c*d + 3*d^2)*d*erf(-sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 30*(4*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(
d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a
*d)/d)/b^2 - 30*(-4*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - 3*sqr
t(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2)*c)/d
```

3.49 $\int (c+dx)^{3/2}\cos^2(a+bx)\,dx$

Optimal. Leaf size=203

$$-\frac{3\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{32b^{5/2}}+\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}}+\frac{3d\sqrt{c+dx}\cos^2(a+bx)}{8b^2}+(c$$

```
[Out] (-3*d*Sqrt[c + d*x])/(16*b^2) + (c + d*x)^(5/2)/(5*d) + (3*d*Sqrt[c + d*x]*
Cos[a + b*x]^2)/(8*b^2) - (3*d^(3/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC
[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(32*b^(5/2)) + (3*d^(3/2)*S
qrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2
*b*c)/d])/(32*b^(5/2)) + ((c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rubi [A] time = 0.342156, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3311, 32, 3312, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{32b^{5/2}}+\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}}+\frac{3d\sqrt{c+dx}\cos^2(a+bx)}{8b^2}+(c$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2,x]
```

```
[Out] (-3*d*Sqrt[c + d*x])/(16*b^2) + (c + d*x)^(5/2)/(5*d) + (3*d*Sqrt[c + d*x]*
Cos[a + b*x]^2)/(8*b^2) - (3*d^(3/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC
[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(32*b^(5/2)) + (3*d^(3/2)*S
qrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2
*b*c)/d])/(32*b^(5/2)) + ((c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
```

```
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\,dx = \frac{3d\sqrt{c+dx}\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)}{2b} + \frac{1}{2}\int (c+dx)^{3/2}\,dx - \frac{(3d^2 \ldots}{\ldots}$$

$$= \frac{(c+dx)^{5/2}}{5d} + \frac{3d\sqrt{c+dx}\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)}{2b} - \frac{(3d^2)\int\left(\frac{\ldots}{2\ldots}\right.}{\ldots}$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} + \frac{3d\sqrt{c+dx}\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+b\ldots}{2b}$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} + \frac{3d\sqrt{c+dx}\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+b\ldots}{2b}$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} + \frac{3d\sqrt{c+dx}\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+b\ldots}{2b}$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} + \frac{3d\sqrt{c+dx}\cos^2(a+bx)}{8b^2} - \frac{3d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+\ldots}}{\sqrt{d}\sqrt{\ldots}}\right.}{32b^{5/2}}$$

Mathematica [A] time = 1.53213, size = 175, normalized size = 0.86

$$\frac{\sqrt{\frac{b}{d}}\left(-15\sqrt{\pi}d^2\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+15\sqrt{\pi}d^2\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+2\sqrt{\frac{b}{d}}\sqrt{c+dx}\left(4b(c+dx)(5\ldots\right.\right.}{160b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(-15*d^2*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqr
t[c + d*x])/Sqrt[Pi]] + 15*d^2*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x]
```

```
)/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] + 2*Sqrt[b/d]*Sqrt[c + d*x]*(15*d^2*Cos[2*
(a + b*x)] + 4*b*(c + d*x)*(4*b*(c + d*x) + 5*d*Sin[2*(a + b*x)]))))/(160*b
^3)
```

Maple [A] time = 0.036, size = 197, normalized size = 1.

$$2\,\frac{1}{d}\left(1/10\,(dx+c)^{5/2}+1/8\,\frac{d\,(dx+c)^{3/2}}{b}\sin\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{da-cb}{d}\right)-3/8\,\frac{d}{b}\left(-1/4\,\frac{d\sqrt{dx+c}}{b}\cos\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{da}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2,x)
```

```
[Out] 2/d*(1/10*(d*x+c)^(5/2)+1/8/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)
/d)-3/8/b*d*(-1/4/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*
d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*
(d*x+c)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+
c)^(1/2)*b/d))))
```

Maxima [C] time = 2.22094, size = 902, normalized size = 4.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/1280*sqrt(2)*(128*sqrt(2)*(d*x + c)^(5/2)*b^2*sqrt(abs(b)/abs(d)) + 160*s
qrt(2)*(d*x + c)^(3/2)*b*d*sqrt(abs(b)/abs(d))*sin(2*((d*x + c)*b - b*c + a
*d)/d) + 120*sqrt(2)*sqrt(d*x + c)*d^2*sqrt(abs(b)/abs(d))*cos(2*((d*x + c)
*b - b*c + a*d)/d) - ((15*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))))*d^2*cos(-2*(b*c - a*d)/d) - (15*I*sqrt(pi)*
cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(p
i)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt
(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqr
t(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*s
in(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - ((15*sqrt(pi)*cos(
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)
*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(
pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*cos
(-2*(b*c - a*d)/d) - (-15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c
)*sqrt(-2*I*b/d)))/(b^2*d*sqrt(abs(b)/abs(d)))
```

Fricas [A] time = 1.92151, size = 479, normalized size = 2.36

$$-\frac{15\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\left(16\,b^3d^2x^2+32\,b^3cd\right.}{160\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/160*(15*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d
*x + c)*sqrt(b/(pi*d))) - 15*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*
x + 16*b^3*c^2 + 30*b*d^2*cos(b*x + a)^2 - 15*b*d^2 + 40*(b^2*d^2*x + b^2*c
*d)*cos(b*x + a)*sin(b*x + a))*sqrt(d*x + c))/(b^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.27892, size = 772, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/960*(192*(d*x + c)^(5/2) - 320*(d*x + c)^(3/2)*c - 20*(-3*I*sqrt(pi)*d^2*
erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 3*I*sqrt(pi)*d^2*erf(-sq
rt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 16*(d*x + c)^(3/2) + 6*I*sqr
t(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b - 6*I*sqrt(d*x +
 c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b)*c + 15*sqrt(pi)*(-4*I
*b*c*d + 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 15*s
qrt(pi)*(4*I*b*c*d + 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)*b^2) - 30*(4*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d
*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^2 - 30*(-4*I*(d*
x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-2*I*
(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2)/d
```

3.50 $\int \sqrt{c+dx}\cos^2(a+bx)\,dx$

Optimal. Leaf size=158

$$-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{8b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}+\frac{\sqrt{c+dx}\sin(2a+2bx)}{4b}+\frac{(c+dx}{3d}$$

```
[Out] (c + d*x)^(3/2)/(3*d) - (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*
Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(8*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]
*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d
])/(8*b^(3/2)) + (Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(4*b)
```

Rubi [A] time = 0.278329, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3312, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{8b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}+\frac{\sqrt{c+dx}\sin(2a+2bx)}{4b}+\frac{(c+dx}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(3/2)/(3*d) - (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*
Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(8*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]
*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d
])/(8*b^(3/2)) + (Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(4*b)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{c+dx}\cos^2(a+bx)\,dx &= \int \left(\frac{1}{2}\sqrt{c+dx}+\frac{1}{2}\sqrt{c+dx}\cos(2a+2bx)\right)dx \\
&= \frac{(c+dx)^{3/2}}{3d}+\frac{1}{2}\int \sqrt{c+dx}\cos(2a+2bx)\,dx \\
&= \frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{c+dx}\sin(2a+2bx)}{4b}-\frac{d\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}\,dx}{8b} \\
&= \frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{c+dx}\sin(2a+2bx)}{4b}-\frac{\left(d\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{8b}-\frac{\left(d\sin\left(2a-\frac{2\ldots}{\ldots}\right.\right.}{} \\
&= \frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{c+dx}\sin(2a+2bx)}{4b}-\frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Subst}\left(\int \sin\left(\frac{2bx^2}{d}\right)dx,x,\sqrt{c+dx}\right)}{4b}\ldots \\
&= \frac{(c+dx)^{3/2}}{3d}-\frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{d}\sqrt{\pi}C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{8b^{3/2}}+\frac{\sqrt{\ldots}}{}
\end{aligned}
$$

Mathematica [C] time = 0.544999, size = 146, normalized size = 0.92

$$
\frac{1}{48}\sqrt{c+dx}\left(-\frac{3i\sqrt{2}e^{2i\left(a-\frac{bc}{d}\right)}\mathrm{Gamma}\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)}{b\sqrt{-\frac{ib(c+dx)}{d}}}+\frac{3i\sqrt{2}e^{-2i\left(a-\frac{bc}{d}\right)}\mathrm{Gamma}\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right)}{b\sqrt{\frac{ib(c+dx)}{d}}}+\frac{16(c+dx)}{d}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2,x]
```

```
[Out] (Sqrt[c + d*x]*((16*(c + d*x))/d - ((3*I)*Sqrt[2]*E^((2*I)*(a - (b*c)/d))*G
amma[3/2, ((-2*I)*b*(c + d*x))/d])/(b*Sqrt[((-I)*b*(c + d*x))/d]) + ((3*I)*
Sqrt[2]*Gamma[3/2, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*Sqrt[
(I*b*(c + d*x))/d])))/48
```

Maple [A] time = 0.036, size = 150, normalized size = 1.

$$
2\frac{1}{d}\left(1/6\,(dx+c)^{3/2}+1/8\,\frac{d\sqrt{dx+c}}{b}\sin\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{da-cb}{d}\right)-1/16\,\frac{d\sqrt{\pi}}{b}\left(\cos\left(2\,\frac{da-cb}{d}\right)\mathrm{FresnelS}\left(2\,\frac{b\sqrt{dx+c}}{d\sqrt{\pi}}\frac{1}{\sqrt{\ldots}}\right.\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2,x)
```

```
[Out] 2/d*(1/6*(d*x+c)^(3/2)+1/8/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/
d)-1/16/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b
/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(
1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 2.28497, size = 824, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/192*sqrt(2)*(32*sqrt(2)*(d*x + c)^(3/2)*b*sqrt(abs(b)/abs(d)) + 24*sqrt(2
)*sqrt(d*x + c)*d*sqrt(abs(b)/abs(d))*sin(2*((d*x + c)*b - b*c + a*d)/d) +
((-3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) - 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) - 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))))*d*cos(-2*(b*c - a*d)/d) - (3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sin(-2*(b*c - a*d)/d))*erf(sqrt(d
*x + c)*sqrt(2*I*b/d)) + ((3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))))*d*cos(-2*(b*c - a*d)/d) - (3*sqrt(pi)*cos(1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*si
n(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*
sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sin(-2*(b
*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/(b*d*sqrt(abs(b)/abs(d)))
```

Fricas [A] time = 1.88309, size = 369, normalized size = 2.34

$$-\frac{3\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-4\left(2\,b^2dx+3\,bd\cos(\right.}{24\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(3*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 3*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)
*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(2*b^2*d*x + 3*b*d*cos(b*x + a)*
sin(b*x + a) + 2*b^2*c)*sqrt(d*x + c))/(b^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c+dx}\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x)*cos(a + b*x)**2, x)
```

Giac [C] time = 1.20211, size = 331, normalized size = 2.09

$$-\frac{-\frac{3i\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{2ibc-2iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+\frac{3i\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-2ibc+2iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}-16(dx+c)^{\frac{3}{2}}+\frac{6i\sqrt{dx+c}de^{\left(\frac{2i(dx+c)b-2ibc+2iad}{d}\right)}}{b}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/48*(-3*I*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) +
 3*I*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 16*
(d*x + c)^(3/2) + 6*I*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a
*d)/d)/b - 6*I*sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)
/b)/d
```

3.51 $\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=130

$$\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}$$

```
[Out] Sqrt[c + d*x]/d + (Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c
 + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi]*FresnelS[(2*S
qrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2*Sqrt[b]*
Sqrt[d])
```

Rubi [A] time = 0.243225, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3312, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/Sqrt[c + d*x],x]
```

```
[Out] Sqrt[c + d*x]/d + (Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c
 + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi]*FresnelS[(2*S
qrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2*Sqrt[b]*
Sqrt[d])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx &= \int \left(\frac{1}{2\sqrt{c+dx}}+\frac{\cos(2a+2bx)}{2\sqrt{c+dx}}\right)dx\\
&= \frac{\sqrt{c+dx}}{d}+\frac{1}{2}\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}\,dx\\
&= \frac{\sqrt{c+dx}}{d}+\frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx-\frac{1}{2}\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx\\
&= \frac{\sqrt{c+dx}}{d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Subst}\left(\int \cos\left(\frac{2bx^2}{d}\right)dx,x,\sqrt{c+dx}\right)}{d}-\frac{\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Subst}\left(\int \sin\left(\frac{2bx^2}{d}\right.\right.}{d}\\
&= \frac{\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\pi}S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{2\sqrt{b}\sqrt{d}}
\end{aligned}$$

Mathematica [C] time = 0.246514, size = 145, normalized size = 1.12

$$\frac{-\frac{i\sqrt{2}e^{2i\left(a-\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\operatorname{Gamma}\left(\frac{1}{2},-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{i\sqrt{2}e^{-2i\left(a-\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\operatorname{Gamma}\left(\frac{1}{2},\frac{2ib(c+dx)}{d}\right)}{b}+8\left(\frac{c}{d}+x\right)}{8\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/Sqrt[c + d*x],x]
```

```
[Out] (8*(c/d + x) - (I*Sqrt[2]*E^((2*I)*(a - (b*c)/d))*Sqrt[((-I)*b*(c + d*x))/d
]*Gamma[1/2, ((-2*I)*b*(c + d*x))/d])/b + (I*Sqrt[2]*Sqrt[(I*b*(c + d*x))/d
]*Gamma[1/2, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))))/(8*Sqrt[c
 + d*x])
```

Maple [A] time = 0.039, size = 108, normalized size = 0.8

$$2\,\frac{1}{d}\left(1/2\,\sqrt{dx+c}+1/4\,\sqrt{\pi}\left(\cos\left(2\,\frac{da-cb}{d}\right)\operatorname{FresnelC}\left(2\,\frac{b\sqrt{dx+c}}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right)-\sin\left(2\,\frac{da-cb}{d}\right)\operatorname{FresnelS}\left(2\,\frac{b\sqrt{dx+c}}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/(d*x+c)^(1/2),x)
```

```
[Out] 2/d*(1/2*(d*x+c)^(1/2)+1/4*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*Fresnel
C(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/P
```

```
i^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 2.13707, size = 747, normalized size = 5.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*(((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/s
qrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/s
qrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))))*cos(-2*(b*c - a*d)/d) - (I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)
*sqrt(2*I*b/d)) + ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
 d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
 d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))))*cos(-2*(b*c - a*d)/d) - (-I*sqrt(pi)*cos(1/4*pi + 1/2*arct
an2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x
 + c)*sqrt(-2*I*b/d)) + 8*sqrt(2)*sqrt(d*x + c)*sqrt(abs(b)/abs(d)))/(d*sqr
t(abs(b)/abs(d)))
```

Fricas [A] time = 1.70371, size = 279, normalized size = 2.15

$$\frac{\pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{2\,(bc-ad)}{d}\right) \mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - \pi d \sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{2\,(bc-ad)}{d}\right) + 2\,\sqrt{dx+c}\,b}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(pi*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*
sqrt(b/(pi*d))) - pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(p
i*d)))*sin(-2*(b*c - a*d)/d) + 2*sqrt(d*x + c)*b)/(b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/(d*x+c)**(1/2),x)
```

```
[Out] Integral(cos(a + b*x)**2/sqrt(c + d*x), x)
```

Giac [C] time = 1.18268, size = 220, normalized size = 1.69

$$-\frac{\frac{\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{2ibc-2iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}+\frac{\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-2ibc+2iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}-4\sqrt{dx+c}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(pi)*
d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c +
 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 4*sqrt(d*x + c))/d
```

3.52 $\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}} \, dx$

Optimal. Leaf size=135

$$-\frac{2\sqrt{\pi}\sqrt{b}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sqrt{\pi}\sqrt{b}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{d^{3/2}}-\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}$$

```
[Out] (-2*Cos[a + b*x]^2)/(d*Sqrt[c + d*x]) - (2*Sqrt[b]*Sqrt[Pi]*Cos[2*a - (2*b*
c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/d^(3/2) - (2*
Sqrt[b]*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin
[2*a - (2*b*c)/d])/d^(3/2)
```

Rubi [A] time = 0.257704, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3313, 12, 3306, 3305, 3351, 3304, 3352}

$$-\frac{2\sqrt{\pi}\sqrt{b}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sqrt{\pi}\sqrt{b}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{d^{3/2}}-\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x]^2)/(d*Sqrt[c + d*x]) - (2*Sqrt[b]*Sqrt[Pi]*Cos[2*a - (2*b*
c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/d^(3/2) - (2*
Sqrt[b]*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin
[2*a - (2*b*c)/d])/d^(3/2)
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}\,dx &= -\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}} + \frac{(4b)\int -\frac{\sin(2a+2bx)}{2\sqrt{c+dx}}\,dx}{d} \\
&= -\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}} - \frac{(2b)\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}\,dx}{d} \\
&= -\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}} - \frac{\left(2b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{d} - \frac{\left(2b\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{d} \\
&= -\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}} - \frac{\left(4b\cos\left(2a-\frac{2bc}{d}\right)\right)\text{Subst}\left(\int \sin\left(\frac{2bx^2}{d}\right)dx, x, \sqrt{c+dx}\right)}{d^2} - \frac{\left(4b\sin\left(2a-\frac{2bc}{d}\right)\right)\text{Su}}{} \\
&= -\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}} - \frac{2\sqrt{b}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{d^{3/2}} - \frac{2\sqrt{b}\sqrt{\pi}C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.633191, size = 133, normalized size = 0.99

$$\frac{2\left(-\sqrt{\pi}\sqrt{\frac{b}{d}}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-\sqrt{\pi}\sqrt{\frac{b}{d}}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-\frac{\cos^2(a+bx)}{\sqrt{c+dx}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/(c + d*x)^(3/2),x]
```

```
[Out] (2*(-(Cos[a + b*x]^2/Sqrt[c + d*x]) - Sqrt[b/d]*Sqrt[Pi]*Cos[2*a - (2*b*c)/
d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - Sqrt[b/d]*Sqrt[Pi]*Fres
nelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d]))/d
```

Maple [A] time = 0.037, size = 146, normalized size = 1.1

$$2\frac{1}{d}\left(-1/2\frac{1}{\sqrt{dx+c}}-1/2\frac{1}{\sqrt{dx+c}}\cos\left(2\frac{(dx+c)b}{d}+2\frac{da-cb}{d}\right)-\frac{b\sqrt{\pi}}{d}\left(\cos\left(2\frac{da-cb}{d}\right)\text{FresnelS}\left(2\frac{b\sqrt{dx+c}}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right)+\text{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/(d*x+c)^(3/2),x)
```

```
[Out] 2/d*(-1/2/(d*x+c)^(1/2)-1/2/(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-
b/d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2
)*(d*x+c)^(1/2)*b/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*
x+c)^(1/2)*b/d)))
```

Maxima [C] time = 1.47252, size = 640, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(2)*(((gamma(-1/2, 2*I*(d*x + c)*b/d) + gamma(-1/2, -2*I*(d*x + c
)*b/d))*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + (ga
mma(-1/2, 2*I*(d*x + c)*b/d) + gamma(-1/2, -2*I*(d*x + c)*b/d))*cos(-1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + (I*gamma(-1/2, 2*I*(d
*x + c)*b/d) - I*gamma(-1/2, -2*I*(d*x + c)*b/d))*sin(1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + (-I*gamma(-1/2, 2*I*(d*x + c)*b/d) +
 I*gamma(-1/2, -2*I*(d*x + c)*b/d))*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))))*cos(-2*(b*c - a*d)/d) + ((-I*gamma(-1/2, 2*I*(d*x
+ c)*b/d) + I*gamma(-1/2, -2*I*(d*x + c)*b/d))*cos(1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + (-I*gamma(-1/2, 2*I*(d*x + c)*b/d) + I*
gamma(-1/2, -2*I*(d*x + c)*b/d))*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) + (gamma(-1/2, 2*I*(d*x + c)*b/d) + gamma(-1/2, -2*I*(
d*x + c)*b/d))*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) - (gamma(-1/2, 2*I*(d*x + c)*b/d) + gamma(-1/2, -2*I*(d*x + c)*b/d))*sin(
-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*sin(-2*(b*c - a
*d)/d))*sqrt((d*x + c)*abs(b)/abs(d)) + 8)/(sqrt(d*x + c)*d)
```

Fricas [A] time = 1.85053, size = 333, normalized size = 2.47

$$-\frac{2\left((\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)S\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}C\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)+\sqrt{dx+}\right.}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*((pi*d*x + pi*c)*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt
(d*x + c)*sqrt(b/(pi*d))) + (pi*d*x + pi*c)*sqrt(b/(pi*d))*fresnel_cos(2*sq
rt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + sqrt(d*x + c)*cos(b*x +
 a)^2)/(d^2*x + c*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos^2(a+bx)}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/(d*x+c)**(3/2),x)
```

```
[Out] Integral(cos(a + b*x)**2/(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^2}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2/(d*x + c)^(3/2), x)
```

3.53 $\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=170

$$-\frac{8\sqrt{\pi}b^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{3d^{5/2}}+\frac{8\sqrt{\pi}b^{3/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{3d^{5/2}}+\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-$$

```
[Out] (-2*Cos[a + b*x]^2)/(3*d*(c + d*x)^(3/2)) - (8*b^(3/2)*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(3*d^(5/2
)) + (8*b^(3/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[P
i])]*Sin[2*a - (2*b*c)/d])/(3*d^(5/2)) + (8*b*Cos[a + b*x]*Sin[a + b*x])/(3
*d^2*Sqrt[c + d*x])
```

Rubi [A] time = 0.312177, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3314, 32, 3312, 3306, 3305, 3351, 3304, 3352}

$$-\frac{8\sqrt{\pi}b^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{3d^{5/2}}+\frac{8\sqrt{\pi}b^{3/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{3d^{5/2}}+\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/(c + d*x)^(5/2),x]
```


```
[Out] (-2*Cos[a + b*x]^2)/(3*d*(c + d*x)^(3/2)) - (8*b^(3/2)*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(3*d^(5/2
)) + (8*b^(3/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[P
i])]*Sin[2*a - (2*b*c)/d])/(3*d^(5/2)) + (8*b*Cos[a + b*x]*Sin[a + b*x])/(3
*d^2*Sqrt[c + d*x])
```


Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
```

```
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}\,dx &= -\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} + \frac{\left(8b^2\right)\int \frac{1}{\sqrt{c+dx}}\,dx}{3d^2} - \frac{\left(16b^2\right)\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx}{3d^2}\\
&= \frac{16b^2\sqrt{c+dx}}{3d^3} - \frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{\left(16b^2\right)\int \left(\frac{1}{2\sqrt{c+dx}} + \frac{\cos(2a+2bx)}{2\sqrt{c+dx}}\right)dx}{3d^2}\\
&= -\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{\left(8b^2\right)\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}\,dx}{3d^2}\\
&= -\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{\left(8b^2\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{3d^2} + \frac{\left(8b^2\sin\left(2\right.\right.}{}\\
&= -\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{\left(16b^2\cos\left(2a-\frac{2bc}{d}\right)\right)\text{Subst}\left(\int\cos\left(\frac{2bx^2}{d}\right)dx, x, \sqrt{\right.}{3d^3}\\
&= -\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{8b^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{3d^{5/2}} + \frac{8b^{3/2}\sqrt{\pi}S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{3d^{5/2}} + \frac{8}{}
\end{aligned}$$

Mathematica [C] time = 1.41751, size = 181, normalized size = 1.06

$$\frac{e^{-\frac{2i(ad+bc)}{d}}\left(-2\sqrt{2}e^{4ia}d\left(-\frac{ib(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{2ib(c+dx)}{d}\right) - 2\sqrt{2}de^{\frac{4ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},\frac{2ib(c+dx)}{d}\right) + 2e^{\frac{2i(ad+bc)}{d}}\left(2b(\right.\right.}{3d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/(c + d*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[2]*d*E^((4*I)*a)*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-2*I)*b
*(c + d*x))/d] - 2*Sqrt[2]*d*E^(((4*I)*b*c)/d)*((I*b*(c + d*x))/d)^(3/2)*Ga
mma[1/2, ((2*I)*b*(c + d*x))/d] + 2*E^(((2*I)*(b*c + a*d))/d)*(-(d*Cos[a +
b*x]^2) + 2*b*(c + d*x)*Sin[2*(a + b*x)]))/(3*d^2*E^(((2*I)*(b*c + a*d))/d)
*(c + d*x)^(3/2))
```

Maple [A] time = 0.036, size = 189, normalized size = 1.1

$$2\frac{1}{d}\left(-1/6\,(dx+c)^{-3/2}-1/6\,\frac{1}{(dx+c)^{3/2}}\cos\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{da-cb}{d}\right)-2/3\,\frac{b}{d}\left(-\frac{1}{\sqrt{dx+c}}\sin\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{da-cb}{d}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/(d*x+c)^(5/2),x)
```

```
[Out] 2/d*(-1/6/(d*x+c)^(3/2)-1/6/(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-
2/3*b/d*(-1/(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+2*b/d*Pi^(1/2)/(
b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d
))))
```

Maxima [C] time = 1.55071, size = 644, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(sqrt(2)*((3*(gamma(-3/2, 2*I*(d*x + c)*b/d) + gamma(-3/2, -2*I*(d*x
+ c)*b/d))*cos(3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) +
3*(gamma(-3/2, 2*I*(d*x + c)*b/d) + gamma(-3/2, -2*I*(d*x + c)*b/d))*cos(-3
/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) + (3*I*gamma(-3/2,
 2*I*(d*x + c)*b/d) - 3*I*gamma(-3/2, -2*I*(d*x + c)*b/d))*sin(3/4*pi + 3/2
*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) + (-3*I*gamma(-3/2, 2*I*(d*x
+ c)*b/d) + 3*I*gamma(-3/2, -2*I*(d*x + c)*b/d))*sin(-3/4*pi + 3/2*arctan2(
0, b) + 3/2*arctan2(0, d/sqrt(d^2))))*cos(-2*(b*c - a*d)/d) + ((-3*I*gamma(
-3/2, 2*I*(d*x + c)*b/d) + 3*I*gamma(-3/2, -2*I*(d*x + c)*b/d))*cos(3/4*pi
+ 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) + (-3*I*gamma(-3/2, 2*I*
(d*x + c)*b/d) + 3*I*gamma(-3/2, -2*I*(d*x + c)*b/d))*cos(-3/4*pi + 3/2*arc
tan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) + 3*(gamma(-3/2, 2*I*(d*x + c)*b/
d) + gamma(-3/2, -2*I*(d*x + c)*b/d))*sin(3/4*pi + 3/2*arctan2(0, b) + 3/2*
arctan2(0, d/sqrt(d^2))) - 3*(gamma(-3/2, 2*I*(d*x + c)*b/d) + gamma(-3/2,
-2*I*(d*x + c)*b/d))*sin(-3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqr
t(d^2))))*sin(-2*(b*c - a*d)/d))*((d*x + c)*abs(b)/abs(d))^(3/2) + 4)/((d*x
 + c)^(3/2)*d)
```

Fricas [A] time = 1.97897, size = 498, normalized size = 2.93

$$-\frac{2\left(4\left(\pi bd^2x^2+2\,\pi bcdx+\pi bc^2\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-4\left(\pi bd^2x^2+2\,\pi bcdx+\pi bc^2\right)\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{d}\right.\right.}{3\left(d^4x^2+2\,cd^3x+c^2d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(4*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos(-2*(b*c
 - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - 4*(pi*b*d^2*x^2 +
2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/
(pi*d)))*sin(-2*(b*c - a*d)/d) + (d*cos(b*x + a)^2 - 4*(b*d*x + b*c)*cos(b*
x + a)*sin(b*x + a))*sqrt(d*x + c))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/(d*x+c)**(5/2),x)
```

```
[Out] Integral(cos(a + b*x)**2/(c + d*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^2}{(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2/(d*x + c)^(5/2), x)
```

3.54 $\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal. Leaf size=216

$$\frac{32\sqrt{\pi}b^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{15d^{7/2}}+\frac{32\sqrt{\pi}b^{5/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{15d^{7/2}}+\frac{32b^2\cos^2(a+bx)}{15d^3\sqrt{c+dx}}+\frac{8b\sin(}{1}$$

```
[Out] (-16*b^2)/(15*d^3*Sqrt[c + d*x]) - (2*Cos[a + b*x]^2)/(5*d*(c + d*x)^(5/2))
 + (32*b^2*Cos[a + b*x]^2)/(15*d^3*Sqrt[c + d*x]) + (32*b^(5/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/
(15*d^(7/2)) + (32*b^(5/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(15*d^(7/2)) + (8*b*Cos[a + b*x]*Sin[
a + b*x])/(15*d^2*(c + d*x)^(3/2))
```

Rubi [A] time = 0.324499, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3314, 32, 3313, 12, 3306, 3305, 3351, 3304, 3352}

$$\frac{32\sqrt{\pi}b^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{15d^{7/2}}+\frac{32\sqrt{\pi}b^{5/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{15d^{7/2}}+\frac{32b^2\cos^2(a+bx)}{15d^3\sqrt{c+dx}}+\frac{8b\sin(}{1}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/(c + d*x)^(7/2),x]
```

```
[Out] (-16*b^2)/(15*d^3*Sqrt[c + d*x]) - (2*Cos[a + b*x]^2)/(5*d*(c + d*x)^(5/2))
 + (32*b^2*Cos[a + b*x]^2)/(15*d^3*Sqrt[c + d*x]) + (32*b^(5/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/
(15*d^(7/2)) + (32*b^(5/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(15*d^(7/2)) + (8*b*Cos[a + b*x]*Sin[
a + b*x])/(15*d^2*(c + d*x)^(3/2))
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{\left(8b^2\right)\int \frac{1}{(c+dx)^{3/2}}\,dx}{15d^2} - \frac{\left(16b^2\right)\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}\,dx}{15d^2}$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\cos^2(a+bx)}{15d^3\sqrt{c+dx}} + \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{\left(64b^3\right)\int -\frac{\sin\cdots}{2\cdots}}{15d^3\cdots}$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\cos^2(a+bx)}{15d^3\sqrt{c+dx}} + \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{\left(32b^3\right)\int \frac{\sin(2\cdots}{\sqrt{c\cdots}}}{15d^3\cdots}$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\cos^2(a+bx)}{15d^3\sqrt{c+dx}} + \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{\left(32b^3\cos\left(2a\cdots\right.\right.}{\cdots}$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\cos^2(a+bx)}{15d^3\sqrt{c+dx}} + \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{\left(64b^3\cos\left(2a\cdots\right.\right.}{\cdots}$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\cos^2(a+bx)}{15d^3\sqrt{c+dx}} + \frac{32b^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{15d^{7/2}} + \frac{32\cdots}{\cdots}$$

Mathematica [A] time = 1.28978, size = 244, normalized size = 1.13

$$16b^2c^2\cos(2(a+bx))+32b^2cdx\cos(2(a+bx))+16b^2d^2x^2\cos(2(a+bx))+32\sqrt{\pi}bd\left(\frac{b}{d}\right)^{3/2}(c+dx)^{5/2}\sin\left(2a-\frac{2bc}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/(c + d*x)^(7/2),x]
```

```
[Out] (-3*d^2 + 16*b^2*c^2*Cos[2*(a + b*x)] - 3*d^2*Cos[2*(a + b*x)] + 32*b^2*c*d
*x*Cos[2*(a + b*x)] + 16*b^2*d^2*x^2*Cos[2*(a + b*x)] + 32*b*(b/d)^(3/2)*d*
Sqrt[Pi]*(c + d*x)^(5/2)*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c
+ d*x])/Sqrt[Pi]] + 32*b*(b/d)^(3/2)*d*Sqrt[Pi]*(c + d*x)^(5/2)*FresnelC[(2
*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] + 4*b*c*d*Sin[2*(a
 + b*x)] + 4*b*d^2*x*Sin[2*(a + b*x)])/(15*d^3*(c + d*x)^(5/2))
```

Maple [A] time = 0.037, size = 230, normalized size = 1.1

$$2\frac{1}{d}\left(-1/10\,(dx+c)^{-5/2}-1/10\,\frac{1}{(dx+c)^{5/2}}\cos\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{da-cb}{d}\right)-2/5\,\frac{b}{d}\left(-1/3\,\frac{1}{(dx+c)^{3/2}}\sin\left(2\,\frac{(dx+c)\,b}{d}+2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/(d*x+c)^(7/2),x)
```

```
[Out] 2/d*(-1/10/(d*x+c)^(5/2)-1/10/(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d
)-2/5*b/d*(-1/3/(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+4/3*b/d*(-1/
(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-2*b/d*Pi^(1/2)/(b/d)^(1/2)*(
cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(2
*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

Maxima [C] time = 1.65191, size = 644, normalized size = 2.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/10*(sqrt(2)*((5*(gamma(-5/2, 2*I*(d*x + c)*b/d) + gamma(-5/2, -2*I*(d*x
+ c)*b/d))*cos(5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) +
5*(gamma(-5/2, 2*I*(d*x + c)*b/d) + gamma(-5/2, -2*I*(d*x + c)*b/d))*cos(-5
/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) + (5*I*gamma(-5/2,
 2*I*(d*x + c)*b/d) - 5*I*gamma(-5/2, -2*I*(d*x + c)*b/d))*sin(5/4*pi + 5/2
*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) + (-5*I*gamma(-5/2, 2*I*(d*x
+ c)*b/d) + 5*I*gamma(-5/2, -2*I*(d*x + c)*b/d))*sin(-5/4*pi + 5/2*arctan2(
0, b) + 5/2*arctan2(0, d/sqrt(d^2))))*cos(-2*(b*c - a*d)/d) + ((-5*I*gamma(
-5/2, 2*I*(d*x + c)*b/d) + 5*I*gamma(-5/2, -2*I*(d*x + c)*b/d))*cos(5/4*pi
+ 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) + (-5*I*gamma(-5/2, 2*I*
(d*x + c)*b/d) + 5*I*gamma(-5/2, -2*I*(d*x + c)*b/d))*cos(-5/4*pi + 5/2*arc
tan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) + 5*(gamma(-5/2, 2*I*(d*x + c)*b/
```

```
d) + gamma(-5/2, -2*I*(d*x + c)*b/d))*sin(5/4*pi + 5/2*arctan2(0, b) + 5/2*
arctan2(0, d/sqrt(d^2))) - 5*(gamma(-5/2, 2*I*(d*x + c)*b/d) + gamma(-5/2,
-2*I*(d*x + c)*b/d))*sin(-5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqr
t(d^2))))*sin(-2*(b*c - a*d)/d))*((d*x + c)*abs(b)/abs(d))^(5/2) + 2)/((d*x
 + c)^(5/2)*d)
```

Fricas [A] time = 2.07002, size = 733, normalized size = 3.39

$$\frac{2\left(16\left(\pi b^2 d^3 x^3+3\,\pi b^2 c d^2 x^2+3\,\pi b^2 c^2 d x+\pi b^2 c^3\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+16\left(\pi b^2 d^3 x^3+3\,\pi b^2 c d^2 x^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(16*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi*b^2*c
^3)*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*sqrt(b
/(pi*d))) + 16*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi
*b^2*c^3)*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2
*(b*c - a*d)/d) - (8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - (16*b^2*d^2*x
^2 + 32*b^2*c*d*x + 16*b^2*c^2 - 3*d^2)*cos(b*x + a)^2 - 4*(b*d^2*x + b*c*d
)*cos(b*x + a)*sin(b*x + a))*sqrt(d*x + c))/(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*
d^4*x + c^3*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^2}{(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2/(d*x + c)^(7/2), x)
```

3.55 $\int \frac{\cos^2(a+bx)}{(c+dx)^{9/2}} dx$

Optimal. Leaf size=247

$$\frac{128\sqrt{\pi}b^{7/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{105d^{9/2}}-\frac{128\sqrt{\pi}b^{7/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{105d^{9/2}}+\frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}}-\frac{128b}{}$$

```
[Out] (-16*b^2)/(105*d^3*(c + d*x)^(3/2)) - (2*Cos[a + b*x]^2)/(7*d*(c + d*x)^(7/
2)) + (32*b^2*Cos[a + b*x]^2)/(105*d^3*(c + d*x)^(3/2)) + (128*b^(7/2)*Sqrt
[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
Pi])])/(105*d^(9/2)) - (128*b^(7/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d
*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(105*d^(9/2)) + (8*b*Cos[a +
 b*x]*Sin[a + b*x])/(35*d^2*(c + d*x)^(5/2)) - (128*b^3*Cos[a + b*x]*Sin[a
+ b*x])/(105*d^4*Sqrt[c + d*x])
```

Rubi [A] time = 0.407135, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3314, 32, 3312, 3306, 3305, 3351, 3304, 3352}

$$\frac{128\sqrt{\pi}b^{7/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{105d^{9/2}}-\frac{128\sqrt{\pi}b^{7/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{105d^{9/2}}+\frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}}-\frac{128b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/(c + d*x)^(9/2),x]
```


```
[Out] (-16*b^2)/(105*d^3*(c + d*x)^(3/2)) - (2*Cos[a + b*x]^2)/(7*d*(c + d*x)^(7/
2)) + (32*b^2*Cos[a + b*x]^2)/(105*d^3*(c + d*x)^(3/2)) + (128*b^(7/2)*Sqrt
[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
Pi])])/(105*d^(9/2)) - (128*b^(7/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d
*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(105*d^(9/2)) + (8*b*Cos[a +
 b*x]*Sin[a + b*x])/(35*d^2*(c + d*x)^(5/2)) - (128*b^3*Cos[a + b*x]*Sin[a
+ b*x])/(105*d^4*Sqrt[c + d*x])
```


Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{9/2}}\,dx = -\frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} + \frac{\left(8b^2\right)\int \frac{1}{(c+dx)^{5/2}}\,dx}{35d^2} - \frac{\left(16b^2\right)\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}\,dx}{35d^2}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3\cos(}{10}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{256b^4\sqrt{c+dx}}{105d^5} - \frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+}{35d^2(c+dx)^{5/2}}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3\cos(}{10}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3\cos(}{10}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3\cos(}{10}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{128b^{7/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{105d^{9/2}}$$

Mathematica [C] time = 0.857093, size = 237, normalized size = 0.96

$$2\left(16\sqrt{2}b^2d(c+dx)^2e^{2i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{2ib(c+dx)}{d}\right)+16\sqrt{2}b^2d(c+dx)^2e^{-2i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/(c + d*x)^(9/2),x]
```

```
[Out] (2*(-8*b^2*d*(c + d*x)^2 - 15*d^3*Cos[a + b*x]^2 + 16*b^2*d*(c + d*x)^2*Cos
[a + b*x]^2 + 16*Sqrt[2]*b^2*d*E^((2*I)*(a - (b*c)/d))*(c + d*x)^2*(((-I)*b
*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-2*I)*b*(c + d*x))/d] + (16*Sqrt[2]*b^2*d
*(c + d*x)^2*((I*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((2*I)*b*(c + d*x))/d])/E
^((2*I)*(a - (b*c)/d)) + 6*b*d^2*(c + d*x)*Sin[2*(a + b*x)] - 32*b^3*(c + d
*x)^3*Sin[2*(a + b*x)]))/(105*d^4*(c + d*x)^(7/2))
```

Maple [A] time = 0.036, size = 273, normalized size = 1.1

$$2\frac{1}{d}\left(-1/14\,(dx+c)^{-7/2}-1/14\,\frac{1}{(dx+c)^{7/2}}\cos\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{da-cb}{d}\right)-2/7\,\frac{b}{d}\left(-1/5\,\frac{1}{(dx+c)^{5/2}}\sin\left(2\,\frac{(dx+c)\,b}{d}+2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/(d*x+c)^(9/2),x)
```

```
[Out] 2/d*(-1/14/(d*x+c)^(7/2)-1/14/(d*x+c)^(7/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d
)-2/7*b/d*(-1/5/(d*x+c)^(5/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+4/5*b/d*(-1/
3/(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-4/3*b/d*(-1/(d*x+c)^(1/2)*
sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+2*b/d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c
)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*
FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 1.71768, size = 644, normalized size = 2.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/7*(sqrt(2)*((7*(gamma(-7/2, 2*I*(d*x + c)*b/d) + gamma(-7/2, -2*I*(d*x +
 c)*b/d))*cos(7/4*pi + 7/2*arctan2(0, b) + 7/2*arctan2(0, d/sqrt(d^2))) + 7
*(gamma(-7/2, 2*I*(d*x + c)*b/d) + gamma(-7/2, -2*I*(d*x + c)*b/d))*cos(-7/
4*pi + 7/2*arctan2(0, b) + 7/2*arctan2(0, d/sqrt(d^2))) + (7*I*gamma(-7/2,
2*I*(d*x + c)*b/d) - 7*I*gamma(-7/2, -2*I*(d*x + c)*b/d))*sin(7/4*pi + 7/2*
arctan2(0, b) + 7/2*arctan2(0, d/sqrt(d^2))) + (-7*I*gamma(-7/2, 2*I*(d*x +
 c)*b/d) + 7*I*gamma(-7/2, -2*I*(d*x + c)*b/d))*sin(-7/4*pi + 7/2*arctan2(0
, b) + 7/2*arctan2(0, d/sqrt(d^2))))*cos(-2*(b*c - a*d)/d) + ((-7*I*gamma(-
7/2, 2*I*(d*x + c)*b/d) + 7*I*gamma(-7/2, -2*I*(d*x + c)*b/d))*cos(7/4*pi +
 7/2*arctan2(0, b) + 7/2*arctan2(0, d/sqrt(d^2))) + (-7*I*gamma(-7/2, 2*I*(
d*x + c)*b/d) + 7*I*gamma(-7/2, -2*I*(d*x + c)*b/d))*cos(-7/4*pi + 7/2*arct
an2(0, b) + 7/2*arctan2(0, d/sqrt(d^2))) + 7*(gamma(-7/2, 2*I*(d*x + c)*b/d
) + gamma(-7/2, -2*I*(d*x + c)*b/d))*sin(7/4*pi + 7/2*arctan2(0, b) + 7/2*a
rctan2(0, d/sqrt(d^2))) - 7*(gamma(-7/2, 2*I*(d*x + c)*b/d) + gamma(-7/2, -
2*I*(d*x + c)*b/d))*sin(-7/4*pi + 7/2*arctan2(0, b) + 7/2*arctan2(0, d/sqrt
(d^2))))*sin(-2*(b*c - a*d)/d))*((d*x + c)*abs(b)/abs(d))^(7/2) + 1)/((d*x
+ c)^(7/2)*d)
```

Fricas [B] time = 2.23948, size = 927, normalized size = 3.75

$$\frac{2\left(64\left(\pi b^3d^4x^4+4\,\pi b^3cd^3x^3+6\,\pi b^3c^2d^2x^2+4\,\pi b^3c^3dx+\pi b^3c^4\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-64\left(\pi b^3d^4x^4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/105*(64*(pi*b^3*d^4*x^4 + 4*pi*b^3*c*d^3*x^3 + 6*pi*b^3*c^2*d^2*x^2 + 4*p
i*b^3*c^3*d*x + pi*b^3*c^4)*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_co
s(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - 64*(pi*b^3*d^4*x^4 + 4*pi*b^3*c*d^3*x^3
 + 6*pi*b^3*c^2*d^2*x^2 + 4*pi*b^3*c^3*d*x + pi*b^3*c^4)*sqrt(b/(pi*d))*fre
snel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - (8*b^2*d^3
*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - (16*b^2*d^3*x^2 + 32*b^2*c*d^2*x + 16
*b^2*c^2*d - 15*d^3)*cos(b*x + a)^2 + 4*(16*b^3*d^3*x^3 + 48*b^3*c*d^2*x^2
+ 16*b^3*c^3 - 3*b*c*d^2 + 3*(16*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)*sin(b*x
 + a))*sqrt(d*x + c))/(d^8*x^4 + 4*c*d^7*x^3 + 6*c^2*d^6*x^2 + 4*c^3*d^5*x
+ c^4*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^2}{(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2/(d*x + c)^(9/2), x)
```

3.56 $\int (c+dx)^{5/2} \cos^3(a+bx)\,dx$

Optimal. Leaf size=410

$$\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{45\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{45\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a\ldots\right.}{16\ldots}$$

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(3*b^2) + (5*d*(c + d*x)^(3/2)*Cos[a + b
*x]^3)/(18*b^2) + (45*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*C
os[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(
144*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c +
d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi/2]
*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16
*b^(7/2)) - (45*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(16*b^3) + (2*(c + d*x)^(5/
2)*Sin[a + b*x])/(3*b) + ((c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x])/(3*b
) - (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(144*b^3)
```

Rubi [A] time = 1.13902, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3311, 3296, 3306, 3305, 3351, 3304, 3352, 3312}

$$\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{45\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{45\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a\ldots\right.}{16\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3,x]
```


```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(3*b^2) + (5*d*(c + d*x)^(3/2)*Cos[a + b
*x]^3)/(18*b^2) + (45*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*C
os[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(
144*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c +
d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi/2]
*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16
*b^(7/2)) - (45*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(16*b^3) + (2*(c + d*x)^(5/
2)*Sin[a + b*x])/(3*b) + ((c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x])/(3*b
) - (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(144*b^3)
```


Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\,dx = \frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2} + \frac{(c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)}{3b} + \frac{2}{3}\int (c+dx)^{5/2}\cos(a$$

$$= \frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2} + \frac{2(c+dx)^{5/2}\sin(a+bx)}{3b} + \frac{(c+dx)^{5/2}\cos^2(a+bx)\sin(a+b}{3b}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{3b^2} + \frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2} + \frac{2(c+dx)^{5/2}\sin(a+bx)}{3b} + \frac{(c+}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{3b^2} + \frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2} - \frac{45d^2\sqrt{c+dx}\sin(a+bx)}{16b^3} + \frac{2(c}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{3b^2} + \frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2} - \frac{45d^2\sqrt{c+dx}\sin(a+bx)}{16b^3} + \frac{2(c}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{3b^2} + \frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2} - \frac{45d^2\sqrt{c+dx}\sin(a+bx)}{16b^3} + \frac{2(c}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{3b^2} + \frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2} + \frac{45d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt{}}\right.}{16b^{7/2}}$$

Mathematica [A] time = 3.23229, size = 542, normalized size = 1.32

$$648b^3c^2\sqrt{c+dx}\sin(a+bx)+72b^3c^2\sqrt{c+dx}\sin(3(a+bx))+648b^3d^2x^2\sqrt{c+dx}\sin(a+bx)+72b^3d^2x^2\sqrt{c+dx}\sin($$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3,x]
```

```
[Out] (1620*b^2*c*d*Sqrt[c + d*x]*Cos[a + b*x] + 1620*b^2*d^2*x*Sqrt[c + d*x]*Cos
[a + b*x] + 60*b^2*c*d*Sqrt[c + d*x]*Cos[3*(a + b*x)] + 60*b^2*d^2*x*Sqrt[c
 + d*x]*Cos[3*(a + b*x)] + 1215*Sqrt[b/d]*d^3*Sqrt[2*Pi]*Cos[a - (b*c)/d]*F
resnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] + 5*Sqrt[b/d]*d^3*Sqrt[6*Pi]*Co
s[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + 5*Sqrt[b/
d]*d^3*Sqrt[6*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3
*b*c)/d] + 1215*Sqrt[b/d]*d^3*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt
[c + d*x]]*Sin[a - (b*c)/d] + 648*b^3*c^2*Sqrt[c + d*x]*Sin[a + b*x] - 2430
*b*d^2*Sqrt[c + d*x]*Sin[a + b*x] + 1296*b^3*c*d*x*Sqrt[c + d*x]*Sin[a + b*
x] + 648*b^3*d^2*x^2*Sqrt[c + d*x]*Sin[a + b*x] + 72*b^3*c^2*Sqrt[c + d*x]*
Sin[3*(a + b*x)] - 30*b*d^2*Sqrt[c + d*x]*Sin[3*(a + b*x)] + 144*b^3*c*d*x*
Sqrt[c + d*x]*Sin[3*(a + b*x)] + 72*b^3*d^2*x^2*Sqrt[c + d*x]*Sin[3*(a + b*
x)])/(864*b^4)
```

Maple [A] time = 0.043, size = 474, normalized size = 1.2

$$2\frac{1}{d}\left(3/8\,\frac{d\,(dx+c)^{5/2}}{b}\sin\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)-\frac{15\,d}{8\,b}\left(-1/2\,\frac{d\,(dx+c)^{3/2}}{b}\cos\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)+3/2\,\frac{d}{b}\left(1/2\,\frac{d\sqrt{}}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3,x)
```

```
[Out] 2/d*(3/8/b*d*(d*x+c)^(5/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-15/8/b*d*(-1/2/b*
d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/
2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos
((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(
(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))+1/
24/b*d*(d*x+c)^(5/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-5/24/b*d*(-1/6/b*d*(d
*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2)
*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 2.62728, size = 1863, normalized size = 4.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3456*sqrt(3)*(80*sqrt(3)*(d*x + c)^(3/2)*b*d^2*abs(b)*cos(3*((d*x + c)*b
- b*c + a*d)/d)/abs(d) + 2160*sqrt(3)*(d*x + c)^(3/2)*b*d^2*abs(b)*cos(((d*
x + c)*b - b*c + a*d)/d)/abs(d) + ((5*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*sin(1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)*sin(-1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-3*(b*
c - a*d)/d) + (5*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d
/sqrt(d^2))) + 5*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) - 5*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))) + 5*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(3*I*b/d)) + (sqrt(3)*(405*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 405*I*sqrt(pi)*cos(-1/4*pi + 1/2*a
rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 405*sqrt(pi)*sin(1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 405*sqrt(pi)*sin(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*c
os(-(b*c - a*d)/d) + sqrt(3)*(405*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) + 405*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) - 405*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 405*I*sqrt(pi)*sin(-1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-(b*
c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + (sqrt(3)*(-405*I*sqrt(pi)*cos
(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 405*I*sqrt(pi)
*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 405*sqrt(
pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 405*sqr
t(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*s
qrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) + sqrt(3)*(405*sqrt(pi)*cos(1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 405*sqrt(pi)*cos(-1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 405*I*sqrt(pi)*sin(1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 405*I*sqrt(pi)*s
in(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs
(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((-5*I*s
qrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I
*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
5*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) -
5*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*
d^3*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*d)/d) + (5*sqrt(pi)*cos(1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*cos(-1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*sin(1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*sin(-1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d
))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + 8*(12*sqrt(3)
*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 5*sqrt(3)*sqrt(d*x + c)*d^3*abs(b)/a
bs(d))*sin(3*((d*x + c)*b - b*c + a*d)/d) + 216*(4*sqrt(3)*(d*x + c)^(5/2)*
b^2*d*abs(b)/abs(d) - 15*sqrt(3)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*sin(((d*x
 + c)*b - b*c + a*d)/d))*abs(d)/(b^3*d*abs(b))
```

Fricas [A] time = 2.13137, size = 921, normalized size = 2.25

$$5\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+1215\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+1215\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sq
rt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 1215*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*co
s(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 1215*
sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*
d)))*sin(-(b*c - a*d)/d) + 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt
(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(10*(b^2*d^2*x
 + b^2*c*d)*cos(b*x + a)^3 + 60*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) + (24*b^
3*d^2*x^2 + 48*b^3*c*d*x + 24*b^3*c^2 - 100*b*d^2 + (12*b^3*d^2*x^2 + 24*b^
3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c)
)/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.60756, size = 2719, normalized size = 6.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/1728*(12*(-I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d
/sqrt(b^2*d^2) + 1)*b) - 27*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b
*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 27*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(
2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d
)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(6)*sqrt(pi)*d^2*erf(
-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I
*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 6*I*sqrt(d*x
+ c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 54*I*sqrt(d*x + c)*d
*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 54*I*sqrt(d*x + c)*d*e^((-I*(d*x
 + c)*b + I*b*c - I*a*d)/d)/b - 6*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b +
3*I*b*c - 3*I*a*d)/d)/b)*c^2 - d^2*((sqrt(6)*sqrt(pi)*(12*I*b^2*c^2*d - 12*
b*c*d^2 - 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
 1)*b^3) - 6*(-12*I*(d*x + c)^(5/2)*b^2*d + 24*I*(d*x + c)^(3/2)*b^2*c*d -
12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 12*sqrt(d*x + c)*
b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/
d)/b^3)/d^2 + 27*(sqrt(2)*sqrt(pi)*(12*I*b^2*c^2*d - 36*b*c*d^2 - 45*I*d^3)
*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(-12*I*(d
*x + c)^(5/2)*b^2*d + 24*I*(d*x + c)^(3/2)*b^2*c*d - 12*I*sqrt(d*x + c)*b^2
*c^2*d - 30*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 + 45*I*sqrt(d*
x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^2 + 27*(sqrt(2)*s
qrt(pi)*(-12*I*b^2*c^2*d - 36*b*c*d^2 + 45*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b
```

```
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(12*I*(d*x + c)^(5/2)*b^2*d - 24
*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d - 30*(d*x + c)^(3
/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 45*I*sqrt(d*x + c)*d^3)*e^((I*(d*x +
 c)*b - I*b*c + I*a*d)/d)/b^3)/d^2 + (sqrt(6)*sqrt(pi)*(-12*I*b^2*c^2*d - 1
2*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d
^2) + 1)*b^3) - 6*(12*I*(d*x + c)^(5/2)*b^2*d - 24*I*(d*x + c)^(3/2)*b^2*c*
d + 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 12*sqrt(d*x +
 c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*
d)/d)/b^3)/d^2) - 12*(sqrt(6)*sqrt(pi)*(-2*I*b*c*d + d^2)*d*erf(-1/2*sqrt(6
)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 9*sqrt(2)*sqrt(pi)*(-6*I*
b*c*d + 9*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2)
 + 9*sqrt(2)*sqrt(pi)*(6*I*b*c*d + 9*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt
(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(
-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + sqrt(6)*sqrt(pi)*(2*I*b*c*d + d^2)*d*erf(-
1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*
b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(2*I*(d*x
+ c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((3*I*(d*x
+ c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2 - 18*(6*I*(d*x + c)^(3/2)*b*d - 6*I*sqrt
(d*x + c)*b*c*d - 9*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d
)/b^2 - 18*(-6*I*(d*x + c)^(3/2)*b*d + 6*I*sqrt(d*x + c)*b*c*d - 9*sqrt(d*x
 + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2 - 6*(-2*I*(d*x + c)^(
3/2)*b*d + 2*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*
b + 3*I*b*c - 3*I*a*d)/d)/b^2)*c)/d
```

3.57 $\int (c+dx)^{3/2} \cos^3(a+bx)\,dx$

Optimal. Leaf size=354

$$-\frac{9\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\right.}{24}$$

```
[Out] (d*Sqrt[c + d*x]*Cos[a + b*x])/b^2 + (d*Sqrt[c + d*x]*Cos[a + b*x]^3)/(6*b^
2) - (9*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/
d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/2)) + (d^
(3/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3
*a - (3*b*c)/d])/(24*b^(5/2)) + (9*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + (2*(c + d*x
)^(3/2)*Sin[a + b*x])/(3*b) + ((c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x])
/(3*b)
```

Rubi [A] time = 0.99067, antiderivative size = 354, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3311, 3296, 3306, 3305, 3351, 3304, 3352, 3312}

$$-\frac{9\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\right.}{24}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3,x]
```


```
[Out] (d*Sqrt[c + d*x]*Cos[a + b*x])/b^2 + (d*Sqrt[c + d*x]*Cos[a + b*x]^3)/(6*b^
2) - (9*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/
d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/2)) + (d^
(3/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3
*a - (3*b*c)/d])/(24*b^(5/2)) + (9*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + (2*(c + d*x
)^(3/2)*Sin[a + b*x])/(3*b) + ((c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x])
/(3*b)
```


Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\,dx = \frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2} + \frac{(c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)}{3b} + \frac{2}{3}\int (c+dx)^{3/2}\cos(a+b\ldots$$

$$= \frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2} + \frac{2(c+dx)^{3/2}\sin(a+bx)}{3b} + \frac{(c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)}{3b} \ldots$$

$$= \frac{d\sqrt{c+dx}\cos(a+bx)}{b^2} + \frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2} + \frac{2(c+dx)^{3/2}\sin(a+bx)}{3b} + \frac{(c+dx)^{3/2}\,c\ldots}{}$$

$$= \frac{d\sqrt{c+dx}\cos(a+bx)}{b^2} + \frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2} + \frac{2(c+dx)^{3/2}\sin(a+bx)}{3b} + \frac{(c+dx)^{3/2}\,c\ldots}{}$$

$$= \frac{d\sqrt{c+dx}\cos(a+bx)}{b^2} + \frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2} + \frac{2(c+dx)^{3/2}\sin(a+bx)}{3b} + \frac{(c+dx)^{3/2}\,c\ldots}{}$$

$$= \frac{d\sqrt{c+dx}\cos(a+bx)}{b^2} + \frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2} - \frac{9d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} - \ldots$$

Mathematica [A] time = 1.6959, size = 390, normalized size = 1.1

$$-81\sqrt{2\pi}d\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)-\sqrt{6\pi}d\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)+\sqrt{6\pi}d\sin\left(3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3,x]
```

```
[Out] (162*Sqrt[b/d]*d*Sqrt[c + d*x]*Cos[a + b*x] + 6*Sqrt[b/d]*d*Sqrt[c + d*x]*C
os[3*(a + b*x)] - 81*d*Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[
2/Pi]*Sqrt[c + d*x]] - d*Sqrt[6*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]
*Sqrt[6/Pi]*Sqrt[c + d*x]] + d*Sqrt[6*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqr
t[c + d*x]]*Sin[3*a - (3*b*c)/d] + 81*d*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[
2/Pi]*Sqrt[c + d*x]]*Sin[a - (b*c)/d] + 108*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Sin
[a + b*x] + 108*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Sin[a + b*x] + 12*b*c*Sqrt[b/
d]*Sqrt[c + d*x]*Sin[3*(a + b*x)] + 12*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Sin[3*
(a + b*x)])/(144*b^2*Sqrt[b/d])
```

Maple [A] time = 0.043, size = 386, normalized size = 1.1

$$2\frac{1}{d}\left(3/8\frac{d(dx+c)^{3/2}}{b}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)-\frac{9d}{8b}\left(-1/2\frac{d\sqrt{dx+c}}{b}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)+1/4\frac{d\sqrt{2}\sqrt{\pi}}{b}\left(\cos\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3,x)
```

```
[Out] 2/d*(3/8/b*d*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-9/8/b*d*(-1/2/b*d
*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)))+1/24/b*d*(d*x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/8/b*d*(-
1/6/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^
(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^
(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

Maxima [C] time = 2.70164, size = 1793, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/576*sqrt(3)*(16*sqrt(3)*(d*x + c)^(3/2)*b*d*abs(b)*sin(3*((d*x + c)*b - b
*c + a*d)/d)/abs(d) + 144*sqrt(3)*(d*x + c)^(3/2)*b*d*abs(b)*sin(((d*x + c)
*b - b*c + a*d)/d)/abs(d) + 8*sqrt(3)*sqrt(d*x + c)*d^2*abs(b)*cos(3*((d*x
+ c)*b - b*c + a*d)/d)/abs(d) + 216*sqrt(3)*sqrt(d*x + c)*d^2*abs(b)*cos(((
d*x + c)*b - b*c + a*d)/d)/abs(d) - ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0,
```

```
b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*
d)/d) - (I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt
(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))))*d^2*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt
(3*I*b/d)) - (sqrt(3)*(27*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) + 27*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) - 27*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) + 27*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d)
 - sqrt(3)*(27*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d
/sqrt(d^2))) + 27*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) + 27*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))) - 27*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(
d*x + c)*sqrt(I*b/d)) - (sqrt(3)*(27*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 27*I*sqrt(pi)*sin(-1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-(b*
c - a*d)/d) - sqrt(3)*(-27*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) - 27*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) + 27*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) - 27*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a
*d)/d) - (-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqr
t(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt
(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(-3*I*b/d)))*abs(d)/(b^2*d*abs(b))
```

Fricas [A] time = 2.06816, size = 763, normalized size = 2.16

$$-\frac{\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+81\,\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-81\,\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{S}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqr
t(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 81*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-
(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 81*sqrt(
2)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*
sin(-(b*c - a*d)/d) - sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqr
t(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*d*cos(b*x + a)^3 +
 6*b*d*cos(b*x + a) + 2*(2*b^2*d*x + 2*b^2*c + (b^2*d*x + b^2*c)*cos(b*x +
```

```
a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.44153, size = 1515, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/288*(2*(-I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)*b) - 27*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)
*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 27*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(6)*sqrt(pi)*d^2*erf(-1
/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b
*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 6*I*sqrt(d*x +
c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 54*I*sqrt(d*x + c)*d*e
^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 54*I*sqrt(d*x + c)*d*e^((-I*(d*x +
 c)*b + I*b*c - I*a*d)/d)/b - 6*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*
I*b*c - 3*I*a*d)/d)/b)*c - sqrt(6)*sqrt(pi)*(-2*I*b*c*d + d^2)*d*erf(-1/2*s
qrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3
*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 9*sqrt(2)*sqrt(pi)*(
-6*I*b*c*d + 9*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(
b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b^2) - 9*sqrt(2)*sqrt(pi)*(6*I*b*c*d + 9*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b
*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - sqrt(6)*sqrt(pi)*(2*I*b*c*d + d^2)*d*
erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 6*(2*I*
(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((3*I*
(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2 + 18*(6*I*(d*x + c)^(3/2)*b*d - 6*I
*sqrt(d*x + c)*b*c*d - 9*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a
*d)/d)/b^2 + 18*(-6*I*(d*x + c)^(3/2)*b*d + 6*I*sqrt(d*x + c)*b*c*d - 9*sqr
t(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2 + 6*(-2*I*(d*x +
 c)^(3/2)*b*d + 2*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((-3*I*(d*x
+ c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d
```

3.58 $\int \sqrt{c+dx}\cos^3(a+bx)\,dx$

Optimal. Leaf size=304

$$-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)S}{4b^{3/2}}$$

```
[Out] (-3*Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
 + d*x])/Sqrt[d]])/(4*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*F
resnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) - (Sqrt[d
]*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a -
 (3*b*c)/d])/(12*b^(3/2)) - (3*Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2)) + (3*Sqrt[c + d*x
]*Sin[a + b*x])/(4*b) + (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(12*b)
```

Rubi [A] time = 0.484469, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3312, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)S}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3,x]
```


```
[Out] (-3*Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
 + d*x])/Sqrt[d]])/(4*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*F
resnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) - (Sqrt[d
]*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a -
 (3*b*c)/d])/(12*b^(3/2)) - (3*Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2)) + (3*Sqrt[c + d*x
]*Sin[a + b*x])/(4*b) + (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(12*b)
```


Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{c+dx}\cos^3(a+bx)\,dx &= \int \left(\frac{3}{4}\sqrt{c+dx}\cos(a+bx)+\frac{1}{4}\sqrt{c+dx}\cos(3a+3bx)\right)dx \\
&= \frac{1}{4}\int \sqrt{c+dx}\cos(3a+3bx)\,dx+\frac{3}{4}\int \sqrt{c+dx}\cos(a+bx)\,dx \\
&= \frac{3\sqrt{c+dx}\sin(a+bx)}{4b}+\frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}-\frac{d\int \frac{\sin(3a+3bx)}{\sqrt{c+dx}}\,dx}{24b}-\frac{(3d)\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{8b} \\
&= \frac{3\sqrt{c+dx}\sin(a+bx)}{4b}+\frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}-\frac{\left(d\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}\,dx}{24b}- \\
&= \frac{3\sqrt{c+dx}\sin(a+bx)}{4b}+\frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Subst}\left(\int \sin\left(\frac{3bx^2}{d}\right)dx\right.}{12b} \\
&= -\frac{3\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}}{}
\end{aligned}$$

Mathematica [C] time = 0.443093, size = 254, normalized size = 0.84

$$\frac{i\sqrt{c+dx}e^{-\frac{3i(ad+bc)}{d}}\left(-27e^{2i\left(2a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\text{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+27e^{2ia+\frac{4ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\text{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)+\sqrt{3}\left(e\right.\right.}{72b\sqrt{\frac{b^2(c+dx)^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3,x]
```

```
[Out] ((I/72)*Sqrt[c + d*x]*(-27*E^((2*I)*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d
]*Gamma[3/2, ((-I)*b*(c + d*x))/d] + 27*E^((2*I)*a + ((4*I)*b*c)/d)*Sqrt[((
-I)*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d] + Sqrt[3]*(-(E^((6*I)*a)*
Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-3*I)*b*(c + d*x))/d]) + E^(((6*I)*b*c
)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((3*I)*b*(c + d*x))/d])))/(b*E^(
((3*I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

Maple [A] time = 0.04, size = 294, normalized size = 1.

$$2\frac{1}{d}\left(3/8\,\frac{d\sqrt{dx+c}}{b}\sin\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)-3/16\,\frac{d\sqrt{2}\sqrt{\pi}}{b}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\sqrt{dx+c}b}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right)+\sin\left(\frac{da-cb}{d}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3,x)
```

```
[Out] 2/d*(3/8/b*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/16/b*d*2^(1/2)*
Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*(d*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)
*(d*x+c)^(1/2)*b/d))+1/24/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d
)-1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresne
lS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/
d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 2.61107, size = 1651, normalized size = 5.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/288*sqrt(3)*(8*sqrt(3)*sqrt(d*x + c)*d*abs(b)*sin(3*((d*x + c)*b - b*c +
a*d)/d)/abs(d) + 72*sqrt(3)*sqrt(d*x + c)*d*abs(b)*sin(((d*x + c)*b - b*c +
 a*d)/d)/abs(d) + ((-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))) + sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (sqrt(pi)*c
os(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*si
n(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*si
n(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)
/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (sqrt(3)
*(-9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) - 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))))*d*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) - sqrt(3)*(9*sqrt(pi)*cos
(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*cos
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*
sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi
)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(ab
s(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + (sqrt(3)
*(9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) + 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))))*d*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) - sqrt(3)*(9*sqrt(pi)*cos(
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(
-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*s
```

```
in(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)
*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs
(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((I*sqrt
(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt
(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(
pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi
)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(ab
s(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d)
/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)))*abs(d)/(b*d*abs(b))
```

Fricas [A] time = 1.99167, size = 645, normalized size = 2.12

$$-\frac{\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+27\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+27\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{C}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6
)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c
- a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi
*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b
*c - a*d)/d) + sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c
)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*cos(b*x + a)^2 + 2*b)*sqrt(
d*x + c)*sin(b*x + a))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c+dx}\cos^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(c + d*x)*cos(a + b*x)**3, x)
```

Giac [C] time = 1.29292, size = 662, normalized size = 2.18

$$-\frac{-\frac{i\sqrt{6}\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{6}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{3ibc-3iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}-\frac{27i\sqrt{2}\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{ibc-iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+\frac{27i\sqrt{2}\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+\right.}{2d}\right.}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/144*(-I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)*b) - 27*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(
I*b*d/sqrt(b^2*d^2) + 1)*b) + 27*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sq
rt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*
sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c
+ 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 6*I*sqrt(d*x + c)*
d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 54*I*sqrt(d*x + c)*d*e^((
I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 54*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)
*b + I*b*c - I*a*d)/d)/b - 6*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b
*c - 3*I*a*d)/d)/b)/d
```

3.59 $\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}} \, dx$

Optimal. Leaf size=257

$$\frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\cdots}{\sqrt{\cdots}}\right)}{2\sqrt{b}\sqrt{d}}$$

```
[Out] (3*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/
Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) + (Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(
Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi/
6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d
])/(2*Sqrt[b]*Sqrt[d]) - (3*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(2*Sqrt[b]*Sqrt[d])
```

Rubi [A] time = 0.4174, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3312, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\cdots}{\sqrt{\cdots}}\right)}{2\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/Sqrt[c + d*x],x]
```

```
[Out] (3*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/
Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) + (Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(
Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi/
6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d
])/(2*Sqrt[b]*Sqrt[d]) - (3*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(2*Sqrt[b]*Sqrt[d])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx = \int \left(\frac{3\cos(a+bx)}{4\sqrt{c+dx}}+\frac{\cos(3a+3bx)}{4\sqrt{c+dx}}\right)dx$$
$$= \frac{1}{4}\int \frac{\cos(3a+3bx)}{\sqrt{c+dx}}\,dx+\frac{3}{4}\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx$$
$$= \frac{1}{4}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}\,dx+\frac{1}{4}\left(3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx-\frac{1}{4}\sin\left(3a-\frac{3bc}{d}\right.$$
$$= \frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Subst}\left(\int \cos\left(\frac{3bx^2}{d}\right)dx,x,\sqrt{c+dx}\right)}{2d}+\frac{\left(3\cos\left(a-\frac{bc}{d}\right)\right)\text{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)dx,x,\sqrt{c\right.}}{2d}$$
$$= \frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{6}}S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3\right.}{2\sqrt{b}\sqrt{d}}$$

Mathematica [C] time = 0.409301, size = 236, normalized size = 0.92

$$\frac{ie^{-\frac{3i(ad+bc)}{d}}\left(-9e^{2i\left(2a+\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\text{Gamma}\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)+9e^{2ia+\frac{4ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\text{Gamma}\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right)+\sqrt{3}\left(e^{\frac{6ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\text{Ga}\right.\right.}{24b\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/Sqrt[c + d*x],x]
```

```
[Out] ((I/24)*(-9*E^((2*I)*(2*a + (b*c)/d))*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/2,
 ((-I)*b*(c + d*x))/d] + 9*E^((2*I)*a + ((4*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))
/d]*Gamma[1/2, (I*b*(c + d*x))/d] + Sqrt[3]*(-(E^((6*I)*a)*Sqrt[((-I)*b*(c
+ d*x))/d]*Gamma[1/2, ((-3*I)*b*(c + d*x))/d]) + E^(((6*I)*b*c)/d)*Sqrt[(I*
b*(c + d*x))/d]*Gamma[1/2, ((3*I)*b*(c + d*x))/d])))/(b*E^(((3*I)*(b*c + a*
d))/d)*Sqrt[c + d*x])
```

Maple [A] time = 0.043, size = 212, normalized size = 0.8

$$2\frac{1}{d}\left(3/8\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}\sqrt{dx+cb}}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right)-\sin\left(\frac{da-cb}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}\sqrt{dx+cb}}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right)\right)\frac{1}{\sqrt{\frac{b}{d}}}+1/24\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/(d*x+c)^(1/2),x)
```

```
[Out] 2/d*(3/8*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi
^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^
(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))+1/24*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 2.57673, size = 1530, normalized size = 5.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/48*sqrt(3)*(((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/s
qrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/s
qrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))))*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (I*sqrt(pi)*cos(1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*sqrt(abs(b)/abs(d))*
sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (sqrt(3)*(3*sqrt(
pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(
pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sq
rt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*
sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*sq
rt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) - sqrt(3)*(3*I*sqrt(pi)*cos(1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*cos(-1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(-1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*sqrt(abs(b)/abs(d))*s
in(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + (sqrt(3)*(3*sqrt(pi)*c
os(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*c
os(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi
)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(
pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*sqrt(ab
s(b)/abs(d))*cos(-(b*c - a*d)/d) - sqrt(3)*(-3*I*sqrt(pi)*cos(1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos(-1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*sqrt(abs(b)/abs(d))*sin(-
(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((sqrt(pi)*cos(1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*sqrt(abs(b)/abs(d))*cos(-3
*(b*c - a*d)/d) - (-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
```

```
, d/sqrt(d^2))))*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x +
c)*sqrt(-3*I*b/d)))*abs(d)/(d*abs(b))
```

Fricas [A] time = 1.71124, size = 551, normalized size = 2.14

$$\frac{\sqrt{6}\pi\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+9\sqrt{2}\pi\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\sqrt{2}\pi\sqrt{\frac{b}{\pi d}}S\left(\sqrt{2}\sqrt{dx+\right.}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(6)*pi*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*s
qrt(d*x + c)*sqrt(b/(pi*d))) + 9*sqrt(2)*pi*sqrt(b/(pi*d))*cos(-(b*c - a*d)
/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*sqrt(b
/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)
/d) - sqrt(6)*pi*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(p
i*d)))*sin(-3*(b*c - a*d)/d))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/(d*x+c)**(1/2),x)
```

```
[Out] Integral(cos(a + b*x)**3/sqrt(c + d*x), x)
```

Giac [C] time = 1.16102, size = 443, normalized size = 1.72

$$-\frac{\frac{\sqrt{6}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{6}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{3ibc-3iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}+\frac{9\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{ibc-iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}+\frac{9\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{-ibc+iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)) + 9*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)) + 9*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/s
qrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(
-I*b*d/sqrt(b^2*d^2) + 1)))/d
```

3.60 $\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=271

$$-\frac{\sqrt{\frac{3\pi}{2}}\sqrt{b}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{b}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(a-\frac{bc}{d}\right.}{d^{3/2}}$$

```
[Out] (-2*Cos[a + b*x]^3)/(d*Sqrt[c + d*x]) - (3*Sqrt[b]*Sqrt[Pi/2]*Cos[a - (b*c)
/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/d^(3/2) - (Sqrt[b
]*Sqrt[(3*Pi)/2]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/d^(3/2) - (Sqrt[b]*Sqrt[(3*Pi)/2]*FresnelC[(Sqrt[b]*Sqrt[6
/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/d^(3/2) - (3*Sqrt[b]*Sqr
t[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/
d])/d^(3/2)
```

Rubi [A] time = 0.564146, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3313, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{3\pi}{2}}\sqrt{b}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{b}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(a-\frac{bc}{d}\right.}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x]^3)/(d*Sqrt[c + d*x]) - (3*Sqrt[b]*Sqrt[Pi/2]*Cos[a - (b*c)
/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/d^(3/2) - (Sqrt[b
]*Sqrt[(3*Pi)/2]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/d^(3/2) - (Sqrt[b]*Sqrt[(3*Pi)/2]*FresnelC[(Sqrt[b]*Sqrt[6
/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/d^(3/2) - (3*Sqrt[b]*Sqr
t[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/
d])/d^(3/2)
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
```

```
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{2\cos^3(a+bx)}{d\sqrt{c+dx}} + \frac{(6b)\int\left(-\frac{\sin(a+bx)}{4\sqrt{c+dx}} - \frac{\sin(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d}$$

$$= -\frac{2\cos^3(a+bx)}{d\sqrt{c+dx}} - \frac{(3b)\int\frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{2d} - \frac{(3b)\int\frac{\sin(3a+3bx)}{\sqrt{c+dx}}\,dx}{2d}$$

$$= -\frac{2\cos^3(a+bx)}{d\sqrt{c+dx}} - \frac{\left(3b\cos\left(3a-\frac{3bc}{d}\right)\right)\int\frac{\sin\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}\,dx}{2d} - \frac{\left(3b\cos\left(a-\frac{bc}{d}\right)\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{2d} - \frac{(3b\sin$$

$$= -\frac{2\cos^3(a+bx)}{d\sqrt{c+dx}} - \frac{\left(3b\cos\left(3a-\frac{3bc}{d}\right)\right)\text{Subst}\left(\int\sin\left(\frac{3bx^2}{d}\right)dx, x, \sqrt{c+dx}\right)}{d^2} - \frac{\left(3b\cos\left(a-\frac{bc}{d}\right)\right)\text{Sub}}{}$$

$$= -\frac{2\cos^3(a+bx)}{d\sqrt{c+dx}} - \frac{3\sqrt{b}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{\sqrt{b}\sqrt{\frac{3\pi}{2}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} -$$

Mathematica [A] time = 1.54831, size = 299, normalized size = 1.1

$$-\frac{\sqrt{6\pi}\sqrt{\frac{b}{d}}\sqrt{c+dx}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)+3\sqrt{2\pi}\sqrt{\frac{b}{d}}\sqrt{c+dx}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/(c + d*x)^(3/2),x]
```

```
[Out] -(3*Cos[a + b*x] + Cos[3*(a + b*x)] + 3*Sqrt[b/d]*Sqrt[2*Pi]*Sqrt[c + d*x]*
Cos[a - (b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] + Sqrt[b/d]*S
qrt[6*Pi]*Sqrt[c + d*x]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*
Sqrt[c + d*x]] + Sqrt[b/d]*Sqrt[6*Pi]*Sqrt[c + d*x]*FresnelC[Sqrt[b/d]*Sqrt
[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] + 3*Sqrt[b/d]*Sqrt[2*Pi]*Sqrt[c
+ d*x]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[a - (b*c)/d])/(2*d*
Sqrt[c + d*x])
```

Maple [A] time = 0.042, size = 286, normalized size = 1.1

$$2\frac{1}{d}\left(-3/4\frac{1}{\sqrt{dx+c}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)-3/4\frac{\sqrt{2}b\sqrt{\pi}}{d}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\sqrt{dx+cb}}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right)+\sin\left(\frac{da-}{d}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/(d*x+c)^(3/2),x)
```

```
[Out] 2/d*(-3/4/(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/4*b/d*2^(1/2)*Pi^(
1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d
*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*
x+c)^(1/2)*b/d))-1/4/(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/4*b/d
*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC
(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 1.89239, size = 1260, normalized size = 4.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*(sqrt(3)*(((gamma(-1/2, 3*I*(d*x + c)*b/d) + gamma(-1/2, -3*I*(d*x +
c)*b/d))*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + (g
amma(-1/2, 3*I*(d*x + c)*b/d) + gamma(-1/2, -3*I*(d*x + c)*b/d))*cos(-1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + (I*gamma(-1/2, 3*I*(
d*x + c)*b/d) - I*gamma(-1/2, -3*I*(d*x + c)*b/d))*sin(1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + (-I*gamma(-1/2, 3*I*(d*x + c)*b/d)
+ I*gamma(-1/2, -3*I*(d*x + c)*b/d))*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))))*cos(-3*(b*c - a*d)/d) + ((-I*gamma(-1/2, 3*I*(d*x
 + c)*b/d) + I*gamma(-1/2, -3*I*(d*x + c)*b/d))*cos(1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))) + (-I*gamma(-1/2, 3*I*(d*x + c)*b/d) + I
*gamma(-1/2, -3*I*(d*x + c)*b/d))*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) + (gamma(-1/2, 3*I*(d*x + c)*b/d) + gamma(-1/2, -3*I*
(d*x + c)*b/d))*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) - (gamma(-1/2, 3*I*(d*x + c)*b/d) + gamma(-1/2, -3*I*(d*x + c)*b/d))*sin
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*sin(-3*(b*c -
a*d)/d))*sqrt((d*x + c)*abs(b)/abs(d)) + ((3*(gamma(-1/2, I*(d*x + c)*b/d)
+ gamma(-1/2, -I*(d*x + c)*b/d))*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) + 3*(gamma(-1/2, I*(d*x + c)*b/d) + gamma(-1/2, -I*(d*x
 + c)*b/d))*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ (3*I*gamma(-1/2, I*(d*x + c)*b/d) - 3*I*gamma(-1/2, -I*(d*x + c)*b/d))*si
n(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + (-3*I*gamma(-
1/2, I*(d*x + c)*b/d) + 3*I*gamma(-1/2, -I*(d*x + c)*b/d))*sin(-1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*cos(-(b*c - a*d)/d) + ((-3*
I*gamma(-1/2, I*(d*x + c)*b/d) + 3*I*gamma(-1/2, -I*(d*x + c)*b/d))*cos(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + (-3*I*gamma(-1/2,
I*(d*x + c)*b/d) + 3*I*gamma(-1/2, -I*(d*x + c)*b/d))*cos(-1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*(gamma(-1/2, I*(d*x + c)*b/d)
 + gamma(-1/2, -I*(d*x + c)*b/d))*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) - 3*(gamma(-1/2, I*(d*x + c)*b/d) + gamma(-1/2, -I*(d*
x + c)*b/d))*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
```

```
)*sin(-(b*c - a*d)/d))*sqrt((d*x + c)*abs(b)/abs(d)))/(sqrt(d*x + c)*d)
```

Fricas [A] time = 1.98379, size = 682, normalized size = 2.52

$$-\frac{\sqrt{6}(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(6)*(pi*d*x + pi*c)*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_
sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*(pi*d*x + pi*c)*sqrt(
b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*
d))) + 3*sqrt(2)*(pi*d*x + pi*c)*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*
x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + sqrt(6)*(pi*d*x + pi*c)*sqrt(b
/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*
d)/d) + 4*sqrt(d*x + c)*cos(b*x + a)^3)/(d^2*x + c*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/(d*x+c)**(3/2),x)
```

```
[Out] Integral(cos(a + b*x)**3/(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^3}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3/(d*x + c)^(3/2), x)
```

3.61 $\int \frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=292

$$-\frac{\sqrt{2\pi}b^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}}-\frac{\sqrt{6\pi}b^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}}+\frac{\sqrt{6\pi}b^{3/2}\sin\left(3a-\right.}{d}$$

```
[Out] (-2*Cos[a + b*x]^3)/(3*d*(c + d*x)^(3/2)) - (b^(3/2)*Sqrt[2*Pi]*Cos[a - (b*
c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/d^(5/2) - (b^(3
/2)*Sqrt[6*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d
*x])/Sqrt[d]])/d^(5/2) + (b^(3/2)*Sqrt[6*Pi]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*S
qrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/d^(5/2) + (b^(3/2)*Sqrt[2*Pi]*
FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/d^(5
/2) + (4*b*Cos[a + b*x]^2*Sin[a + b*x])/(d^2*Sqrt[c + d*x])
```

Rubi [A] time = 0.737646, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3314, 3306, 3305, 3351, 3304, 3352, 3312}

$$-\frac{\sqrt{2\pi}b^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}}-\frac{\sqrt{6\pi}b^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}}+\frac{\sqrt{6\pi}b^{3/2}\sin\left(3a-\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/(c + d*x)^(5/2),x]
```



```
[Out] (-2*Cos[a + b*x]^3)/(3*d*(c + d*x)^(3/2)) - (b^(3/2)*Sqrt[2*Pi]*Cos[a - (b*
c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/d^(5/2) - (b^(3
/2)*Sqrt[6*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d
*x])/Sqrt[d]])/d^(5/2) + (b^(3/2)*Sqrt[6*Pi]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*S
qrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/d^(5/2) + (b^(3/2)*Sqrt[2*Pi]*
FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/d^(5
/2) + (4*b*Cos[a + b*x]^2*Sin[a + b*x])/(d^2*Sqrt[c + d*x])
```


Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{d^2\sqrt{c+dx}} + \frac{(8b^2)\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx}{d^2} - \frac{(12b^2)\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx}{d^2}$$

$$= -\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{d^2\sqrt{c+dx}} - \frac{(12b^2)\int\left(\frac{3\cos(a+bx)}{4\sqrt{c+dx}} + \frac{\cos(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d^2} + \frac{(8b^2\cos(}{}$$

$$= -\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{d^2\sqrt{c+dx}} - \frac{(3b^2)\int \frac{\cos(3a+3bx)}{\sqrt{c+dx}}\,dx}{d^2} - \frac{(9b^2)\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx}{d^2} + \frac{(16}{}$$

$$= -\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{8b^{3/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} - \frac{8b^{3/2}\sqrt{2\pi}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{d^{5/2}} +$$

$$= -\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{8b^{3/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} - \frac{8b^{3/2}\sqrt{2\pi}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{d^{5/2}} +$$

$$= -\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{b^{3/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} - \frac{b^{3/2}\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} +$$

Mathematica [C] time = 2.05878, size = 268, normalized size = 0.92

$$-3de^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right) - 3de^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right) - 3\sqrt{3}de^{3i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/(c + d*x)^(5/2),x]
```

```
[Out] (-4*d*Cos[a + b*x]^3 - 3*d*E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(3/2)
*Gamma[1/2, ((-I)*b*(c + d*x))/d] - (3*d*((I*b*(c + d*x))/d)^(3/2)*Gamma[1/
2, (I*b*(c + d*x))/d])/E^(I*(a - (b*c)/d)) - 3*Sqrt[3]*d*E^((3*I)*(a - (b*c
)/d))*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-3*I)*b*(c + d*x))/d] - (3*
Sqrt[3]*d*((I*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((3*I)*b*(c + d*x))/d])/E^((
3*I)*(a - (b*c)/d)) + 24*b*(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(6*d^2*(c
 + d*x)^(3/2))
```

Maple [A] time = 0.043, size = 368, normalized size = 1.3

$$2\,\frac{1}{d}\left(-1/4\,\frac{1}{(dx+c)^{3/2}}\cos\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)-1/2\,\frac{b}{d}\left(-\frac{1}{\sqrt{dx+c}}\sin\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)+\frac{\sqrt{2}b\sqrt{\pi}}{d}\left(\cos\left(\frac{da-c}{d}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/(d*x+c)^(5/2),x)
```

```
[Out] 2/d*(-1/4/(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/2*b/d*(-1/(d*x+c)^
(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)+b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(
(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin((
a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))-1/12
/(d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/2*b/d*(-1/(d*x+c)^(1/2)*s
in(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+b/d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(c
os(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d
*x+c)^(1/2)*b/d))))
```

Maxima [C] time = 1.73488, size = 1261, normalized size = 4.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/16*(3*sqrt(3)*(((gamma(-3/2, 3*I*(d*x + c)*b/d) + gamma(-3/2, -3*I*(d*x
+ c)*b/d))*cos(3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) +
(gamma(-3/2, 3*I*(d*x + c)*b/d) + gamma(-3/2, -3*I*(d*x + c)*b/d))*cos(-3/4
*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) + (I*gamma(-3/2, 3*I
*(d*x + c)*b/d) - I*gamma(-3/2, -3*I*(d*x + c)*b/d))*sin(3/4*pi + 3/2*arcta
n2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) + (-I*gamma(-3/2, 3*I*(d*x + c)*b/d
) + I*gamma(-3/2, -3*I*(d*x + c)*b/d))*sin(-3/4*pi + 3/2*arctan2(0, b) + 3/
2*arctan2(0, d/sqrt(d^2))))*cos(-3*(b*c - a*d)/d) + ((-I*gamma(-3/2, 3*I*(d
*x + c)*b/d) + I*gamma(-3/2, -3*I*(d*x + c)*b/d))*cos(3/4*pi + 3/2*arctan2(
0, b) + 3/2*arctan2(0, d/sqrt(d^2))) + (-I*gamma(-3/2, 3*I*(d*x + c)*b/d) +
 I*gamma(-3/2, -3*I*(d*x + c)*b/d))*cos(-3/4*pi + 3/2*arctan2(0, b) + 3/2*a
rctan2(0, d/sqrt(d^2))) + (gamma(-3/2, 3*I*(d*x + c)*b/d) + gamma(-3/2, -3*
I*(d*x + c)*b/d))*sin(3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^
2))) - (gamma(-3/2, 3*I*(d*x + c)*b/d) + gamma(-3/2, -3*I*(d*x + c)*b/d))*s
in(-3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))))*sin(-3*(b*c
- a*d)/d))*((d*x + c)*abs(b)/abs(d))^(3/2) + ((3*(gamma(-3/2, I*(d*x + c)*b
```

```
/d) + gamma(-3/2, -I*(d*x + c)*b/d))*cos(3/4*pi + 3/2*arctan2(0, b) + 3/2*a
rctan2(0, d/sqrt(d^2))) + 3*(gamma(-3/2, I*(d*x + c)*b/d) + gamma(-3/2, -I*
(d*x + c)*b/d))*cos(-3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2
))) + (3*I*gamma(-3/2, I*(d*x + c)*b/d) - 3*I*gamma(-3/2, -I*(d*x + c)*b/d)
)*sin(3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) + (-3*I*gam
ma(-3/2, I*(d*x + c)*b/d) + 3*I*gamma(-3/2, -I*(d*x + c)*b/d))*sin(-3/4*pi
+ 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))))*cos(-(b*c - a*d)/d) + (
(-3*I*gamma(-3/2, I*(d*x + c)*b/d) + 3*I*gamma(-3/2, -I*(d*x + c)*b/d))*cos
(3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) + (-3*I*gamma(-3
/2, I*(d*x + c)*b/d) + 3*I*gamma(-3/2, -I*(d*x + c)*b/d))*cos(-3/4*pi + 3/2
*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^2))) + 3*(gamma(-3/2, I*(d*x + c)*
b/d) + gamma(-3/2, -I*(d*x + c)*b/d))*sin(3/4*pi + 3/2*arctan2(0, b) + 3/2*
arctan2(0, d/sqrt(d^2))) - 3*(gamma(-3/2, I*(d*x + c)*b/d) + gamma(-3/2, -I
*(d*x + c)*b/d))*sin(-3/4*pi + 3/2*arctan2(0, b) + 3/2*arctan2(0, d/sqrt(d^
2))))*sin(-(b*c - a*d)/d))*((d*x + c)*abs(b)/abs(d))^(3/2))/((d*x + c)^(3/2
)*d)
```

Fricas [A] time = 2.32907, size = 914, normalized size = 3.13

$$-\frac{3\sqrt{6}\left(\pi bd^2x^2 + 2\pi bcdx + \pi bc^2\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) + 3\sqrt{2}\left(\pi bd^2x^2 + 2\pi bcdx + \pi bc^2\right)\sqrt{\frac{b}{\pi d}}\cos\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*sqrt(6)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos
(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sq
rt(2)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos(-(b*c - a
*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 3*sqrt(2)*(pi*b*
d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(
d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 3*sqrt(6)*(pi*b*d^2*x^2 + 2*
pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqr
t(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 2*(d*cos(b*x + a)^3 - 6*(b*d*x + b*c)*
cos(b*x + a)^2*sin(b*x + a))*sqrt(d*x + c))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/(d*x+c)**(5/2),x)
```

```
[Out] Integral(cos(a + b*x)**3/(c + d*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^3}{(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3/(d*x + c)^(5/2), x)
```

3.62 $\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal. Leaf size=356

$$\frac{6\sqrt{6\pi}b^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{2\sqrt{2\pi}b^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{2\sqrt{2\pi}b^{5/2}\cos\left(a-\frac{bc}{d}\right.}{5d^{7/2}}$$

```
[Out] (-16*b^2*Cos[a + b*x])/(5*d^3*Sqrt[c + d*x]) - (2*Cos[a + b*x]^3)/(5*d*(c +
 d*x)^(5/2)) + (24*b^2*Cos[a + b*x]^3)/(5*d^3*Sqrt[c + d*x]) + (2*b^(5/2)*S
qrt[2*Pi]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt
[d]])/(5*d^(7/2)) + (6*b^(5/2)*Sqrt[6*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sq
rt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/2)) + (6*b^(5/2)*Sqrt[6*P
i]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d
])/(5*d^(7/2)) + (2*b^(5/2)*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(5*d^(7/2)) + (4*b*Cos[a + b*x]^2*Sin[a
+ b*x])/(5*d^2*(c + d*x)^(3/2))
```

Rubi [A] time = 0.831403, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3314, 3297, 3306, 3305, 3351, 3304, 3352, 3313}

$$\frac{6\sqrt{6\pi}b^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{2\sqrt{2\pi}b^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{2\sqrt{2\pi}b^{5/2}\cos\left(a-\frac{bc}{d}\right.}{5d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/(c + d*x)^(7/2),x]
```


```
[Out] (-16*b^2*Cos[a + b*x])/(5*d^3*Sqrt[c + d*x]) - (2*Cos[a + b*x]^3)/(5*d*(c +
 d*x)^(5/2)) + (24*b^2*Cos[a + b*x]^3)/(5*d^3*Sqrt[c + d*x]) + (2*b^(5/2)*S
qrt[2*Pi]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt
[d]])/(5*d^(7/2)) + (6*b^(5/2)*Sqrt[6*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sq
rt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/2)) + (6*b^(5/2)*Sqrt[6*P
i]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d
])/(5*d^(7/2)) + (2*b^(5/2)*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(5*d^(7/2)) + (4*b*Cos[a + b*x]^2*Sin[a
+ b*x])/(5*d^2*(c + d*x)^(3/2))
```


Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

]

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{5d^2(c+dx)^{3/2}} + \frac{(8b^2)\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}dx}{5d^2} - \frac{(12b^2)\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}dx}{5d^2}$$

$$= -\frac{16b^2\cos(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{24b^2\cos^3(a+bx)}{5d^3\sqrt{c+dx}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{(16b^3)\int}{5}$$

$$= -\frac{16b^2\cos(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{24b^2\cos^3(a+bx)}{5d^3\sqrt{c+dx}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{5d^2(c+dx)^{3/2}} + \frac{(18b^3)\int}{5}$$

$$= -\frac{16b^2\cos(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{24b^2\cos^3(a+bx)}{5d^3\sqrt{c+dx}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{5d^2(c+dx)^{3/2}} + \frac{(18b^3\cos}{}$$

$$= -\frac{16b^2\cos(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{24b^2\cos^3(a+bx)}{5d^3\sqrt{c+dx}} - \frac{16b^{5/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}$$

$$= -\frac{16b^2\cos(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{24b^2\cos^3(a+bx)}{5d^3\sqrt{c+dx}} + \frac{2b^{5/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} +$$

Mathematica [B] time = 6.33348, size = 1429, normalized size = 4.01

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/(c + d*x)^(7/2),x]
```

```
[Out] (3*(-(Sin[a]*((2*(b/d)^(5/2)*Sin[(b*c)/d]*(Cos[(b*(c + d*x))/d]/((b/d)^(5/2
)*(c + d*x)^(5/2)) - (2*(2*(Cos[(b*(c + d*x))/d]/(Sqrt[b/d]*Sqrt[c + d*x])
+ Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]) + Sin[(b*(c + d*
x))/d]/((b/d)^(3/2)*(c + d*x)^(3/2))))/3))/(5*d) - (2*(b/d)^(5/2)*Cos[(b*c)
/d]*(Sin[(b*(c + d*x))/d]/((b/d)^(5/2)*(c + d*x)^(5/2)) + (2*(Cos[(b*(c + d
*x))/d]/((b/d)^(3/2)*(c + d*x)^(3/2)) - 2*(-(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*
Sqrt[2/Pi]*Sqrt[c + d*x]]) + Sin[(b*(c + d*x))/d]/(Sqrt[b/d]*Sqrt[c + d*x])
)))/3))/(5*d))) + Cos[a]*((-2*(b/d)^(5/2)*Cos[(b*c)/d]*(Cos[(b*(c + d*x))/d
]/((b/d)^(5/2)*(c + d*x)^(5/2)) - (2*(2*(Cos[(b*(c + d*x))/d]/(Sqrt[b/d]*Sq
rt[c + d*x]) + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]) + S
in[(b*(c + d*x))/d]/((b/d)^(3/2)*(c + d*x)^(3/2))))/3))/(5*d) - (2*(b/d)^(5
/2)*Sin[(b*c)/d]*(Sin[(b*(c + d*x))/d]/((b/d)^(5/2)*(c + d*x)^(5/2)) + (2*(
Cos[(b*(c + d*x))/d]/((b/d)^(3/2)*(c + d*x)^(3/2)) - 2*(-(Sqrt[2*Pi]*Fresne
lC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]) + Sin[(b*(c + d*x))/d]/(Sqrt[b/d]*S
qrt[c + d*x]))))/3))/(5*d))))/4 + (-(Sin[3*a]*((18*Sqrt[3]*(b/d)^(5/2)*Sin[
(3*b*c)/d]*(Cos[(3*b*(c + d*x))/d]/(9*Sqrt[3]*(b/d)^(5/2)*(c + d*x)^(5/2))
- (2*(2*(Cos[(3*b*(c + d*x))/d]/(Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]) + Sqrt[2*
Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) + Sin[(3*b*(c + d*x))/d]/
(3*Sqrt[3]*(b/d)^(3/2)*(c + d*x)^(3/2))))/3))/(5*d) - (18*Sqrt[3]*(b/d)^(5/
2)*Cos[(3*b*c)/d]*(Sin[(3*b*(c + d*x))/d]/(9*Sqrt[3]*(b/d)^(5/2)*(c + d*x)^
(5/2)) + (2*(Cos[(3*b*(c + d*x))/d]/(3*Sqrt[3]*(b/d)^(3/2)*(c + d*x)^(3/2))
 - 2*(-(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) + Sin[(3*b
*(c + d*x))/d]/(Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]))))/3))/(5*d))) + Cos[3*a]*
((-18*Sqrt[3]*(b/d)^(5/2)*Cos[(3*b*c)/d]*(Cos[(3*b*(c + d*x))/d]/(9*Sqrt[3]
*(b/d)^(5/2)*(c + d*x)^(5/2)) - (2*(2*(Cos[(3*b*(c + d*x))/d]/(Sqrt[3]*Sqrt
[b/d]*Sqrt[c + d*x]) + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*
x]]) + Sin[(3*b*(c + d*x))/d]/(3*Sqrt[3]*(b/d)^(3/2)*(c + d*x)^(3/2))))/3))
```

```
/(5*d) - (18*Sqrt[3]*(b/d)^(5/2)*Sin[(3*b*c)/d]*(Sin[(3*b*(c + d*x))/d]/(9*
Sqrt[3]*(b/d)^(5/2)*(c + d*x)^(5/2)) + (2*(Cos[(3*b*(c + d*x))/d]/(3*Sqrt[3
]*(b/d)^(3/2)*(c + d*x)^(3/2)) - 2*(-(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/
Pi]*Sqrt[c + d*x]]) + Sin[(3*b*(c + d*x))/d]/(Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*
x]))))/3))/(5*d)))/4
```

Maple [A] time = 0.042, size = 450, normalized size = 1.3

$$2\frac{1}{d}\left(-\frac{3}{20\,(dx+c)^{5/2}}\cos\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)-3/10\,\frac{b}{d}\left(-1/3\,\frac{1}{(dx+c)^{3/2}}\sin\left(\frac{(dx+c)\,b}{d}+\frac{da-cb}{d}\right)+2/3\,\frac{b}{d}\left(-\frac{1}{\sqrt{dx}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/(d*x+c)^(7/2),x)
```

```
[Out] 2/d*(-3/20/(d*x+c)^(5/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/10*b/d*(-1/3/(d*x
+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)+2/3*b/d*(-1/(d*x+c)^(1/2)*cos(1/d*
(d*x+c)*b+(a*d-b*c)/d)-b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*F
resnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*Fr
esnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))-1/20/(d*x+c)^(5/2
)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-3/10*b/d*(-1/3/(d*x+c)^(3/2)*sin(3/d*(d*
x+c)*b+3*(a*d-b*c)/d)+2*b/d*(-1/(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)
/d)-b/d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2
^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*F
resnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 1.70576, size = 1261, normalized size = 3.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/16*(9*sqrt(3)*(((gamma(-5/2, 3*I*(d*x + c)*b/d) + gamma(-5/2, -3*I*(d*x
+ c)*b/d))*cos(5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) +
(gamma(-5/2, 3*I*(d*x + c)*b/d) + gamma(-5/2, -3*I*(d*x + c)*b/d))*cos(-5/4
*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) + (I*gamma(-5/2, 3*I
*(d*x + c)*b/d) - I*gamma(-5/2, -3*I*(d*x + c)*b/d))*sin(5/4*pi + 5/2*arcta
n2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) + (-I*gamma(-5/2, 3*I*(d*x + c)*b/d
) + I*gamma(-5/2, -3*I*(d*x + c)*b/d))*sin(-5/4*pi + 5/2*arctan2(0, b) + 5/
2*arctan2(0, d/sqrt(d^2))))*cos(-3*(b*c - a*d)/d) + ((-I*gamma(-5/2, 3*I*(d
*x + c)*b/d) + I*gamma(-5/2, -3*I*(d*x + c)*b/d))*cos(5/4*pi + 5/2*arctan2(
0, b) + 5/2*arctan2(0, d/sqrt(d^2))) + (-I*gamma(-5/2, 3*I*(d*x + c)*b/d) +
 I*gamma(-5/2, -3*I*(d*x + c)*b/d))*cos(-5/4*pi + 5/2*arctan2(0, b) + 5/2*a
rctan2(0, d/sqrt(d^2))) + (gamma(-5/2, 3*I*(d*x + c)*b/d) + gamma(-5/2, -3*
I*(d*x + c)*b/d))*sin(5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^
2))) - (gamma(-5/2, 3*I*(d*x + c)*b/d) + gamma(-5/2, -3*I*(d*x + c)*b/d))*s
in(-5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))))*sin(-3*(b*c
- a*d)/d))*((d*x + c)*abs(b)/abs(d))^(5/2) + ((3*(gamma(-5/2, I*(d*x + c)*b
/d) + gamma(-5/2, -I*(d*x + c)*b/d))*cos(5/4*pi + 5/2*arctan2(0, b) + 5/2*a
rctan2(0, d/sqrt(d^2))) + 3*(gamma(-5/2, I*(d*x + c)*b/d) + gamma(-5/2, -I*
(d*x + c)*b/d))*cos(-5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2
```

```
))) + (3*I*gamma(-5/2, I*(d*x + c)*b/d) - 3*I*gamma(-5/2, -I*(d*x + c)*b/d)
)*sin(5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) + (-3*I*gam
ma(-5/2, I*(d*x + c)*b/d) + 3*I*gamma(-5/2, -I*(d*x + c)*b/d))*sin(-5/4*pi
+ 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))))*cos(-(b*c - a*d)/d) + (
(-3*I*gamma(-5/2, I*(d*x + c)*b/d) + 3*I*gamma(-5/2, -I*(d*x + c)*b/d))*cos
(5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) + (-3*I*gamma(-5
/2, I*(d*x + c)*b/d) + 3*I*gamma(-5/2, -I*(d*x + c)*b/d))*cos(-5/4*pi + 5/2
*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^2))) + 3*(gamma(-5/2, I*(d*x + c)*
b/d) + gamma(-5/2, -I*(d*x + c)*b/d))*sin(5/4*pi + 5/2*arctan2(0, b) + 5/2*
arctan2(0, d/sqrt(d^2))) - 3*(gamma(-5/2, I*(d*x + c)*b/d) + gamma(-5/2, -I
*(d*x + c)*b/d))*sin(-5/4*pi + 5/2*arctan2(0, b) + 5/2*arctan2(0, d/sqrt(d^
2))))*sin(-(b*c - a*d)/d))*((d*x + c)*abs(b)/abs(d))^(5/2))/((d*x + c)^(5/2
)*d)
```

Fricas [A] time = 2.531, size = 1229, normalized size = 3.45

$$\frac{2\left(3\sqrt{6}\left(\pi b^2 d^3 x^3+3\,\pi b^2 c d^2 x^2+3\,\pi b^2 c^2 d x+\pi b^2 c^3\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+\sqrt{2}\left(\pi b^2 d^3 x^3+3\,\pi b^2 c d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(3*sqrt(6)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi
*b^2*c^3)*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi
*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(s
qrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c
*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*fresnel_cos(sqrt(2
)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 3*sqrt(6)*(pi*b^2*d^3
*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*f
resnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + ((
12*b^2*d^2*x^2 + 24*b^2*c*d*x + 12*b^2*c^2 - d^2)*cos(b*x + a)^3 + 2*(b*d^2
*x + b*c*d)*cos(b*x + a)^2*sin(b*x + a) - 8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2)*cos(b*x + a))*sqrt(d*x + c))/(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x +
c^3*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(bx+a\right)^3}{(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3/(d*x + c)^(7/2), x)
```

3.63 $\int x^{3/2} \cos(x) \, dx$

Optimal. Leaf size=49

$$-\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)+x^{3/2}\sin(x)+\frac{3}{2}\sqrt{x}\cos(x)$$

```
[Out] (3*Sqrt[x]*Cos[x])/2 - (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[x]])/2 + x^(3
/2)*Sin[x]
```

Rubi [A] time = 0.0572119, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3296, 3304, 3352}

$$-\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)+x^{3/2}\sin(x)+\frac{3}{2}\sqrt{x}\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Cos[x],x]
```

```
[Out] (3*Sqrt[x]*Cos[x])/2 - (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[x]])/2 + x^(3
/2)*Sin[x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int x^{3/2}\cos(x)\,dx &= x^{3/2}\sin(x)-\frac{3}{2}\int\sqrt{x}\sin(x)\,dx\\
&= \frac{3}{2}\sqrt{x}\cos(x)+x^{3/2}\sin(x)-\frac{3}{4}\int\frac{\cos(x)}{\sqrt{x}}\,dx\\
&= \frac{3}{2}\sqrt{x}\cos(x)+x^{3/2}\sin(x)-\frac{3}{2}\,\text{Subst}\left(\int\cos\left(x^2\right)\,dx,x,\sqrt{x}\right)\\
&= \frac{3}{2}\sqrt{x}\cos(x)-\frac{3}{2}\sqrt{\frac{\pi}{2}}C\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)+x^{3/2}\sin(x)
\end{aligned}$$

Mathematica [C] time = 0.0133136, size = 55, normalized size = 1.12

$$\frac{\sqrt{x}\text{Gamma}\left(\frac{5}{2},-ix\right)}{2\sqrt{-ix}}+\frac{\sqrt{x}\text{Gamma}\left(\frac{5}{2},ix\right)}{2\sqrt{ix}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Cos[x],x]
```

```
[Out] (Sqrt[x]*Gamma[5/2, (-I)*x])/(2*Sqrt[(-I)*x]) + (Sqrt[x]*Gamma[5/2, I*x])/(
2*Sqrt[I*x])
```

Maple [A] time = 0.027, size = 34, normalized size = 0.7

$$x^{\frac{3}{2}}\sin\left(x\right)-\frac{3\sqrt{2}\sqrt{\pi}}{4}\text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\sqrt{x}\right)+\frac{3\cos\left(x\right)}{2}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*cos(x),x)
```

```
[Out] x^(3/2)*sin(x)-3/4*FresnelC(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)+3/2*
x^(1/2)*cos(x)
```

Maxima [C] time = 1.78026, size = 100, normalized size = 2.04

$$x^{\frac{3}{2}}\sin\left(x\right)+\frac{1}{32}\sqrt{\pi}\left((3i-3)\sqrt{2}\operatorname{erf}\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right)+(3i+3)\sqrt{2}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right)-(3i+3)\sqrt{2}\operatorname{erf}\left(\sqrt{-i}\sqrt{x}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*cos(x),x, algorithm="maxima")
```

```
[Out] x^(3/2)*sin(x) + 1/32*sqrt(pi)*((3*I - 3)*sqrt(2)*erf((1/2*I + 1/2)*sqrt(2)
*sqrt(x)) + (3*I + 3)*sqrt(2)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) - (3*I + 3
)*sqrt(2)*erf(sqrt(-I)*sqrt(x)) + (3*I - 3)*sqrt(2)*erf((-1)^(1/4)*sqrt(x))
) + 3/2*sqrt(x)*cos(x)
```

Fricas [A] time = 1.5896, size = 134, normalized size = 2.73

$$-\frac{3}{4}\sqrt{2}\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)+\frac{1}{2}\left(2x\sin\left(x\right)+3\cos\left(x\right)\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*cos(x),x, algorithm="fricas")
```

```
[Out] -3/4*sqrt(2)*sqrt(pi)*fresnel_cos(sqrt(2)*sqrt(x)/sqrt(pi)) + 1/2*(2*x*sin(
x) + 3*cos(x))*sqrt(x)
```

Sympy [A] time = 16.2414, size = 83, normalized size = 1.69

$$\frac{5x^{\frac{3}{2}}\sin(x)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{15\sqrt{x}\cos(x)\Gamma\left(\frac{5}{4}\right)}{8\Gamma\left(\frac{9}{4}\right)}-\frac{15\sqrt{2}\sqrt{\pi}C\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{5}{4}\right)}{16\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*cos(x),x)
```

```
[Out] 5*x**(3/2)*sin(x)*gamma(5/4)/(4*gamma(9/4)) + 15*sqrt(x)*cos(x)*gamma(5/4)/
(8*gamma(9/4)) - 15*sqrt(2)*sqrt(pi)*fresnelc(sqrt(2)*sqrt(x)/sqrt(pi))*gam
ma(5/4)/(16*gamma(9/4))
```

Giac [C] time = 1.14162, size = 93, normalized size = 1.9

$$\left(\frac{3}{16}i+\frac{3}{16}\right)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right)-\left(\frac{3}{16}i-\frac{3}{16}\right)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right)-\frac{1}{4}\left(2i\,x^{\frac{3}{2}}-3\sqrt{x}\right)e^{(i\,x)}-\frac{1}{4}\left(-2i\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*cos(x),x, algorithm="giac")
```

```
[Out] (3/16*I + 3/16)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) - (3/16
*I - 3/16)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(x)) - 1/4*(2*I*
x^(3/2) - 3*sqrt(x))*e^(I*x) - 1/4*(-2*I*x^(3/2) - 3*sqrt(x))*e^(-I*x)
```

3.64 $\int \sqrt{x}\cos(x)\,dx$

Optimal. Leaf size=36

$$\sqrt{x}\sin(x)-\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)$$

```
[Out] -(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[x]]) + Sqrt[x]*Sin[x]
```

Rubi [A] time = 0.034927, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3296, 3305, 3351}

$$\sqrt{x}\sin(x)-\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Cos[x],x]
```

```
[Out] -(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[x]]) + Sqrt[x]*Sin[x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\cos(x)\,dx &= \sqrt{x}\sin(x)-\frac{1}{2}\int \frac{\sin(x)}{\sqrt{x}}\,dx\\ &= \sqrt{x}\sin(x)-\text{Subst}\left(\int \sin\left(x^2\right)\,dx,x,\sqrt{x}\right)\\ &= -\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)+\sqrt{x}\sin(x)\end{aligned}$$

Mathematica [C] time = 0.0061006, size = 48, normalized size = 1.33

$$\frac{\sqrt{-ix}\,\text{Gamma}\left(\frac{3}{2},-ix\right)+\sqrt{ix}\,\text{Gamma}\left(\frac{3}{2},ix\right)}{2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Cos[x],x]
```

```
[Out] (Sqrt[(-I)*x]*Gamma[3/2, (-I)*x] + Sqrt[I*x]*Gamma[3/2, I*x])/(2*Sqrt[x])
```

Maple [A] time = 0.028, size = 27, normalized size = 0.8

$$-\frac{\sqrt{2}\sqrt{\pi}}{2}\mathrm{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\sqrt{x}\right)+\sin(x)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*cos(x),x)
```

```
[Out] -1/2*FresnelS(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)+sin(x)*x^(1/2)
```

Maxima [C] time = 1.74941, size = 90, normalized size = 2.5

$$-\frac{1}{16}\sqrt{\pi}\left((i+1)\sqrt{2}\operatorname{erf}\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right)+(i-1)\sqrt{2}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right)-(i-1)\sqrt{2}\operatorname{erf}\left(\sqrt{-i}\sqrt{x}\right)+(i+1)\sqrt{2}\operatorname{erf}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*cos(x),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(pi)*((I + 1)*sqrt(2)*erf((1/2*I + 1/2)*sqrt(2)*sqrt(x)) + (I - 1
)*sqrt(2)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) - (I - 1)*sqrt(2)*erf(sqrt(-I)
*sqrt(x)) + (I + 1)*sqrt(2)*erf((-1)^(1/4)*sqrt(x))) + sqrt(x)*sin(x)
```

Fricas [A] time = 1.62922, size = 105, normalized size = 2.92

$$-\frac{1}{2}\sqrt{2}\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)+\sqrt{x}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*cos(x),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt(pi)*fresnel_sin(sqrt(2)*sqrt(x)/sqrt(pi)) + sqrt(x)*sin(x
)
```

Sympy [A] time = 1.19293, size = 61, normalized size = 1.69

$$\frac{3\sqrt{x}\sin(x)\Gamma\left(\frac{3}{4}\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{3\sqrt{2}\sqrt{\pi}S\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{3}{4}\right)}{8\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*cos(x),x)
```

```
[Out] 3*sqrt(x)*sin(x)*gamma(3/4)/(4*gamma(7/4)) - 3*sqrt(2)*sqrt(pi)*fresnels(sq
rt(2)*sqrt(x)/sqrt(pi))*gamma(3/4)/(8*gamma(7/4))
```

Giac [C] time = 1.13463, size = 72, normalized size = 2.

$$-\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right)+\left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right)-\frac{1}{2}i\sqrt{x}e^{(ix)}+\frac{1}{2}i\sqrt{x}e^{(-ix)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*cos(x),x, algorithm="giac")
```

```
[Out] -(1/8*I - 1/8)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) + (1/8*I
 + 1/8)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(x)) - 1/2*I*sqrt(x
)*e^(I*x) + 1/2*I*sqrt(x)*e^(-I*x)
```

3.65 $\int \frac{\cos(x)}{\sqrt{x}} \, dx$

Optimal. Leaf size=24

$$\sqrt{2\pi}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)$$

```
[Out] Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*Sqrt[x]]
```

Rubi [A] time = 0.019832, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3304, 3352}

$$\sqrt{2\pi}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/Sqrt[x],x]
```

```
[Out] Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*Sqrt[x]]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{\sqrt{x}} \, dx &= 2\,\text{Subst}\left(\int \cos\left(x^2\right) \, dx, x, \sqrt{x}\right)\\ &= \sqrt{2\pi}C\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)\end{aligned}$$

Mathematica [C] time = 0.0066152, size = 51, normalized size = 2.12

$$-\frac{i\left(\sqrt{-ix}\text{Gamma}\left(\frac{1}{2}, -ix\right) - \sqrt{ix}\text{Gamma}\left(\frac{1}{2}, ix\right)\right)}{2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/Sqrt[x],x]
```

```
[Out] ((-I/2)*(Sqrt[(-I)*x]*Gamma[1/2, (-I)*x] - Sqrt[I*x]*Gamma[1/2, I*x]))/Sqrt
[x]
```

Maple [A] time = 0.026, size = 19, normalized size = 0.8

$$\mathrm{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\sqrt{x}\right)\sqrt{2}\sqrt{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/x^(1/2),x)
```

```
[Out] FresnelC(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)
```

Maxima [C] time = 1.8776, size = 81, normalized size = 3.38

$$-\frac{1}{8}\sqrt{\pi}\left((i-1)\sqrt{2}\,\mathrm{erf}\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right)+(i+1)\sqrt{2}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right)-(i+1)\sqrt{2}\,\mathrm{erf}\left(\sqrt{-i}\sqrt{x}\right)+(i-1)\sqrt{2}\,\mathrm{er}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*((I - 1)*sqrt(2)*erf((1/2*I + 1/2)*sqrt(2)*sqrt(x)) + (I + 1)
*sqrt(2)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) - (I + 1)*sqrt(2)*erf(sqrt(-I)*
sqrt(x)) + (I - 1)*sqrt(2)*erf((-1)^(1/4)*sqrt(x)))
```

Fricas [A] time = 1.6563, size = 76, normalized size = 3.17

$$\sqrt{2}\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*sqrt(pi)*fresnel_cos(sqrt(2)*sqrt(x)/sqrt(pi))
```

Sympy [A] time = 0.996044, size = 37, normalized size = 1.54

$$\frac{\sqrt{2}\sqrt{\pi}C\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(pi)*fresnelc(sqrt(2)*sqrt(x)/sqrt(pi))*gamma(1/4)/(4*gamma(5/4
))
```

Giac [C] time = 1.13746, size = 47, normalized size = 1.96

$$-\left(\frac{1}{4}i+\frac{1}{4}\right)\sqrt{2}\sqrt{\pi}\,\text{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right)+\left(\frac{1}{4}i-\frac{1}{4}\right)\sqrt{2}\sqrt{\pi}\,\text{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x^(1/2),x, algorithm="giac")
```

```
[Out] -(1/4*I + 1/4)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) + (1/4*I
 - 1/4)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(x))
```

3.66 $\int \frac{\cos(x)}{x^{3/2}}\,dx$

Optimal. Leaf size=35

$$-2\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)-\frac{2\cos(x)}{\sqrt{x}}$$

```
[Out] (-2*Cos[x])/Sqrt[x] - 2*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[x]]
```

Rubi [A] time = 0.0366712, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3297, 3305, 3351}

$$-2\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)-\frac{2\cos(x)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/x^(3/2),x]
```

```
[Out] (-2*Cos[x])/Sqrt[x] - 2*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{x^{3/2}}\,dx &= -\frac{2\cos(x)}{\sqrt{x}}-2\int \frac{\sin(x)}{\sqrt{x}}\,dx\\ &= -\frac{2\cos(x)}{\sqrt{x}}-4\,\text{Subst}\left(\int \sin\left(x^2\right)\,dx,x,\sqrt{x}\right)\\ &= -\frac{2\cos(x)}{\sqrt{x}}-2\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)\end{aligned}$$

Mathematica [C] time = 0.0423232, size = 63, normalized size = 1.8

$$\frac{\sqrt{-ix}\,\text{Gamma}\left(\frac{1}{2},-ix\right)+\sqrt{ix}\,\text{Gamma}\left(\frac{1}{2},ix\right)-e^{-ix}\left(1+e^{2ix}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/x^(3/2),x]
```

```
[Out] (-((1 + E^((2*I)*x))/E^(I*x)) + Sqrt[(-I)*x]*Gamma[1/2, (-I)*x] + Sqrt[I*x]
*Gamma[1/2, I*x])/Sqrt[x]
```

Maple [A] time = 0.028, size = 28, normalized size = 0.8

$$-2\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\sqrt{\pi}-2\,\frac{\cos(x)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/x^(3/2),x)
```

```
[Out] -2*FresnelS(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)-2*cos(x)/x^(1/2)
```

Maxima [C] time = 2.43777, size = 28, normalized size = 0.8

$$-\left(\frac{1}{4}i+\frac{1}{4}\right)\sqrt{2}\Gamma\left(-\frac{1}{2},ix\right)+\left(\frac{1}{4}i-\frac{1}{4}\right)\sqrt{2}\Gamma\left(-\frac{1}{2},-ix\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -(1/4*I + 1/4)*sqrt(2)*gamma(-1/2, I*x) + (1/4*I - 1/4)*sqrt(2)*gamma(-1/2,
 -I*x)
```

Fricas [A] time = 1.66506, size = 111, normalized size = 3.17

$$-\frac{2\left(\sqrt{2}\sqrt{\pi}x\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)+\sqrt{x}\cos(x)\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(2)*sqrt(pi)*x*fresnel_sin(sqrt(2)*sqrt(x)/sqrt(pi)) + sqrt(x)*cos(
x))/x
```

Sympy [A] time = 4.17337, size = 61, normalized size = 1.74

$$\frac{\sqrt{2}\sqrt{\pi}S\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(-\frac{1}{4}\right)}{2\Gamma\left(\frac{3}{4}\right)}+\frac{\cos(x)\Gamma\left(-\frac{1}{4}\right)}{2\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x**(3/2),x)
```

```
[Out] sqrt(2)*sqrt(pi)*fresnels(sqrt(2)*sqrt(x)/sqrt(pi))*gamma(-1/4)/(2*gamma(3/
4)) + cos(x)*gamma(-1/4)/(2*sqrt(x)*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(x)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(x)/x^(3/2), x)
```

3.67 $\int (c+dx)^{4/3}\cos(a+bx)\,dx$

Optimal. Leaf size=183

$$\frac{2id^2e^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gamma}\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{9b^3(c+dx)^{2/3}}-\frac{2id^2e^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gamma}\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{9b^3(c+dx)^{2/3}}+\frac{4d\sqrt[3]{c+dx}\cos(a+bx)}{3b^2}$$

```
[Out] (4*d*(c + d*x)^(1/3)*Cos[a + b*x])/(3*b^2) + (((2*I)/9)*d^2*E^(I*(a - (b*c)
/d))*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[1/3, ((-I)*b*(c + d*x))/d])/(b^3*(c
 + d*x)^(2/3)) - (((2*I)/9)*d^2*((I*b*(c + d*x))/d)^(2/3)*Gamma[1/3, (I*b*(
c + d*x))/d])/(b^3*E^(I*(a - (b*c)/d))*(c + d*x)^(2/3)) + ((c + d*x)^(4/3)*
Sin[a + b*x])/b
```

Rubi [A] time = 0.2404, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3296, 3307, 2181}

$$\frac{2id^2e^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gamma}\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{9b^3(c+dx)^{2/3}}-\frac{2id^2e^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gamma}\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{9b^3(c+dx)^{2/3}}+\frac{4d\sqrt[3]{c+dx}\cos(a+bx)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(4/3)*Cos[a + b*x],x]
```

```
[Out] (4*d*(c + d*x)^(1/3)*Cos[a + b*x])/(3*b^2) + (((2*I)/9)*d^2*E^(I*(a - (b*c)
/d))*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[1/3, ((-I)*b*(c + d*x))/d])/(b^3*(c
 + d*x)^(2/3)) - (((2*I)/9)*d^2*((I*b*(c + d*x))/d)^(2/3)*Gamma[1/3, (I*b*(
c + d*x))/d])/(b^3*E^(I*(a - (b*c)/d))*(c + d*x)^(2/3)) + ((c + d*x)^(4/3)*
Sin[a + b*x])/b
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int (c+dx)^{4/3}\cos(a+bx)\,dx = \frac{(c+dx)^{4/3}\sin(a+bx)}{b} - \frac{(4d)\int \sqrt[3]{c+dx}\sin(a+bx)\,dx}{3b}$$
$$= \frac{4d\sqrt[3]{c+dx}\cos(a+bx)}{3b^2} + \frac{(c+dx)^{4/3}\sin(a+bx)}{b} - \frac{(4d^2)\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}dx}{9b^2}$$
$$= \frac{4d\sqrt[3]{c+dx}\cos(a+bx)}{3b^2} + \frac{(c+dx)^{4/3}\sin(a+bx)}{b} - \frac{(2d^2)\int \frac{e^{-i(a+bx)}}{(c+dx)^{2/3}}dx}{9b^2} - \frac{(2d^2)\int \frac{e^{i(a+bx}}{(c+dx)^{2}}}{9b^2}$$
$$= \frac{4d\sqrt[3]{c+dx}\cos(a+bx)}{3b^2} + \frac{2id^2e^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{9b^3(c+dx)^{2/3}} - \frac{2id^2e^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+}{d}\right.}{9b^3(c}$$

Mathematica [A] time = 0.107263, size = 122, normalized size = 0.67

$$\frac{d\sqrt[3]{c+dx}e^{-\frac{i(ad+bc)}{d}}\left(\frac{e^{2ia}\text{Gamma}\left(\frac{7}{3},-\frac{ib(c+dx)}{d}\right)}{\sqrt[3]{-\frac{ib(c+dx)}{d}}}+\frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{7}{3},\frac{ib(c+dx)}{d}\right)}{\sqrt[3]{\frac{ib(c+dx)}{d}}}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(4/3)*Cos[a + b*x],x]
```

```
[Out] (d*(c + d*x)^(1/3)*((E^((2*I)*a)*Gamma[7/3, ((-I)*b*(c + d*x))/d])/(((-I)*b
*(c + d*x))/d)^(1/3) + (E^(((2*I)*b*c)/d)*Gamma[7/3, (I*b*(c + d*x))/d])/((
I*b*(c + d*x))/d)^(1/3)))/(2*b^2*E^((I*(b*c + a*d))/d))
```

Maple [F] time = 0.191, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{4}{3}}\cos(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(4/3)*cos(b*x+a),x)
```

```
[Out] int((d*x+c)^(4/3)*cos(b*x+a),x)
```

Maxima [B] time = 1.5645, size = 770, normalized size = 4.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(4/3)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/9*(9*(d*x + c)^(4/3)*b*d*((d*x + c)*abs(b)/abs(d))^(1/3)*sin(((d*x + c)*b
 - b*c + a*d)/d) + 12*(d*x + c)^(1/3)*d^2*((d*x + c)*abs(b)/abs(d))^(1/3)*c
os(((d*x + c)*b - b*c + a*d)/d) + (((gamma(1/3, I*(d*x + c)*b/d) + gamma(1/
3, -I*(d*x + c)*b/d))*cos(1/6*pi + 1/3*arctan2(0, b) + 1/3*arctan2(0, d/sqr
t(d^2))) + (gamma(1/3, I*(d*x + c)*b/d) + gamma(1/3, -I*(d*x + c)*b/d))*cos
(-1/6*pi + 1/3*arctan2(0, b) + 1/3*arctan2(0, d/sqrt(d^2))) + (-I*gamma(1/3
```

```
, I*(d*x + c)*b/d) + I*gamma(1/3, -I*(d*x + c)*b/d))*sin(1/6*pi + 1/3*arcta
n2(0, b) + 1/3*arctan2(0, d/sqrt(d^2))) + (I*gamma(1/3, I*(d*x + c)*b/d) -
I*gamma(1/3, -I*(d*x + c)*b/d))*sin(-1/6*pi + 1/3*arctan2(0, b) + 1/3*arcta
n2(0, d/sqrt(d^2))))*d^2*cos(-(b*c - a*d)/d) + ((-I*gamma(1/3, I*(d*x + c)*
b/d) + I*gamma(1/3, -I*(d*x + c)*b/d))*cos(1/6*pi + 1/3*arctan2(0, b) + 1/3
*arctan2(0, d/sqrt(d^2))) + (-I*gamma(1/3, I*(d*x + c)*b/d) + I*gamma(1/3,
-I*(d*x + c)*b/d))*cos(-1/6*pi + 1/3*arctan2(0, b) + 1/3*arctan2(0, d/sqrt(
d^2))) - (gamma(1/3, I*(d*x + c)*b/d) + gamma(1/3, -I*(d*x + c)*b/d))*sin(1
/6*pi + 1/3*arctan2(0, b) + 1/3*arctan2(0, d/sqrt(d^2))) + (gamma(1/3, I*(d
*x + c)*b/d) + gamma(1/3, -I*(d*x + c)*b/d))*sin(-1/6*pi + 1/3*arctan2(0, b
) + 1/3*arctan2(0, d/sqrt(d^2))))*d^2*sin(-(b*c - a*d)/d))*(d*x + c)^(1/3))
/(b^2*d*((d*x + c)*abs(b)/abs(d))^(1/3))
```

Fricas [A] time = 1.75519, size = 331, normalized size = 1.81

$$\frac{-2i\,d^2\left(\frac{i\,b}{d}\right)^{\frac{2}{3}}e^{\left(\frac{i\,bc-i\,ad}{d}\right)}\Gamma\left(\frac{1}{3},\frac{i\,bdx+i\,bc}{d}\right)+2i\,d^2\left(-\frac{i\,b}{d}\right)^{\frac{2}{3}}e^{\left(\frac{-i\,bc+i\,ad}{d}\right)}\Gamma\left(\frac{1}{3},\frac{-i\,bdx-i\,bc}{d}\right)+3\left(4\,bd\cos\left(bx+a\right)+3\left(b^2dx+b^2c\right)\sin\left(bx\right.\right.}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(4/3)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/9*(-2*I*d^2*(I*b/d)^(2/3)*e^((I*b*c - I*a*d)/d)*gamma(1/3, (I*b*d*x + I*b
*c)/d) + 2*I*d^2*(-I*b/d)^(2/3)*e^((-I*b*c + I*a*d)/d)*gamma(1/3, (-I*b*d*x
 - I*b*c)/d) + 3*(4*b*d*cos(b*x + a) + 3*(b^2*d*x + b^2*c)*sin(b*x + a))*(d
*x + c)^(1/3))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(4/3)*cos(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{4}{3}}\cos(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(4/3)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(4/3)*cos(b*x + a), x)
```

3.68 $\int (c+dx)^{2/3}\cos(a+bx)\,dx$

Optimal. Leaf size=152

$$\frac{de^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{3b^2\sqrt[3]{c+dx}}+\frac{de^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{3b^2\sqrt[3]{c+dx}}+\frac{(c+dx)^{2/3}\sin(a+bx)}{b}$$

```
[Out] (d*E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[2/3, ((-I)*b*(c +
 d*x))/d])/(3*b^2*(c + d*x)^(1/3)) + (d*((I*b*(c + d*x))/d)^(1/3)*Gamma[2/3
, (I*b*(c + d*x))/d])/(3*b^2*E^(I*(a - (b*c)/d))*(c + d*x)^(1/3)) + ((c + d
*x)^(2/3)*Sin[a + b*x])/b
```

Rubi [A] time = 0.148958, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3296, 3308, 2181}

$$\frac{de^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{3b^2\sqrt[3]{c+dx}}+\frac{de^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{3b^2\sqrt[3]{c+dx}}+\frac{(c+dx)^{2/3}\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(2/3)*Cos[a + b*x],x]
```

```
[Out] (d*E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[2/3, ((-I)*b*(c +
 d*x))/d])/(3*b^2*(c + d*x)^(1/3)) + (d*((I*b*(c + d*x))/d)^(1/3)*Gamma[2/3
, (I*b*(c + d*x))/d])/(3*b^2*E^(I*(a - (b*c)/d))*(c + d*x)^(1/3)) + ((c + d
*x)^(2/3)*Sin[a + b*x])/b
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int (c+dx)^{2/3}\cos(a+bx)\,dx = \frac{(c+dx)^{2/3}\sin(a+bx)}{b} - \frac{(2d)\int \frac{\sin(a+bx)}{\sqrt[3]{c+dx}}\,dx}{3b}$$
$$= \frac{(c+dx)^{2/3}\sin(a+bx)}{b} - \frac{(id)\int \frac{e^{-i(a+bx)}}{\sqrt[3]{c+dx}}\,dx}{3b} + \frac{(id)\int \frac{e^{i(a+bx)}}{\sqrt[3]{c+dx}}\,dx}{3b}$$
$$= \frac{de^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{3b^2\sqrt[3]{c+dx}} + \frac{de^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{3b^2\sqrt[3]{c+dx}} + \frac{(c+dx)^{2/3}\sin(a+}{b}$$

Mathematica [A] time = 0.118327, size = 124, normalized size = 0.82

$$-\frac{i(c+dx)^{2/3}e^{-\frac{i(ad+bc)}{d}}\left(\frac{e^{2ia}\text{Gamma}\left(\frac{5}{3},-\frac{ib(c+dx)}{d}\right)}{\left(-\frac{ib(c+dx)}{d}\right)^{2/3}}-\frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{5}{3},\frac{ib(c+dx)}{d}\right)}{\left(\frac{ib(c+dx)}{d}\right)^{2/3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(2/3)*Cos[a + b*x],x]
```

```
[Out] ((-I/2)*(c + d*x)^(2/3)*((E^((2*I)*a)*Gamma[5/3, ((-I)*b*(c + d*x))/d])/(((
-I)*b*(c + d*x))/d)^(2/3) - (E^(((2*I)*b*c)/d)*Gamma[5/3, (I*b*(c + d*x))/d
])/((I*b*(c + d*x))/d)^(2/3)))/(b*E^((I*(b*c + a*d))/d))
```

Maple [F] time = 0.178, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{2}{3}}\cos(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(2/3)*cos(b*x+a),x)
```

```
[Out] int((d*x+c)^(2/3)*cos(b*x+a),x)
```

Maxima [B] time = 1.59708, size = 701, normalized size = 4.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(2/3)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(6*(d*x + c)^(2/3)*d*((d*x + c)*abs(b)/abs(d))^(2/3)*sin(((d*x + c)*b -
 b*c + a*d)/d) + (((I*gamma(2/3, I*(d*x + c)*b/d) - I*gamma(2/3, -I*(d*x +
c)*b/d))*cos(1/3*pi + 2/3*arctan2(0, b) + 2/3*arctan2(0, d/sqrt(d^2))) + (I
*gamma(2/3, I*(d*x + c)*b/d) - I*gamma(2/3, -I*(d*x + c)*b/d))*cos(-1/3*pi
+ 2/3*arctan2(0, b) + 2/3*arctan2(0, d/sqrt(d^2))) + (gamma(2/3, I*(d*x + c
)*b/d) + gamma(2/3, -I*(d*x + c)*b/d))*sin(1/3*pi + 2/3*arctan2(0, b) + 2/3
*arctan2(0, d/sqrt(d^2))) - (gamma(2/3, I*(d*x + c)*b/d) + gamma(2/3, -I*(d
*x + c)*b/d))*sin(-1/3*pi + 2/3*arctan2(0, b) + 2/3*arctan2(0, d/sqrt(d^2))
```

```
))*d*cos(-(b*c - a*d)/d) + ((gamma(2/3, I*(d*x + c)*b/d) + gamma(2/3, -I*(d
*x + c)*b/d))*cos(1/3*pi + 2/3*arctan2(0, b) + 2/3*arctan2(0, d/sqrt(d^2)))
 + (gamma(2/3, I*(d*x + c)*b/d) + gamma(2/3, -I*(d*x + c)*b/d))*cos(-1/3*pi
 + 2/3*arctan2(0, b) + 2/3*arctan2(0, d/sqrt(d^2))) + (-I*gamma(2/3, I*(d*x
 + c)*b/d) + I*gamma(2/3, -I*(d*x + c)*b/d))*sin(1/3*pi + 2/3*arctan2(0, b)
 + 2/3*arctan2(0, d/sqrt(d^2))) + (I*gamma(2/3, I*(d*x + c)*b/d) - I*gamma(
2/3, -I*(d*x + c)*b/d))*sin(-1/3*pi + 2/3*arctan2(0, b) + 2/3*arctan2(0, d/
sqrt(d^2))))*d*sin(-(b*c - a*d)/d))*(d*x + c)^(2/3))/(b*d*((d*x + c)*abs(b)
/abs(d))^(2/3))
```

Fricas [A] time = 1.72673, size = 258, normalized size = 1.7

$$\frac{d\left(\frac{ib}{d}\right)^{\frac{1}{3}}e^{\left(\frac{ibc-iad}{d}\right)}\Gamma\left(\frac{2}{3},\frac{ibdx+ibc}{d}\right)+d\left(-\frac{ib}{d}\right)^{\frac{1}{3}}e^{\left(\frac{-ibc+iad}{d}\right)}\Gamma\left(\frac{2}{3},\frac{-ibdx-ibc}{d}\right)+3\,(dx+c)^{\frac{2}{3}}b\sin\left(bx+a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(2/3)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/3*(d*(I*b/d)^(1/3)*e^((I*b*c - I*a*d)/d)*gamma(2/3, (I*b*d*x + I*b*c)/d)
+ d*(-I*b/d)^(1/3)*e^((-I*b*c + I*a*d)/d)*gamma(2/3, (-I*b*d*x - I*b*c)/d)
+ 3*(d*x + c)^(2/3)*b*sin(b*x + a))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^{\frac{2}{3}}\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(2/3)*cos(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(2/3)*cos(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{2}{3}}\cos\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(2/3)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(2/3)*cos(b*x + a), x)
```

3.69 $\int \sqrt[3]{c+dx}\cos(a+bx)\,dx$

Optimal. Leaf size=152

$$\frac{de^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gamma}\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{6b^2(c+dx)^{2/3}}+\frac{de^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gamma}\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{6b^2(c+dx)^{2/3}}+\frac{\sqrt[3]{c+dx}\sin(a+bx)}{b}$$

```
[Out] (d*E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[1/3, ((-I)*b*(c +
 d*x))/d])/(6*b^2*(c + d*x)^(2/3)) + (d*((I*b*(c + d*x))/d)^(2/3)*Gamma[1/3
, (I*b*(c + d*x))/d])/(6*b^2*E^(I*(a - (b*c)/d))*(c + d*x)^(2/3)) + ((c + d
*x)^(1/3)*Sin[a + b*x])/b
```

Rubi [A] time = 0.161746, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3296, 3308, 2181}

$$\frac{de^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gamma}\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{6b^2(c+dx)^{2/3}}+\frac{de^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gamma}\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{6b^2(c+dx)^{2/3}}+\frac{\sqrt[3]{c+dx}\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)*Cos[a + b*x],x]
```


```
[Out] (d*E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[1/3, ((-I)*b*(c +
 d*x))/d])/(6*b^2*(c + d*x)^(2/3)) + (d*((I*b*(c + d*x))/d)^(2/3)*Gamma[1/3
, (I*b*(c + d*x))/d])/(6*b^2*E^(I*(a - (b*c)/d))*(c + d*x)^(2/3)) + ((c + d
*x)^(1/3)*Sin[a + b*x])/b
```


Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int \sqrt[3]{c+dx}\cos(a+bx)\,dx = \frac{\sqrt[3]{c+dx}\sin(a+bx)}{b} - \frac{d\int \frac{\sin(a+bx)}{(c+dx)^{2/3}}dx}{3b}$$
$$= \frac{\sqrt[3]{c+dx}\sin(a+bx)}{b} - \frac{(id)\int \frac{e^{-i(a+bx)}}{(c+dx)^{2/3}}dx}{6b} + \frac{(id)\int \frac{e^{i(a+bx)}}{(c+dx)^{2/3}}dx}{6b}$$
$$= \frac{de^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{6b^2(c+dx)^{2/3}} + \frac{de^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{6b^2(c+dx)^{2/3}} + \frac{\sqrt[3]{c+dx}\,\mathrm{si}}{b}$$

Mathematica [A] time = 0.0995348, size = 124, normalized size = 0.82

$$-\frac{i\sqrt[3]{c+dx}e^{-\frac{i(ad+bc)}{d}}\left(\frac{e^{2ia}\mathrm{Gamma}\left(\frac{4}{3},-\frac{ib(c+dx)}{d}\right)}{\sqrt[3]{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}}\mathrm{Gamma}\left(\frac{4}{3},\frac{ib(c+dx)}{d}\right)}{\sqrt[3]{\frac{ib(c+dx)}{d}}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)*Cos[a + b*x],x]
```

```
[Out] ((-I/2)*(c + d*x)^(1/3)*((E^((2*I)*a)*Gamma[4/3, ((-I)*b*(c + d*x))/d])/(((
-I)*b*(c + d*x))/d)^(1/3) - (E^(((2*I)*b*c)/d)*Gamma[4/3, (I*b*(c + d*x))/d
])/((I*b*(c + d*x))/d)^(1/3)))/(b*E^((I*(b*c + a*d))/d))
```

Maple [F] time = 0.174, size = 0, normalized size = 0.

$$\int \sqrt[3]{dx+c}\cos(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)*cos(b*x+a),x)
```

```
[Out] int((d*x+c)^(1/3)*cos(b*x+a),x)
```

Maxima [B] time = 1.558, size = 701, normalized size = 4.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/12*(12*(d*x + c)^(1/3)*d*((d*x + c)*abs(b)/abs(d))^(1/3)*sin(((d*x + c)*b
 - b*c + a*d)/d) + (((I*gamma(1/3, I*(d*x + c)*b/d) - I*gamma(1/3, -I*(d*x
+ c)*b/d))*cos(1/6*pi + 1/3*arctan2(0, b) + 1/3*arctan2(0, d/sqrt(d^2))) +
(I*gamma(1/3, I*(d*x + c)*b/d) - I*gamma(1/3, -I*(d*x + c)*b/d))*cos(-1/6*p
i + 1/3*arctan2(0, b) + 1/3*arctan2(0, d/sqrt(d^2))) + (gamma(1/3, I*(d*x +
 c)*b/d) + gamma(1/3, -I*(d*x + c)*b/d))*sin(1/6*pi + 1/3*arctan2(0, b) + 1
/3*arctan2(0, d/sqrt(d^2))) - (gamma(1/3, I*(d*x + c)*b/d) + gamma(1/3, -I*
(d*x + c)*b/d))*sin(-1/6*pi + 1/3*arctan2(0, b) + 1/3*arctan2(0, d/sqrt(d^2
```

```
))))*d*cos(-(b*c - a*d)/d) + ((gamma(1/3, I*(d*x + c)*b/d) + gamma(1/3, -I*
(d*x + c)*b/d))*cos(1/6*pi + 1/3*arctan2(0, b) + 1/3*arctan2(0, d/sqrt(d^2)
)) + (gamma(1/3, I*(d*x + c)*b/d) + gamma(1/3, -I*(d*x + c)*b/d))*cos(-1/6*
pi + 1/3*arctan2(0, b) + 1/3*arctan2(0, d/sqrt(d^2))) + (-I*gamma(1/3, I*(d
*x + c)*b/d) + I*gamma(1/3, -I*(d*x + c)*b/d))*sin(1/6*pi + 1/3*arctan2(0,
b) + 1/3*arctan2(0, d/sqrt(d^2))) + (I*gamma(1/3, I*(d*x + c)*b/d) - I*gamm
a(1/3, -I*(d*x + c)*b/d))*sin(-1/6*pi + 1/3*arctan2(0, b) + 1/3*arctan2(0,
d/sqrt(d^2))))*d*sin(-(b*c - a*d)/d))*(d*x + c)^(1/3))/(b*d*((d*x + c)*abs(
b)/abs(d))^(1/3))
```

Fricas [A] time = 1.78942, size = 258, normalized size = 1.7

$$\frac{d\left(\frac{ib}{d}\right)^{\frac{2}{3}} e^{\left(\frac{ibc-iad}{d}\right)}\Gamma\left(\frac{1}{3}, \frac{ibdx+ibc}{d}\right) + d\left(-\frac{ib}{d}\right)^{\frac{2}{3}} e^{\left(\frac{-ibc+iad}{d}\right)}\Gamma\left(\frac{1}{3}, \frac{-ibdx-ibc}{d}\right) + 6\,(dx+c)^{\frac{1}{3}}b\sin\left(bx+a\right)}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(d*(I*b/d)^(2/3)*e^((I*b*c - I*a*d)/d)*gamma(1/3, (I*b*d*x + I*b*c)/d)
+ d*(-I*b/d)^(2/3)*e^((-I*b*c + I*a*d)/d)*gamma(1/3, (-I*b*d*x - I*b*c)/d)
+ 6*(d*x + c)^(1/3)*b*sin(b*x + a))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{c+dx}\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)*cos(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(1/3)*cos(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{1}{3}}\cos\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)*cos(b*x + a), x)
```

3.70 $\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}} \, dx$

Optimal. Leaf size=135

$$\frac{ie^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\text{Gamma}\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}}-\frac{ie^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\text{Gamma}\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}}$$

```
[Out] ((-I/2)*E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[2/3, ((-I)*b
*(c + d*x))/d])/(b*(c + d*x)^(1/3)) + ((I/2)*((I*b*(c + d*x))/d)^(1/3)*Gamm
a[2/3, (I*b*(c + d*x))/d])/(b*E^(I*(a - (b*c)/d))*(c + d*x)^(1/3))
```

Rubi [A] time = 0.116229, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3307, 2181}

$$\frac{ie^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\text{Gamma}\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}}-\frac{ie^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\text{Gamma}\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x)^(1/3),x]
```

```
[Out] ((-I/2)*E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[2/3, ((-I)*b
*(c + d*x))/d])/(b*(c + d*x)^(1/3)) + ((I/2)*((I*b*(c + d*x))/d)^(1/3)*Gamm
a[2/3, (I*b*(c + d*x))/d])/(b*E^(I*(a - (b*c)/d))*(c + d*x)^(1/3))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}} \, dx &= \frac{1}{2}\int \frac{e^{-i(a+bx)}}{\sqrt[3]{c+dx}} \, dx+\frac{1}{2}\int \frac{e^{i(a+bx)}}{\sqrt[3]{c+dx}} \, dx \\
&= -\frac{ie^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}}+\frac{ie^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}}
\end{aligned}$$

Mathematica [A] time = 0.0601455, size = 124, normalized size = 0.92

$$\frac{ie^{-\frac{i(ad+bc)}{d}}\left(e^{\frac{2ibc}{d}}\sqrt[3]{\frac{ib(c+dx)}{d}}\mathrm{Gamma}\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)-e^{2ia}\sqrt[3]{-\frac{ib(c+dx)}{d}}\mathrm{Gamma}\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)\right)}{2b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(1/3),x]
```

```
[Out] ((I/2)*(-(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[2/3, ((-I)*b*(c +
d*x))/d]) + E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(1/3)*Gamma[2/3, (I*b*(c
+ d*x))/d]))/(b*E^((I*(b*c + a*d))/d)*(c + d*x)^(1/3))
```

Maple [F] time = 0.177, size = 0, normalized size = 0.

$$\int \cos(bx+a)\frac{1}{\sqrt[3]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^(1/3),x)
```

```
[Out] int(cos(b*x+a)/(d*x+c)^(1/3),x)
```

Maxima [B] time = 1.62036, size = 636, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/4*(d*x + c)^(2/3)*(((gamma(2/3, I*(d*x + c)*b/d) + gamma(2/3, -I*(d*x +
c)*b/d))*cos(1/3*pi + 2/3*arctan2(0, b) + 2/3*arctan2(0, d/sqrt(d^2))) + (g
amma(2/3, I*(d*x + c)*b/d) + gamma(2/3, -I*(d*x + c)*b/d))*cos(-1/3*pi + 2/
3*arctan2(0, b) + 2/3*arctan2(0, d/sqrt(d^2))) - (I*gamma(2/3, I*(d*x + c)*
b/d) - I*gamma(2/3, -I*(d*x + c)*b/d))*sin(1/3*pi + 2/3*arctan2(0, b) + 2/3
*arctan2(0, d/sqrt(d^2))) - (-I*gamma(2/3, I*(d*x + c)*b/d) + I*gamma(2/3,
-I*(d*x + c)*b/d))*sin(-1/3*pi + 2/3*arctan2(0, b) + 2/3*arctan2(0, d/sqrt(
d^2))))*cos(-(b*c - a*d)/d) - ((I*gamma(2/3, I*(d*x + c)*b/d) - I*gamma(2/3
, -I*(d*x + c)*b/d))*cos(1/3*pi + 2/3*arctan2(0, b) + 2/3*arctan2(0, d/sqrt
(d^2))) + (I*gamma(2/3, I*(d*x + c)*b/d) - I*gamma(2/3, -I*(d*x + c)*b/d))*
cos(-1/3*pi + 2/3*arctan2(0, b) + 2/3*arctan2(0, d/sqrt(d^2))) + (gamma(2/3
, I*(d*x + c)*b/d) + gamma(2/3, -I*(d*x + c)*b/d))*sin(1/3*pi + 2/3*arctan2
(0, b) + 2/3*arctan2(0, d/sqrt(d^2))) - (gamma(2/3, I*(d*x + c)*b/d) + gamm
a(2/3, -I*(d*x + c)*b/d))*sin(-1/3*pi + 2/3*arctan2(0, b) + 2/3*arctan2(0,
d/sqrt(d^2))))*sin(-(b*c - a*d)/d))/(d*((d*x + c)*abs(b)/abs(d))^(2/3))
```

Fricas [A] time = 1.73929, size = 208, normalized size = 1.54

$$\frac{i\left(\frac{ib}{d}\right)^{\frac{1}{3}}e^{\left(\frac{ibc-iad}{d}\right)}\Gamma\left(\frac{2}{3},\frac{ibdx+ibc}{d}\right)-i\left(-\frac{ib}{d}\right)^{\frac{1}{3}}e^{\left(\frac{-ibc+iad}{d}\right)}\Gamma\left(\frac{2}{3},\frac{-ibdx-ibc}{d}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(I*(I*b/d)^(1/3)*e^((I*b*c - I*a*d)/d)*gamma(2/3, (I*b*d*x + I*b*c)/d)
- I*(-I*b/d)^(1/3)*e^((-I*b*c + I*a*d)/d)*gamma(2/3, (-I*b*d*x - I*b*c)/d))
/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(d*x + c)^(1/3), x)
```

3.71 $\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=135

$$\frac{ie^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gamma}\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}}-\frac{ie^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gamma}\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}}$$

```
[Out] ((-I/2)*E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[1/3, ((-I)*b
*(c + d*x))/d])/(b*(c + d*x)^(2/3)) + ((I/2)*((I*b*(c + d*x))/d)^(2/3)*Gamm
a[1/3, (I*b*(c + d*x))/d])/(b*E^(I*(a - (b*c)/d))*(c + d*x)^(2/3))
```

Rubi [A] time = 0.119584, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3307, 2181}

$$\frac{ie^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gamma}\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}}-\frac{ie^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gamma}\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x)^(2/3),x]
```

```
[Out] ((-I/2)*E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[1/3, ((-I)*b
*(c + d*x))/d])/(b*(c + d*x)^(2/3)) + ((I/2)*((I*b*(c + d*x))/d)^(2/3)*Gamm
a[1/3, (I*b*(c + d*x))/d])/(b*E^(I*(a - (b*c)/d))*(c + d*x)^(2/3))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx &= \frac{1}{2}\int \frac{e^{-i(a+bx)}}{(c+dx)^{2/3}}\,dx+\frac{1}{2}\int \frac{e^{i(a+bx)}}{(c+dx)^{2/3}}\,dx\\ &= -\frac{ie^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}}+\frac{ie^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.0638611, size = 124, normalized size = 0.92

$$\frac{ie^{-\frac{i(ad+bc)}{d}}\left(e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gamma}\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)-e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gamma}\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)\right)}{2b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(2/3),x]
```

```
[Out] ((I/2)*(-(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[1/3, ((-I)*b*(c +
d*x))/d]) + E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(2/3)*Gamma[1/3, (I*b*(c
+ d*x))/d]))/(b*E^((I*(b*c + a*d))/d)*(c + d*x)^(2/3))
```

Maple [F] time = 0.165, size = 0, normalized size = 0.

$$\int \cos(bx+a)(dx+c)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^(2/3),x)
```

```
[Out] int(cos(b*x+a)/(d*x+c)^(2/3),x)
```

Maxima [B] time = 1.61575, size = 636, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/4*(d*x + c)^(1/3)*(((gamma(1/3, I*(d*x + c)*b/d) + gamma(1/3, -I*(d*x +
c)*b/d))*cos(1/6*pi + 1/3*arctan2(0, b) + 1/3*arctan2(0, d/sqrt(d^2))) + (g
amma(1/3, I*(d*x + c)*b/d) + gamma(1/3, -I*(d*x + c)*b/d))*cos(-1/6*pi + 1/
3*arctan2(0, b) + 1/3*arctan2(0, d/sqrt(d^2))) - (I*gamma(1/3, I*(d*x + c)*
b/d) - I*gamma(1/3, -I*(d*x + c)*b/d))*sin(1/6*pi + 1/3*arctan2(0, b) + 1/3
*arctan2(0, d/sqrt(d^2))) - (-I*gamma(1/3, I*(d*x + c)*b/d) + I*gamma(1/3,
-I*(d*x + c)*b/d))*sin(-1/6*pi + 1/3*arctan2(0, b) + 1/3*arctan2(0, d/sqrt(
d^2))))*cos(-(b*c - a*d)/d) - ((I*gamma(1/3, I*(d*x + c)*b/d) - I*gamma(1/3
, -I*(d*x + c)*b/d))*cos(1/6*pi + 1/3*arctan2(0, b) + 1/3*arctan2(0, d/sqrt
(d^2))) + (I*gamma(1/3, I*(d*x + c)*b/d) - I*gamma(1/3, -I*(d*x + c)*b/d))*
cos(-1/6*pi + 1/3*arctan2(0, b) + 1/3*arctan2(0, d/sqrt(d^2))) + (gamma(1/3
, I*(d*x + c)*b/d) + gamma(1/3, -I*(d*x + c)*b/d))*sin(1/6*pi + 1/3*arctan2
(0, b) + 1/3*arctan2(0, d/sqrt(d^2))) - (gamma(1/3, I*(d*x + c)*b/d) + gamm
a(1/3, -I*(d*x + c)*b/d))*sin(-1/6*pi + 1/3*arctan2(0, b) + 1/3*arctan2(0,
d/sqrt(d^2))))*sin(-(b*c - a*d)/d))/(d*((d*x + c)*abs(b)/abs(d))^(1/3))
```

Fricas [A] time = 1.66481, size = 208, normalized size = 1.54

$$\frac{i\left(\frac{ib}{d}\right)^{\frac{2}{3}}e^{\left(\frac{ibc-iad}{d}\right)}\Gamma\left(\frac{1}{3},\frac{ibdx+ibc}{d}\right)-i\left(-\frac{ib}{d}\right)^{\frac{2}{3}}e^{\left(\frac{-ibc+iad}{d}\right)}\Gamma\left(\frac{1}{3},\frac{-ibdx-ibc}{d}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/2*(I*(I*b/d)^(2/3)*e^((I*b*c - I*a*d)/d)*gamma(1/3, (I*b*d*x + I*b*c)/d)
- I*(-I*b/d)^(2/3)*e^((-I*b*c + I*a*d)/d)*gamma(1/3, (-I*b*d*x - I*b*c)/d))
/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)}{(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(d*x + c)^(2/3), x)
```

3.72 $\int \frac{\cos(a+bx)}{(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=151

$$\frac{3e^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2d\sqrt[3]{c+dx}}+\frac{3e^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{2d\sqrt[3]{c+dx}}-\frac{3\cos(a+bx)}{d\sqrt[3]{c+dx}}$$

```
[Out] (-3*Cos[a + b*x])/(d*(c + d*x)^(1/3)) + (3*E^(I*(a - (b*c)/d))*(((-I)*b*(c
+ d*x))/d)^(1/3)*Gamma[2/3, ((-I)*b*(c + d*x))/d])/(2*d*(c + d*x)^(1/3)) +
(3*((I*b*(c + d*x))/d)^(1/3)*Gamma[2/3, (I*b*(c + d*x))/d])/(2*d*E^(I*(a -
(b*c)/d))*(c + d*x)^(1/3))
```

Rubi [A] time = 0.146239, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3297, 3308, 2181}

$$\frac{3e^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2d\sqrt[3]{c+dx}}+\frac{3e^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{2d\sqrt[3]{c+dx}}-\frac{3\cos(a+bx)}{d\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x)^(4/3),x]
```

```
[Out] (-3*Cos[a + b*x])/(d*(c + d*x)^(1/3)) + (3*E^(I*(a - (b*c)/d))*(((-I)*b*(c
+ d*x))/d)^(1/3)*Gamma[2/3, ((-I)*b*(c + d*x))/d])/(2*d*(c + d*x)^(1/3)) +
(3*((I*b*(c + d*x))/d)^(1/3)*Gamma[2/3, (I*b*(c + d*x))/d])/(2*d*E^(I*(a -
(b*c)/d))*(c + d*x)^(1/3))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{(c+dx)^{4/3}}\,dx = -\frac{3\cos(a+bx)}{d\sqrt[3]{c+dx}} - \frac{(3b)\int \frac{\sin(a+bx)}{\sqrt[3]{c+dx}}\,dx}{d}$$
$$= -\frac{3\cos(a+bx)}{d\sqrt[3]{c+dx}} - \frac{(3ib)\int \frac{e^{-i(a+bx)}}{\sqrt[3]{c+dx}}\,dx}{2d} + \frac{(3ib)\int \frac{e^{i(a+bx)}}{\sqrt[3]{c+dx}}\,dx}{2d}$$
$$= -\frac{3\cos(a+bx)}{d\sqrt[3]{c+dx}} + \frac{3e^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2d\sqrt[3]{c+dx}} + \frac{3e^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{2d\sqrt[3]{c+dx}}$$

Mathematica [A] time = 0.0539572, size = 121, normalized size = 0.8

$$-\frac{e^{-\frac{i(ad+bc)}{d}}\left(e^{2ia}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\mathrm{Gamma}\left(-\frac{1}{3},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\mathrm{Gamma}\left(-\frac{1}{3},\frac{ib(c+dx)}{d}\right)\right)}{2d\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(4/3),x]
```

```
[Out] -(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[-1/3, ((-I)*b*(c + d*x))/d
] + E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(1/3)*Gamma[-1/3, (I*b*(c + d*x))
/d])/(2*d*E^((I*(b*c + a*d))/d)*(c + d*x)^(1/3))
```

Maple [F] time = 0.181, size = 0, normalized size = 0.

$$\int \cos(bx+a)(dx+c)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^(4/3),x)
```

```
[Out] int(cos(b*x+a)/(d*x+c)^(4/3),x)
```

Maxima [B] time = 1.47262, size = 632, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] -1/4*(((gamma(-1/3, I*(d*x + c)*b/d) + gamma(-1/3, -I*(d*x + c)*b/d))*cos(1
/6*pi + 1/3*arctan2(0, b) + 1/3*arctan2(0, d/sqrt(d^2))) + (gamma(-1/3, I*(
d*x + c)*b/d) + gamma(-1/3, -I*(d*x + c)*b/d))*cos(-1/6*pi + 1/3*arctan2(0,
 b) + 1/3*arctan2(0, d/sqrt(d^2))) + (I*gamma(-1/3, I*(d*x + c)*b/d) - I*ga
mma(-1/3, -I*(d*x + c)*b/d))*sin(1/6*pi + 1/3*arctan2(0, b) + 1/3*arctan2(0
, d/sqrt(d^2))) + (-I*gamma(-1/3, I*(d*x + c)*b/d) + I*gamma(-1/3, -I*(d*x
+ c)*b/d))*sin(-1/6*pi + 1/3*arctan2(0, b) + 1/3*arctan2(0, d/sqrt(d^2))))*
cos(-(b*c - a*d)/d) + ((-I*gamma(-1/3, I*(d*x + c)*b/d) + I*gamma(-1/3, -I*
```

```
(d*x + c)*b/d))*cos(1/6*pi + 1/3*arctan2(0, b) + 1/3*arctan2(0, d/sqrt(d^2)
)) + (-I*gamma(-1/3, I*(d*x + c)*b/d) + I*gamma(-1/3, -I*(d*x + c)*b/d))*co
s(-1/6*pi + 1/3*arctan2(0, b) + 1/3*arctan2(0, d/sqrt(d^2))) + (gamma(-1/3,
 I*(d*x + c)*b/d) + gamma(-1/3, -I*(d*x + c)*b/d))*sin(1/6*pi + 1/3*arctan2
(0, b) + 1/3*arctan2(0, d/sqrt(d^2))) - (gamma(-1/3, I*(d*x + c)*b/d) + gam
ma(-1/3, -I*(d*x + c)*b/d))*sin(-1/6*pi + 1/3*arctan2(0, b) + 1/3*arctan2(0
, d/sqrt(d^2))))*sin(-(b*c - a*d)/d))*((d*x + c)*abs(b)/abs(d))^(1/3)/((d*x
 + c)^(1/3)*d)
```

Fricas [A] time = 1.7487, size = 290, normalized size = 1.92

$$\frac{3\left((dx+c)\left(\frac{ib}{d}\right)^{\frac{1}{3}}e^{\left(\frac{ibc-iad}{d}\right)}\Gamma\left(\frac{2}{3},\frac{ibdx+ibc}{d}\right)+(dx+c)\left(-\frac{ib}{d}\right)^{\frac{1}{3}}e^{\left(\frac{-ibc+iad}{d}\right)}\Gamma\left(\frac{2}{3},\frac{-ibdx-ibc}{d}\right)-2\,(dx+c)^{\frac{2}{3}}\cos\left(bx+a\right)\right)}{2\left(d^2x+cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/2*((d*x + c)*(I*b/d)^(1/3)*e^((I*b*c - I*a*d)/d)*gamma(2/3, (I*b*d*x + I*
b*c)/d) + (d*x + c)*(-I*b/d)^(1/3)*e^((-I*b*c + I*a*d)/d)*gamma(2/3, (-I*b*
d*x - I*b*c)/d) - 2*(d*x + c)^(2/3)*cos(b*x + a))/(d^2*x + c*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(a+bx\right)}{(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)**(4/3),x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x)**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(bx+a\right)}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(d*x + c)^(4/3), x)
```

3.73 $\int \frac{\cos(a+bx)}{(c+dx)^{5/3}} dx$

Optimal. Leaf size=153

$$\frac{3e^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gamma}\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{4d(c+dx)^{2/3}}+\frac{3e^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gamma}\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{4d(c+dx)^{2/3}}-\frac{3\cos(a+bx)}{2d(c+dx)^{2/3}}$$

```
[Out] (-3*Cos[a + b*x])/(2*d*(c + d*x)^(2/3)) + (3*E^(I*(a - (b*c)/d))*(((-I)*b*(
c + d*x))/d)^(2/3)*Gamma[1/3, ((-I)*b*(c + d*x))/d])/(4*d*(c + d*x)^(2/3))
+ (3*((I*b*(c + d*x))/d)^(2/3)*Gamma[1/3, (I*b*(c + d*x))/d])/(4*d*E^(I*(a
- (b*c)/d))*(c + d*x)^(2/3))
```

Rubi [A] time = 0.154681, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3297, 3308, 2181}

$$\frac{3e^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gamma}\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{4d(c+dx)^{2/3}}+\frac{3e^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gamma}\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{4d(c+dx)^{2/3}}-\frac{3\cos(a+bx)}{2d(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x)^(5/3),x]
```

```
[Out] (-3*Cos[a + b*x])/(2*d*(c + d*x)^(2/3)) + (3*E^(I*(a - (b*c)/d))*(((-I)*b*(
c + d*x))/d)^(2/3)*Gamma[1/3, ((-I)*b*(c + d*x))/d])/(4*d*(c + d*x)^(2/3))
+ (3*((I*b*(c + d*x))/d)^(2/3)*Gamma[1/3, (I*b*(c + d*x))/d])/(4*d*E^(I*(a
- (b*c)/d))*(c + d*x)^(2/3))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/3}}\,dx = -\frac{3\cos(a+bx)}{2d(c+dx)^{2/3}} - \frac{(3b)\int \frac{\sin(a+bx)}{(c+dx)^{2/3}}\,dx}{2d}$$
$$= -\frac{3\cos(a+bx)}{2d(c+dx)^{2/3}} - \frac{(3ib)\int \frac{e^{-i(a+bx)}}{(c+dx)^{2/3}}\,dx}{4d} + \frac{(3ib)\int \frac{e^{i(a+bx)}}{(c+dx)^{2/3}}\,dx}{4d}$$
$$= -\frac{3\cos(a+bx)}{2d(c+dx)^{2/3}} + \frac{3e^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{4d(c+dx)^{2/3}} + \frac{3e^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{4d(c+dx)^{2/3}}$$

Mathematica [A] time = 0.0569638, size = 121, normalized size = 0.79

$$-\frac{e^{-\frac{i(ad+bc)}{d}}\left(e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gamma}\left(-\frac{2}{3},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\text{Gamma}\left(-\frac{2}{3},\frac{ib(c+dx)}{d}\right)\right)}{2d(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(5/3),x]
```

```
[Out] -(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[-2/3, ((-I)*b*(c + d*x))/d
] + E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(2/3)*Gamma[-2/3, (I*b*(c + d*x))
/d])/(2*d*E^((I*(b*c + a*d))/d)*(c + d*x)^(2/3))
```

Maple [F] time = 0.183, size = 0, normalized size = 0.

$$\int \cos(bx+a)\,(dx+c)^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^(5/3),x)
```

```
[Out] int(cos(b*x+a)/(d*x+c)^(5/3),x)
```

Maxima [B] time = 1.55815, size = 632, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(5/3),x, algorithm="maxima")
```

```
[Out] -1/4*(((gamma(-2/3, I*(d*x + c)*b/d) + gamma(-2/3, -I*(d*x + c)*b/d))*cos(1
/3*pi + 2/3*arctan2(0, b) + 2/3*arctan2(0, d/sqrt(d^2))) + (gamma(-2/3, I*(
d*x + c)*b/d) + gamma(-2/3, -I*(d*x + c)*b/d))*cos(-1/3*pi + 2/3*arctan2(0,
 b) + 2/3*arctan2(0, d/sqrt(d^2))) + (I*gamma(-2/3, I*(d*x + c)*b/d) - I*ga
mma(-2/3, -I*(d*x + c)*b/d))*sin(1/3*pi + 2/3*arctan2(0, b) + 2/3*arctan2(0
, d/sqrt(d^2))) + (-I*gamma(-2/3, I*(d*x + c)*b/d) + I*gamma(-2/3, -I*(d*x
+ c)*b/d))*sin(-1/3*pi + 2/3*arctan2(0, b) + 2/3*arctan2(0, d/sqrt(d^2))))*
cos(-(b*c - a*d)/d) + ((-I*gamma(-2/3, I*(d*x + c)*b/d) + I*gamma(-2/3, -I*
(d*x + c)*b/d))*cos(1/3*pi + 2/3*arctan2(0, b) + 2/3*arctan2(0, d/sqrt(d^2)
```

```
)) + (-I*gamma(-2/3, I*(d*x + c)*b/d) + I*gamma(-2/3, -I*(d*x + c)*b/d))*co
s(-1/3*pi + 2/3*arctan2(0, b) + 2/3*arctan2(0, d/sqrt(d^2))) + (gamma(-2/3,
 I*(d*x + c)*b/d) + gamma(-2/3, -I*(d*x + c)*b/d))*sin(1/3*pi + 2/3*arctan2
(0, b) + 2/3*arctan2(0, d/sqrt(d^2))) - (gamma(-2/3, I*(d*x + c)*b/d) + gam
ma(-2/3, -I*(d*x + c)*b/d))*sin(-1/3*pi + 2/3*arctan2(0, b) + 2/3*arctan2(0
, d/sqrt(d^2))))*sin(-(b*c - a*d)/d))*((d*x + c)*abs(b)/abs(d))^(2/3)/((d*x
 + c)^(2/3)*d)
```

Fricas [A] time = 1.73329, size = 290, normalized size = 1.9

$$\frac{3\left((dx+c)\left(\frac{ib}{d}\right)^{\frac{2}{3}}e^{\left(\frac{ibc-iad}{d}\right)}\Gamma\left(\frac{1}{3},\frac{ibdx+ibc}{d}\right)+(dx+c)\left(-\frac{ib}{d}\right)^{\frac{2}{3}}e^{\left(\frac{-ibc+iad}{d}\right)}\Gamma\left(\frac{1}{3},\frac{-ibdx-ibc}{d}\right)-2(dx+c)^{\frac{1}{3}}\cos(bx+a)\right)}{4\left(d^2x+cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(5/3),x, algorithm="fricas")
```

```
[Out] 3/4*((d*x + c)*(I*b/d)^(2/3)*e^((I*b*c - I*a*d)/d)*gamma(1/3, (I*b*d*x + I*
b*c)/d) + (d*x + c)*(-I*b/d)^(2/3)*e^((-I*b*c + I*a*d)/d)*gamma(1/3, (-I*b*
d*x - I*b*c)/d) - 2*(d*x + c)^(1/3)*cos(b*x + a))/(d^2*x + c*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)}{(c+dx)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)**(5/3),x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x)**(5/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)}{(dx+c)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(5/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(d*x + c)^(5/3), x)
```

3.74 $\int \frac{\cos(a+bx)}{(c+dx)^{7/3}}\,dx$

Optimal. Leaf size=182

$$\frac{9ibe^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\text{Gamma}\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{8d^2\sqrt[3]{c+dx}}-\frac{9ibe^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\text{Gamma}\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{8d^2\sqrt[3]{c+dx}}+\frac{9b\sin(a+bx)}{4d^2\sqrt[3]{c+dx}}-\frac{3\cos(a+}{4d(c+dx}$$

```
[Out] (-3*Cos[a + b*x])/(4*d*(c + d*x)^(4/3)) + (((9*I)/8)*b*E^(I*(a - (b*c)/d))*
(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[2/3, ((-I)*b*(c + d*x))/d])/(d^2*(c + d*
x)^(1/3)) - (((9*I)/8)*b*((I*b*(c + d*x))/d)^(1/3)*Gamma[2/3, (I*b*(c + d*x
))/d])/(d^2*E^(I*(a - (b*c)/d))*(c + d*x)^(1/3)) + (9*b*Sin[a + b*x])/(4*d^
2*(c + d*x)^(1/3))
```

Rubi [A] time = 0.204047, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3297, 3307, 2181}

$$\frac{9ibe^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\text{Gamma}\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{8d^2\sqrt[3]{c+dx}}-\frac{9ibe^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\text{Gamma}\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{8d^2\sqrt[3]{c+dx}}+\frac{9b\sin(a+bx)}{4d^2\sqrt[3]{c+dx}}-\frac{3\cos(a+}{4d(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x)^(7/3),x]
```


```
[Out] (-3*Cos[a + b*x])/(4*d*(c + d*x)^(4/3)) + (((9*I)/8)*b*E^(I*(a - (b*c)/d))*
(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[2/3, ((-I)*b*(c + d*x))/d])/(d^2*(c + d*
x)^(1/3)) - (((9*I)/8)*b*((I*b*(c + d*x))/d)^(1/3)*Gamma[2/3, (I*b*(c + d*x
))/d])/(d^2*E^(I*(a - (b*c)/d))*(c + d*x)^(1/3)) + (9*b*Sin[a + b*x])/(4*d^
2*(c + d*x)^(1/3))
```


Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/3}}\,dx = -\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}} - \frac{(3b)\int \frac{\sin(a+bx)}{(c+dx)^{4/3}}\,dx}{4d}$$

$$= -\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}} + \frac{9b\sin(a+bx)}{4d^2\sqrt[3]{c+dx}} - \frac{\left(9b^2\right)\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx}{4d^2}$$

$$= -\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}} + \frac{9b\sin(a+bx)}{4d^2\sqrt[3]{c+dx}} - \frac{\left(9b^2\right)\int \frac{e^{-i(a+bx)}}{\sqrt[3]{c+dx}}\,dx}{8d^2} - \frac{\left(9b^2\right)\int \frac{e^{i(a+bx)}}{\sqrt[3]{c+dx}}\,dx}{8d^2}$$

$$= -\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}} + \frac{9ibe^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{8d^2\sqrt[3]{c+dx}} - \frac{9ibe^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{8d^2\sqrt[3]{c+dx}} + \frac{9b\sin}{4d^2\sqrt[3]{}}$$

Mathematica [A] time = 0.053829, size = 125, normalized size = 0.69

$$\frac{ibe^{-\frac{i(ad+bc)}{d}}\left(e^{2ia}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(-\frac{4}{3},-\frac{ib(c+dx)}{d}\right)-e^{\frac{2ibc}{d}}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\text{Gamma}\left(-\frac{4}{3},\frac{ib(c+dx)}{d}\right)\right)}{2d^2\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(7/3),x]
```

```
[Out] ((I/2)*b*(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[-4/3, ((-I)*b*(c +
 d*x))/d] - E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(1/3)*Gamma[-4/3, (I*b*(c
 + d*x))/d]))/(d^2*E^((I*(b*c + a*d))/d)*(c + d*x)^(1/3))
```

Maple [F] time = 0.186, size = 0, normalized size = 0.

$$\int \cos(bx+a)\,(dx+c)^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x+c)^(7/3),x)
```

```
[Out] int(cos(b*x+a)/(d*x+c)^(7/3),x)
```

Maxima [B] time = 1.49109, size = 632, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(7/3),x, algorithm="maxima")
```

```
[Out] -1/4*(((gamma(-4/3, I*(d*x + c)*b/d) + gamma(-4/3, -I*(d*x + c)*b/d))*cos(2
/3*pi + 4/3*arctan2(0, b) + 4/3*arctan2(0, d/sqrt(d^2))) + (gamma(-4/3, I*(
d*x + c)*b/d) + gamma(-4/3, -I*(d*x + c)*b/d))*cos(-2/3*pi + 4/3*arctan2(0,
 b) + 4/3*arctan2(0, d/sqrt(d^2))) + (I*gamma(-4/3, I*(d*x + c)*b/d) - I*ga
mma(-4/3, -I*(d*x + c)*b/d))*sin(2/3*pi + 4/3*arctan2(0, b) + 4/3*arctan2(0
, d/sqrt(d^2))) + (-I*gamma(-4/3, I*(d*x + c)*b/d) + I*gamma(-4/3, -I*(d*x
```

```
+ c)*b/d))*sin(-2/3*pi + 4/3*arctan2(0, b) + 4/3*arctan2(0, d/sqrt(d^2))))*
cos(-(b*c - a*d)/d) + ((-I*gamma(-4/3, I*(d*x + c)*b/d) + I*gamma(-4/3, -I*
(d*x + c)*b/d))*cos(2/3*pi + 4/3*arctan2(0, b) + 4/3*arctan2(0, d/sqrt(d^2)
)) + (-I*gamma(-4/3, I*(d*x + c)*b/d) + I*gamma(-4/3, -I*(d*x + c)*b/d))*co
s(-2/3*pi + 4/3*arctan2(0, b) + 4/3*arctan2(0, d/sqrt(d^2))) + (gamma(-4/3,
 I*(d*x + c)*b/d) + gamma(-4/3, -I*(d*x + c)*b/d))*sin(2/3*pi + 4/3*arctan2
(0, b) + 4/3*arctan2(0, d/sqrt(d^2))) - (gamma(-4/3, I*(d*x + c)*b/d) + gam
ma(-4/3, -I*(d*x + c)*b/d))*sin(-2/3*pi + 4/3*arctan2(0, b) + 4/3*arctan2(0
, d/sqrt(d^2))))*sin(-(b*c - a*d)/d))*((d*x + c)*abs(b)/abs(d))^(4/3)/((d*x
 + c)^(4/3)*d)
```

Fricas [A] time = 1.86642, size = 452, normalized size = 2.48

$$\frac{\left(-9i\,bd^2x^2 - 18i\,bcdx - 9i\,bc^2\right)\left(\frac{i\,b}{d}\right)^{\frac{1}{3}} e^{\left(\frac{i\,bc - i\,ad}{d}\right)}\Gamma\left(\frac{2}{3}, \frac{i\,bdx + i\,bc}{d}\right) + \left(9i\,bd^2x^2 + 18i\,bcdx + 9i\,bc^2\right)\left(-\frac{i\,b}{d}\right)^{\frac{1}{3}} e^{\left(\frac{-i\,bc + i\,ad}{d}\right)}\Gamma\left(\frac{2}{3}, \frac{-i\,ba}{}\right.}{8\left(d^4x^2 + 2\,cd^3x + c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(7/3),x, algorithm="fricas")
```

```
[Out] 1/8*((-9*I*b*d^2*x^2 - 18*I*b*c*d*x - 9*I*b*c^2)*(I*b/d)^(1/3)*e^((I*b*c -
I*a*d)/d)*gamma(2/3, (I*b*d*x + I*b*c)/d) + (9*I*b*d^2*x^2 + 18*I*b*c*d*x +
 9*I*b*c^2)*(-I*b/d)^(1/3)*e^((-I*b*c + I*a*d)/d)*gamma(2/3, (-I*b*d*x - I*
b*c)/d) - 6*(d*x + c)^(2/3)*(d*cos(b*x + a) - 3*(b*d*x + b*c)*sin(b*x + a))
)/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)**(7/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(bx + a\right)}{\left(dx + c\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x+c)^(7/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(d*x + c)^(7/3), x)
```

3.75 $\int x\sqrt{\cos(a+bx)}\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(x\sqrt{\cos(a+bx)},x\right)$$

```
[Out] Unintegrable[x*Sqrt[Cos[a + b*x]], x]
```

Rubi [A] time = 0.0173204, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\sqrt{\cos(a+bx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*Sqrt[Cos[a + b*x]],x]
```

```
[Out] Defer[Int][x*Sqrt[Cos[a + b*x]], x]
```

Rubi steps

$$\int x\sqrt{\cos(a+bx)}\,dx=\int x\sqrt{\cos(a+bx)}\,dx$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[x*Sqrt[Cos[a + b*x]],x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.464, size = 0, normalized size = 0.

$$\int x\sqrt{\cos(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)^(1/2),x)
```

```
[Out] int(x*cos(b*x+a)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\cos(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(cos(b*x + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\cos{(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sqrt(cos(a + b*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\cos{(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(cos(b*x + a)), x)
```

3.76 $\int \sqrt{\cos(a+bx)}\,dx$

Optimal. Leaf size=16

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

```
[Out] (2*EllipticE[(a + b*x)/2, 2])/b
```

Rubi [A] time = 0.008924, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2639}

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[a + b*x]],x]
```

```
[Out] (2*EllipticE[(a + b*x)/2, 2])/b
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(a+bx)}\,dx = \frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Mathematica [A] time = 0.0276763, size = 16, normalized size = 1.

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[a + b*x]],x]
```

```
[Out] (2*EllipticE[(a + b*x)/2, 2])/b
```

Maple [B] time = 1.411, size = 133, normalized size = 8.3

$$2\,\frac{\sqrt{\left(2\,\left(\cos\left(1/2\,bx+a/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,bx+a/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,bx+a/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/2\,bx+a/2\right)\right)^{2}+1}\mathrm{EllipticE}\left(\cos\left(1\right.\right.}{\sqrt{-2\,\left(\sin\left(1/2\,bx+a/2\right)\right)^{4}+\left(\sin\left(1/2\,bx+a/2\right)\right)^{2}}\sin\left(1/2\,bx+a/2\right)\sqrt{2\,\left(\cos\left(1/2\,bx+a/2\right)\right)^{2}-1}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(1/2),x)
```

```
[Out] 2*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(sin(1/2*b*x+1/2*
a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),
2^(1/2))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/sin(1/2*b*x+1
/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\cos(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{\cos(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\cos(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(cos(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\cos(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(b*x + a)), x)
```

3.77 $\int \frac{\sqrt{\cos(a+bx)}}{x} dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\sqrt{\cos(a+bx)}}{x},x\right)$$

```
[Out] Unintegrable[Sqrt[Cos[a + b*x]]/x, x]
```

Rubi [A] time = 0.0288276, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\cos(a+bx)}}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[Cos[a + b*x]]/x,x]
```

```
[Out] Defer[Int][Sqrt[Cos[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(a+bx)}}{x} dx = \int \frac{\sqrt{\cos(a+bx)}}{x} dx$$

Mathematica [A] time = 0.70402, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(a+bx)}}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[Cos[a + b*x]]/x,x]
```

```
[Out] Integrate[Sqrt[Cos[a + b*x]]/x, x]
```

Maple [A] time = 0.141, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{\cos(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(1/2)/x,x)
```

```
[Out] int(cos(b*x+a)^(1/2)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(bx+a)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(b*x + a))/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(a+bx)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(cos(a + b*x))/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(bx+a)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(b*x + a))/x, x)
```

3.78 $\int x \cos^{\frac{3}{2}}(a + bx)\, dx$

Optimal. Leaf size=60

$$\frac{1}{3}\text{Unintegrable}\left(\frac{x}{\sqrt{\cos(a+bx)}},x\right)+\frac{4\cos^{\frac{3}{2}}(a+bx)}{9b^2}+\frac{2x\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}$$

```
[Out] (4*Cos[a + b*x]^(3/2))/(9*b^2) + (2*x*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(3*b
) + Unintegrable[x/Sqrt[Cos[a + b*x]], x]/3
```

Rubi [A] time = 0.0377989, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\cos^{\frac{3}{2}}(a+bx)\, dx$$

Verification is Not applicable to the result.

```
[In] Int[x*Cos[a + b*x]^(3/2),x]
```

```
[Out] (4*Cos[a + b*x]^(3/2))/(9*b^2) + (2*x*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(3*b
) + Defer[Int][x/Sqrt[Cos[a + b*x]], x]/3
```

Rubi steps

$$\int x\cos^{\frac{3}{2}}(a+bx)\, dx=\frac{4\cos^{\frac{3}{2}}(a+bx)}{9b^2}+\frac{2x\sqrt{\cos(a+bx)}\sin(a+bx)}{3b}+\frac{1}{3}\int\frac{x}{\sqrt{\cos(a+bx)}}\, dx$$

Mathematica [A] time = 2.02598, size = 0, normalized size = 0.

$$\int x\cos^{\frac{3}{2}}(a+bx)\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*Cos[a + b*x]^(3/2),x]
```

```
[Out] Integrate[x*Cos[a + b*x]^(3/2), x]
```

Maple [A] time = 0.122, size = 0, normalized size = 0.

$$\int x\left(\cos\left(bx+a\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)^(3/2),x)
```

```
[Out] int(x*cos(b*x+a)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x\cos(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x\cos(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)^(3/2), x)
```

3.79 $\int \cos^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=42

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b}+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}$$

```
[Out] (2*EllipticF[(a + b*x)/2, 2])/(3*b) + (2*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(
3*b)
```

Rubi [A] time = 0.0203448, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2635, 2641}

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b}+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(3/2),x]
```

```
[Out] (2*EllipticF[(a + b*x)/2, 2])/(3*b) + (2*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(
3*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{3}{2}}(a+bx)\,dx &= \frac{2\sqrt{\cos(a+bx)}\sin(a+bx)}{3b}+\frac{1}{3}\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx\\ &= \frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b}+\frac{2\sqrt{\cos(a+bx)}\sin(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0310163, size = 36, normalized size = 0.86

$$\frac{2\left(F\left(\frac{1}{2}(a+bx)\middle|2\right)+\sin(a+bx)\sqrt{\cos(a+bx)}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(3/2),x]
```

```
[Out] (2*(EllipticF[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*Sin[a + b*x]))/(3*b)
```

Maple [B] time = 2.089, size = 179, normalized size = 4.3

$$-\frac{2}{3b}\sqrt{\left(2\left(\cos\left(1/2\,bx+a/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^{2}}\left(4\cos\left(1/2\,bx+a/2\right)\left(\sin\left(1/2\,bx+a/2\right)\right)^{4}+\sqrt{\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^{2}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(3/2),x)
```

```
[Out] -2/3*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(4*cos(1/2*b*x
+1/2*a)*sin(1/2*b*x+1/2*a)^4+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/
2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))-2*sin(1/2*b*x+1/2*a)^
2*cos(1/2*b*x+1/2*a))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/
sin(1/2*b*x+1/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(bx+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cos\left(bx+a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(3/2), x)
```

3.80 $\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x} dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\cos^{\frac{3}{2}}(a+bx)}{x},x\right)$$

```
[Out] Unintegrable[Cos[a + b*x]^(3/2)/x, x]
```

Rubi [A] time = 0.0279345, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[a + b*x]^(3/2)/x,x]
```

```
[Out] Defer[Int][Cos[a + b*x]^(3/2)/x, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x} dx = \int \frac{\cos^{\frac{3}{2}}(a+bx)}{x} dx$$

Mathematica [A] time = 16.4978, size = 0, normalized size = 0.

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[a + b*x]^(3/2)/x,x]
```

```
[Out] Integrate[Cos[a + b*x]^(3/2)/x, x]
```

Maple [A] time = 0.122, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\cos(bx+a))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(3/2)/x,x)
```

```
[Out] int(cos(b*x+a)^(3/2)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(3/2)/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(3/2)/x, x)
```

3.81 $\int \left(-\frac{x}{3\sqrt{\cos(a+bx)}} + x\cos^{\frac{3}{2}}(a+bx)\right) dx$

Optimal. Leaf size=42

$$\frac{4\cos^{\frac{3}{2}}(a+bx)}{9b^2} + \frac{2x\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}$$

```
[Out] (4*Cos[a + b*x]^(3/2))/(9*b^2) + (2*x*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(3*b
)
```

Rubi [A] time = 0.0586445, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {3310}

$$\frac{4\cos^{\frac{3}{2}}(a+bx)}{9b^2} + \frac{2x\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[-x/(3*Sqrt[Cos[a + b*x]]) + x*Cos[a + b*x]^(3/2),x]
```

```
[Out] (4*Cos[a + b*x]^(3/2))/(9*b^2) + (2*x*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(3*b
)
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int \left(-\frac{x}{3\sqrt{\cos(a+bx)}} + x\cos^{\frac{3}{2}}(a+bx)\right) dx &= -\left(\frac{1}{3}\int \frac{x}{\sqrt{\cos(a+bx)}}\, dx\right) + \int x\cos^{\frac{3}{2}}(a+bx)\, dx\\ &= \frac{4\cos^{\frac{3}{2}}(a+bx)}{9b^2} + \frac{2x\sqrt{\cos(a+bx)}\sin(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.4149, size = 40, normalized size = 0.95

$$\frac{\sqrt{\cos(a+bx)}\left(4x\sin(a+bx) + \frac{8\cos(a+bx)}{3b}\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[-x/(3*Sqrt[Cos[a + b*x]]) + x*Cos[a + b*x]^(3/2),x]
```

```
[Out] (Sqrt[Cos[a + b*x]]*((8*Cos[a + b*x])/(3*b) + 4*x*Sin[a + b*x]))/(6*b)
```

Maple [F] time = 0.275, size = 0, normalized size = 0.

$$\int x\left(\cos\left(bx+a\right)\right)^{\frac{3}{2}}-\frac{x}{3}\frac{1}{\sqrt{\cos\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)^(3/2)-1/3*x/cos(b*x+a)^(1/2),x)
```

```
[Out] int(x*cos(b*x+a)^(3/2)-1/3*x/cos(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos\left(bx+a\right)^{\frac{3}{2}}-\frac{x}{3\sqrt{\cos\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2)-1/3*x/cos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)^(3/2) - 1/3*x/sqrt(cos(b*x + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2)-1/3*x/cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)**(3/2)-1/3*x/cos(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos\left(bx+a\right)^{\frac{3}{2}}-\frac{x}{3\sqrt{\cos\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2)-1/3*x/cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)^(3/2) - 1/3*x/sqrt(cos(b*x + a)), x)
```

3.82 $\int \frac{\cos^{\frac{3}{2}}(x)}{x^3}\,dx$

Optimal. Leaf size=61

$$-\frac{9}{8}\text{Unintegrable}\left(\frac{\cos^{\frac{3}{2}}(x)}{x},x\right)+\frac{3}{8}\text{Unintegrable}\left(\frac{1}{x\sqrt{\cos(x)}},x\right)-\frac{\cos^{\frac{3}{2}}(x)}{2x^2}+\frac{3\sin(x)\sqrt{\cos(x)}}{4x}$$

```
[Out] -Cos[x]^(3/2)/(2*x^2) + (3*Sqrt[Cos[x]]*Sin[x])/(4*x) + (3*Unintegrable[1/(
x*Sqrt[Cos[x]]), x])/8 - (9*Unintegrable[Cos[x]^(3/2)/x, x])/8
```

Rubi [A] time = 0.0768357, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[x]^(3/2)/x^3,x]
```

```
[Out] -Cos[x]^(3/2)/(2*x^2) + (3*Sqrt[Cos[x]]*Sin[x])/(4*x) + (3*Defer[Int][1/(x*
Sqrt[Cos[x]]), x])/8 - (9*Defer[Int][Cos[x]^(3/2)/x, x])/8
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3}\,dx=-\frac{\cos^{\frac{3}{2}}(x)}{2x^2}+\frac{3\sqrt{\cos(x)}\sin(x)}{4x}+\frac{3}{8}\int \frac{1}{x\sqrt{\cos(x)}}\,dx-\frac{9}{8}\int \frac{\cos^{\frac{3}{2}}(x)}{x}\,dx$$

Mathematica [A] time = 5.12425, size = 0, normalized size = 0.

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[x]^(3/2)/x^3,x]
```

```
[Out] Integrate[Cos[x]^(3/2)/x^3, x]
```

Maple [A] time = 0.085, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}(\cos(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^(3/2)/x^3,x)
```

[Out] int(cos(x)^(3/2)/x^3,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(x)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^(3/2)/x^3,x, algorithm="maxima")

[Out] integrate(cos(x)^(3/2)/x^3, x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^(3/2)/x^3,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**(3/2)/x**3,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(x)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^(3/2)/x^3,x, algorithm="giac")

[Out] integrate(cos(x)^(3/2)/x^3, x)

3.83 $\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{x}{\sqrt{\cos(a+bx)}},x\right)$$

```
[Out] Unintegrable[x/Sqrt[Cos[a + b*x]], x]
```

Rubi [A] time = 0.0165909, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/Sqrt[Cos[a + b*x]],x]
```

```
[Out] Defer[Int][x/Sqrt[Cos[a + b*x]], x]
```

Rubi steps

$$\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx=\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx$$

Mathematica [A] time = 0.298686, size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/Sqrt[Cos[a + b*x]],x]
```

```
[Out] Integrate[x/Sqrt[Cos[a + b*x]], x]
```

Maple [A] time = 0.125, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt{\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(b*x+a)^(1/2),x)
```

```
[Out] int(x/cos(b*x+a)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\cos(bx + a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(cos(b*x + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\cos(a + bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)**(1/2),x)
```

```
[Out] Integral(x/sqrt(cos(a + b*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\cos(bx + a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(cos(b*x + a)), x)
```

3.84 $\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx$

Optimal. Leaf size=16

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

```
[Out] (2*EllipticF[(a + b*x)/2, 2])/b
```

Rubi [A] time = 0.0091666, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2641}

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Cos[a + b*x]],x]
```

```
[Out] (2*EllipticF[(a + b*x)/2, 2])/b
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx = \frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Mathematica [A] time = 0.0154516, size = 16, normalized size = 1.

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Cos[a + b*x]],x]
```

```
[Out] (2*EllipticF[(a + b*x)/2, 2])/b
```

Maple [C] time = 0.03, size = 18, normalized size = 1.1

$$2\,\frac{\mathrm{InverseJacobiAM}\left(1/2\,bx+a/2,\sqrt{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a)^(1/2),x)
```

```
[Out] 2/b*InverseJacobiAM(1/2*b*x+1/2*a,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(cos(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{\cos(bx+a)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(cos(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(cos(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(cos(b*x + a)), x)
```

3.85 $\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{\cos(a+bx)}},x\right)$$

```
[Out] Unintegrable[1/(x*Sqrt[Cos[a + b*x]]), x]
```

Rubi [A] time = 0.0284183, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[Cos[a + b*x]]),x]
```

```
[Out] Defer[Int][1/(x*Sqrt[Cos[a + b*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx = \int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx$$

Mathematica [A] time = 0.198933, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[Cos[a + b*x]]),x]
```

```
[Out] Integrate[1/(x*Sqrt[Cos[a + b*x]]), x]
```

Maple [A] time = 0.132, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt{\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/cos(b*x+a)^(1/2),x)
```

```
[Out] int(1/x/cos(b*x+a)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(cos(b*x + a))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(cos(a + b*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt(cos(b*x + a))), x)
```

3.86 $\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=54

$$-\text{Unintegrable}\left(x\sqrt{\cos(a+bx)},x\right)+\frac{4\sqrt{\cos(a+bx)}}{b^2}+\frac{2x\sin(a+bx)}{b\sqrt{\cos(a+bx)}}$$

```
[Out] (4*Sqrt[Cos[a + b*x]])/b^2 + (2*x*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]]) - Un
integrable[x*Sqrt[Cos[a + b*x]], x]
```

Rubi [A] time = 0.0363469, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/Cos[a + b*x]^(3/2),x]
```

```
[Out] (4*Sqrt[Cos[a + b*x]])/b^2 + (2*x*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]]) - De
fer[Int][x*Sqrt[Cos[a + b*x]], x]
```

Rubi steps

$$\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx=\frac{4\sqrt{\cos(a+bx)}}{b^2}+\frac{2x\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\int x\sqrt{\cos(a+bx)}\,dx$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[x/Cos[a + b*x]^(3/2),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.123, size = 0, normalized size = 0.

$$\int x\left(\cos\left(bx+a\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(b*x+a)^(3/2),x)
```

```
[Out] int(x/cos(b*x+a)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\cos(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/cos(b*x + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\cos(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/cos(b*x + a)^(3/2), x)
```

3.87 $\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=38

$$\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

```
[Out] (-2*EllipticE[(a + b*x)/2, 2])/b + (2*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])
```

Rubi [A] time = 0.0170415, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2636, 2639}

$$\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(-3/2),x]
```

```
[Out] (-2*EllipticE[(a + b*x)/2, 2])/b + (2*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx &= \frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\int \sqrt{\cos(a+bx)}\,dx\\ &= -\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}+\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0535599, size = 38, normalized size = 1.

$$\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(-3/2),x]
```

```
[Out] (-2*EllipticE[(a + b*x)/2, 2])/b + (2*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])
```

Maple [A] time = 2.207, size = 101, normalized size = 2.7

$$-2\,\frac{\sqrt{2\,\left(\sin\left(1/2\,bx+a/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(1/2\,bx+a/2\right)\right)^{2}}\mathrm{EllipticE}\left(\cos\left(1/2\,bx+a/2\right),\sqrt{2}\right)-2\,\left(\sin\left(1/2\,bx+a/2\right)\right)^{2}\cos\left(1\right.}{\sin\left(1/2\,bx+a/2\right)\sqrt{2\,\left(\cos\left(1/2\,bx+a/2\right)\right)^{2}-1}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a)^(3/2),x)
```

```
[Out] -2*((2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE
(cos(1/2*b*x+1/2*a),2^(1/2))-2*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))/sin
(1/2*b*x+1/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\cos\left(bx+a\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^(-3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(-3/2), x)
```

3.88 $\int \frac{1}{x\cos^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{x\cos^{\frac{3}{2}}(a+bx)},x\right)$$

```
[Out] Unintegrable[1/(x*Cos[a + b*x]^(3/2)), x]
```

Rubi [A] time = 0.0275547, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\cos^{\frac{3}{2}}(a+bx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Cos[a + b*x]^(3/2)),x]
```

```
[Out] Defer[Int][1/(x*Cos[a + b*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{x\cos^{\frac{3}{2}}(a+bx)}\,dx$$

Mathematica [A] time = 15.3432, size = 0, normalized size = 0.

$$\int \frac{1}{x\cos^{\frac{3}{2}}(a+bx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Cos[a + b*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x*Cos[a + b*x]^(3/2)), x]
```

Maple [F] time = 180., size = 0, normalized size = 0.

hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/cos(b*x+a)^(3/2),x)
```

```
[Out] int(1/x/cos(b*x+a)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\cos(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*cos(b*x + a)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\cos(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*cos(b*x + a)^(3/2)), x)
```

3.89 $\int \left(\frac{x}{\cos^{\frac{3}{2}}(a+bx)} + x\sqrt{\cos(a+bx)} \right) dx$

Optimal. Leaf size=38

$$\frac{4\sqrt{\cos(a+bx)}}{b^2} + \frac{2x\sin(a+bx)}{b\sqrt{\cos(a+bx)}}$$

```
[Out] (4*Sqrt[Cos[a + b*x]])/b^2 + (2*x*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])
```

Rubi [A] time = 0.055347, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {3315}

$$\frac{4\sqrt{\cos(a+bx)}}{b^2} + \frac{2x\sin(a+bx)}{b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Cos[a + b*x]^(3/2) + x*Sqrt[Cos[a + b*x]],x]
```

```
[Out] (4*Sqrt[Cos[a + b*x]])/b^2 + (2*x*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])
```

Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[((c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[(d*(b*Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{x}{\cos^{\frac{3}{2}}(a+bx)} + x\sqrt{\cos(a+bx)} \right) dx &= \int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\, dx + \int x\sqrt{\cos(a+bx)}\, dx \\
&= \frac{4\sqrt{\cos(a+bx)}}{b^2} + \frac{2x\sin(a+bx)}{b\sqrt{\cos(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.387592, size = 33, normalized size = 0.87

$$\frac{2(bx\sin(a+bx) + 2\cos(a+bx))}{b^2\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Cos[a + b*x]^(3/2) + x*Sqrt[Cos[a + b*x]],x]
```

```
[Out] (2*(2*Cos[a + b*x] + b*x*Sin[a + b*x]))/(b^2*Sqrt[Cos[a + b*x]])
```

Maple [F] time = 0.22, size = 0, normalized size = 0.

$$\int x\left(\cos\left(bx+a\right)\right)^{-\frac{3}{2}}+x\sqrt{\cos\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(b*x+a)^(3/2)+x*cos(b*x+a)^(1/2),x)
```

```
[Out] int(x/cos(b*x+a)^(3/2)+x*cos(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\cos\left(bx+a\right)}+\frac{x}{\cos\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)^(3/2)+x*cos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(cos(b*x + a)) + x/cos(b*x + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)^(3/2)+x*cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\left(\cos^{2}\left(a+bx\right)+1\right)}{\cos^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)**(3/2)+x*cos(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*(cos(a + b*x)**2 + 1)/cos(a + b*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\cos\left(bx+a\right)}+\frac{x}{\cos\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(b*x+a)^(3/2)+x*cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(cos(b*x + a)) + x/cos(b*x + a)^(3/2), x)
```

3.90 $\int \left(\frac{x}{\cos^{\frac{3}{2}}(x)}+x\sqrt{\cos(x)}\right)\,dx$

Optimal. Leaf size=20

$$4\sqrt{\cos(x)}+\frac{2x\sin(x)}{\sqrt{\cos(x)}}$$

```
[Out] 4*Sqrt[Cos[x]] + (2*x*Sin[x])/Sqrt[Cos[x]]
```

Rubi [A] time = 0.042698, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3315}

$$4\sqrt{\cos(x)}+\frac{2x\sin(x)}{\sqrt{\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Cos[x]^(3/2) + x*Sqrt[Cos[x]],x]

[Out] 4*Sqrt[Cos[x]] + (2*x*Sin[x])/Sqrt[Cos[x]]
```

Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[((c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[(d*(b*Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\cos^{\frac{3}{2}}(x)}+x\sqrt{\cos(x)}\right)dx &= \int \frac{x}{\cos^{\frac{3}{2}}(x)}\,dx+\int x\sqrt{\cos(x)}\,dx\\ &= 4\sqrt{\cos(x)}+\frac{2x\sin(x)}{\sqrt{\cos(x)}}\end{aligned}$$

Mathematica [A] time = 0.0858378, size = 17, normalized size = 0.85

$$\frac{2(x\sin(x)+2\cos(x))}{\sqrt{\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Cos[x]^(3/2) + x*Sqrt[Cos[x]],x]

[Out] (2*(2*Cos[x] + x*Sin[x]))/Sqrt[Cos[x]]
```

Maple [F] time = 0.22, size = 0, normalized size = 0.

$$\int x (\cos (x))^{-\frac{3}{2}} + x\sqrt{\cos (x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(x)^(3/2)+x*cos(x)^(1/2),x)
```

```
[Out] int(x/cos(x)^(3/2)+x*cos(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\cos (x)} + \frac{x}{\cos (x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)^(3/2)+x*cos(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(cos(x)) + x/cos(x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)^(3/2)+x*cos(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \left(\cos^{2}(x) + 1\right)}{\cos^{\frac{3}{2}}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)**(3/2)+x*cos(x)**(1/2),x)
```

```
[Out] Integral(x*(cos(x)**2 + 1)/cos(x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\cos (x)} + \frac{x}{\cos (x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)^(3/2)+x*cos(x)^(1/2),x, algorithm="giac")

[Out] integrate(x*sqrt(cos(x)) + x/cos(x)^(3/2), x)
```

3.91 $\int \left(\frac{x}{\cos^{\frac{5}{2}}(x)}-\frac{x}{3\sqrt{\cos(x)}}\right) dx$

Optimal. Leaf size=24

$$\frac{2x\sin(x)}{3\cos^{\frac{3}{2}}(x)}-\frac{4}{3\sqrt{\cos(x)}}$$

```
[Out] -4/(3*Sqrt[Cos[x]]) + (2*x*Sin[x])/(3*Cos[x]^(3/2))
```

Rubi [A] time = 0.0459401, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {3315}

$$\frac{2x\sin(x)}{3\cos^{\frac{3}{2}}(x)}-\frac{4}{3\sqrt{\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Cos[x]^(5/2) - x/(3*Sqrt[Cos[x]]),x]
```

```
[Out] -4/(3*Sqrt[Cos[x]]) + (2*x*Sin[x])/(3*Cos[x]^(3/2))
```

Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[((c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[(d*(b*Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\cos^{\frac{5}{2}}(x)}-\frac{x}{3\sqrt{\cos(x)}}\right) dx &= -\left(\frac{1}{3}\int \frac{x}{\sqrt{\cos(x)}}\,dx\right)+\int \frac{x}{\cos^{\frac{5}{2}}(x)}\,dx\\ &= -\frac{4}{3\sqrt{\cos(x)}}+\frac{2x\sin(x)}{3\cos^{\frac{3}{2}}(x)}\end{aligned}$$

Mathematica [A] time = 0.0751972, size = 17, normalized size = 0.71

$$-\frac{8-4x\tan(x)}{6\sqrt{\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Cos[x]^(5/2) - x/(3*Sqrt[Cos[x]]),x]
```

```
[Out] -(8 - 4*x*Tan[x])/(6*Sqrt[Cos[x]])
```

Maple [F] time = 0.213, size = 0, normalized size = 0.

$$\int x\left(\cos\left(x\right)\right)^{-\frac{5}{2}}-\frac{x}{3}\frac{1}{\sqrt{\cos\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(x)^(5/2)-1/3*x/cos(x)^(1/2),x)
```

```
[Out] int(x/cos(x)^(5/2)-1/3*x/cos(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{3\sqrt{\cos\left(x\right)}}+\frac{x}{\cos\left(x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)^(5/2)-1/3*x/cos(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-1/3*x/sqrt(cos(x)) + x/cos(x)^(5/2), x)
```

Fricas [A] time = 1.59847, size = 54, normalized size = 2.25

$$\frac{2\left(x\sin\left(x\right)-2\cos\left(x\right)\right)}{3\cos\left(x\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)^(5/2)-1/3*x/cos(x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x*sin(x) - 2*cos(x))/cos(x)^(3/2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)**(5/2)-1/3*x/cos(x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{3\sqrt{\cos\left(x\right)}}+\frac{x}{\cos\left(x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)^(5/2)-1/3*x/cos(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/3*x/sqrt(cos(x)) + x/cos(x)^(5/2), x)
```

3.92 $\int \left(\frac{x}{\cos^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cos(x)}\right) dx$

Optimal. Leaf size=47

$$-\frac{4}{15\cos^{\frac{3}{2}}(x)}+\frac{12\sqrt{\cos(x)}}{5}+\frac{2x\sin(x)}{5\cos^{\frac{5}{2}}(x)}+\frac{6x\sin(x)}{5\sqrt{\cos(x)}}$$

```
[Out] -4/(15*Cos[x]^(3/2)) + (12*Sqrt[Cos[x]])/5 + (2*x*Sin[x])/(5*Cos[x]^(5/2))
+ (6*x*Sin[x])/(5*Sqrt[Cos[x]])
```

Rubi [A] time = 0.0610706, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {3315}

$$-\frac{4}{15\cos^{\frac{3}{2}}(x)}+\frac{12\sqrt{\cos(x)}}{5}+\frac{2x\sin(x)}{5\cos^{\frac{5}{2}}(x)}+\frac{6x\sin(x)}{5\sqrt{\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Cos[x]^(7/2) + (3*x*Sqrt[Cos[x]])/5,x]
```

```
[Out] -4/(15*Cos[x]^(3/2)) + (12*Sqrt[Cos[x]])/5 + (2*x*Sin[x])/(5*Cos[x]^(5/2))
+ (6*x*Sin[x])/(5*Sqrt[Cos[x]])
```

Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[((c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[(d*(b*Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{x}{\cos^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cos(x)}\right) dx &= \frac{3}{5}\int x\sqrt{\cos(x)}\,dx+\int \frac{x}{\cos^{\frac{7}{2}}(x)}\,dx\\
&= -\frac{4}{15\cos^{\frac{3}{2}}(x)}+\frac{2x\sin(x)}{5\cos^{\frac{5}{2}}(x)}+\frac{3}{5}\int \frac{x}{\cos^{\frac{3}{2}}(x)}\,dx+\frac{3}{5}\int x\sqrt{\cos(x)}\,dx\\
&= -\frac{4}{15\cos^{\frac{3}{2}}(x)}+\frac{12\sqrt{\cos(x)}}{5}+\frac{2x\sin(x)}{5\cos^{\frac{5}{2}}(x)}+\frac{6x\sin(x)}{5\sqrt{\cos(x)}}
\end{aligned}$$

Mathematica [A] time = 0.129401, size = 33, normalized size = 0.7

$$\frac{21x\sin(x)+9x\sin(3x)+46\cos(x)+18\cos(3x)}{30\cos^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Cos[x]^(7/2) + (3*x*Sqrt[Cos[x]])/5,x]
```

```
[Out] (46*Cos[x] + 18*Cos[3*x] + 21*x*Sin[x] + 9*x*Sin[3*x])/(30*Cos[x]^(5/2))
```

Maple [F] time = 0.298, size = 0, normalized size = 0.

$$\int x(\cos(x))^{-\frac{7}{2}} + \frac{3x}{5}\sqrt{\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(x)^(7/2)+3/5*x*cos(x)^(1/2),x)
```

```
[Out] int(x/cos(x)^(7/2)+3/5*x*cos(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3}{5}x\sqrt{\cos(x)} + \frac{x}{\cos(x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)^(7/2)+3/5*x*cos(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(3/5*x*sqrt(cos(x)) + x/cos(x)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)^(7/2)+3/5*x*cos(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)**(7/2)+3/5*x*cos(x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3}{5}x\sqrt{\cos(x)} + \frac{x}{\cos(x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cos(x)^(7/2)+3/5*x*cos(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(3/5*x*sqrt(cos(x)) + x/cos(x)^(7/2), x)
```

3.93 $\int \left(\frac{x^2}{\cos^{\frac{3}{2}}(x)}+x^2\sqrt{\cos(x)}\right) dx$

Optimal. Leaf size=32

$$\frac{2x^2\sin(x)}{\sqrt{\cos(x)}}+8x\sqrt{\cos(x)}-16E\left(\frac{x}{2}\middle|2\right)$$

```
[Out] 8*x*Sqrt[Cos[x]] - 16*EllipticE[x/2, 2] + (2*x^2*Sin[x])/Sqrt[Cos[x]]
```

Rubi [A] time = 0.0859179, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3316, 2639}

$$\frac{2x^2\sin(x)}{\sqrt{\cos(x)}}+8x\sqrt{\cos(x)}-16E\left(\frac{x}{2}\middle|2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Cos[x]^(3/2) + x^2*Sqrt[Cos[x]],x]
```

```
[Out] 8*x*Sqrt[Cos[x]] - 16*EllipticE[x/2, 2] + (2*x^2*Sin[x])/Sqrt[Cos[x]]
```

Rule 3316

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[((c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1
)), x] + (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n +
 2), x], x] + Dist[(d^2*m*(m - 1))/(b^2*f^2*(n + 1)*(n + 2)), Int[(c + d*x)
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] - Simp[(d*m*(c + d*x)^(m - 1)*(b*
Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; FreeQ[{b, c, d, e,
 f}, x] && LtQ[n, -1] && NeQ[n, -2] && GtQ[m, 1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x^2}{\cos^{\frac{3}{2}}(x)}+x^2\sqrt{\cos(x)}\right) dx &= \int \frac{x^2}{\cos^{\frac{3}{2}}(x)}\, dx+\int x^2\sqrt{\cos(x)}\, dx\\ &= 8x\sqrt{\cos(x)}+\frac{2x^2\sin(x)}{\sqrt{\cos(x)}}-8\int \sqrt{\cos(x)}\, dx\\ &= 8x\sqrt{\cos(x)}-16E\left(\frac{x}{2}\middle|2\right)+\frac{2x^2\sin(x)}{\sqrt{\cos(x)}}\end{aligned}$$

Mathematica [A] time = 0.145364, size = 29, normalized size = 0.91

$$2\left(\frac{x(x\sin(x)+4\cos(x))}{\sqrt{\cos(x)}}-8E\left(\frac{x}{2}\middle|2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Cos[x]^(3/2) + x^2*Sqrt[Cos[x]],x]
```

```
[Out] 2*(-8*EllipticE[x/2, 2] + (x*(4*Cos[x] + x*Sin[x]))/Sqrt[Cos[x]])
```

Maple [F] time = 0.226, size = 0, normalized size = 0.

$$\int x^2 (\cos (x))^{-\frac{3}{2}} + x^2\sqrt{\cos (x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/cos(x)^(3/2)+x^2*cos(x)^(1/2),x)
```

```
[Out] int(x^2/cos(x)^(3/2)+x^2*cos(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{\cos (x)} + \frac{x^2}{\cos (x)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/cos(x)^(3/2)+x^2*cos(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*sqrt(cos(x)) + x^2/cos(x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/cos(x)^(3/2)+x^2*cos(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(\cos^2 (x) + 1\right)}{\cos^{\frac{3}{2}} (x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/cos(x)**(3/2)+x**2*cos(x)**(1/2),x)
```

```
[Out] Integral(x**2*(cos(x)**2 + 1)/cos(x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{\cos(x)} + \frac{x^2}{\cos(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/cos(x)^(3/2)+x^2*cos(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*sqrt(cos(x)) + x^2/cos(x)^(3/2), x)
```

3.94 $\int \left(\frac{x}{\sec^{\frac{3}{2}}(x)}-\frac{1}{3}x\sqrt{\sec(x)}\right)dx$

Optimal. Leaf size=24

$$\frac{4}{9\sec^{\frac{3}{2}}(x)}+\frac{2x\sin(x)}{3\sqrt{\sec(x)}}$$

```
[Out] 4/(9*Sec[x]^(3/2)) + (2*x*Sin[x])/(3*Sqrt[Sec[x]])
```

Rubi [A] time = 0.0814213, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4187, 4189}

$$\frac{4}{9\sec^{\frac{3}{2}}(x)}+\frac{2x\sin(x)}{3\sqrt{\sec(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sec[x]^(3/2) - (x*Sqrt[Sec[x]])/3,x]
```

```
[Out] 4/(9*Sec[x]^(3/2)) + (2*x*Sin[x])/(3*Sqrt[Sec[x]])
```

Rule 4187

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(d*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[((c + d*x)*Cos[e + f*x]*(b*C
sc[e + f*x])^(n + 1))/(b*f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{x}{\sec^{\frac{3}{2}}(x)}-\frac{1}{3}x\sqrt{\sec(x)}\right)dx &= -\left(\frac{1}{3}\int x\sqrt{\sec(x)}\,dx\right)+\int \frac{x}{\sec^{\frac{3}{2}}(x)}\,dx\\
&= \frac{4}{9\sec^{\frac{3}{2}}(x)}+\frac{2x\sin(x)}{3\sqrt{\sec(x)}}+\frac{1}{3}\int x\sqrt{\sec(x)}\,dx-\frac{1}{3}\left(\sqrt{\cos(x)}\sqrt{\sec(x)}\right)\int \frac{x}{\sqrt{\cos(x)}}\,dx\\
&= \frac{4}{9\sec^{\frac{3}{2}}(x)}+\frac{2x\sin(x)}{3\sqrt{\sec(x)}}
\end{aligned}$$

Mathematica [A] time = 0.0904513, size = 17, normalized size = 0.71

$$\frac{2(3x\tan(x)+2)}{9\sec^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sec[x]^(3/2) - (x*Sqrt[Sec[x]])/3,x]
```

```
[Out] (2*(2 + 3*x*Tan[x]))/(9*Sec[x]^(3/2))
```

Maple [F] time = 0.166, size = 0, normalized size = 0.

$$\int x\left(\sec\left(x\right)\right)^{-\frac{3}{2}} - \frac{x}{3}\sqrt{\sec\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sec(x)^(3/2)-1/3*x*sec(x)^(1/2),x)
```

```
[Out] int(x/sec(x)^(3/2)-1/3*x*sec(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{3}\,x\sqrt{\sec\left(x\right)} + \frac{x}{\sec\left(x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)^(3/2)-1/3*x*sec(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-1/3*x*sqrt(sec(x)) + x/sec(x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)^(3/2)-1/3*x*sec(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int -\frac{3x}{\sec^{\frac{3}{2}}\left(x\right)}\,dx + \int x\sqrt{\sec\left(x\right)}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)**(3/2)-1/3*x*sec(x)**(1/2),x)
```

```
[Out] -(Integral(-3*x/sec(x)**(3/2), x) + Integral(x*sqrt(sec(x)), x))/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{3}x\sqrt{\sec(x)} + \frac{x}{\sec(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)^(3/2)-1/3*x*sec(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/3*x*sqrt(sec(x)) + x/sec(x)^(3/2), x)
```

3.95 $\int \left(\frac{x}{\sec^{\frac{5}{2}}(x)} - \frac{3x}{5\sqrt{\sec(x)}}\right) dx$

Optimal. Leaf size=24

$$\frac{4}{25\sec^{\frac{5}{2}}(x)} + \frac{2x\sin(x)}{5\sec^{\frac{3}{2}}(x)}$$

```
[Out] 4/(25*Sec[x]^(5/2)) + (2*x*Sin[x])/(5*Sec[x]^(3/2))
```

Rubi [A] time = 0.0791018, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4187, 4189}

$$\frac{4}{25\sec^{\frac{5}{2}}(x)} + \frac{2x\sin(x)}{5\sec^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sec[x]^(5/2) - (3*x)/(5*Sqrt[Sec[x]]),x]
```

```
[Out] 4/(25*Sec[x]^(5/2)) + (2*x*Sin[x])/(5*Sec[x]^(3/2))
```

Rule 4187

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(d*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[((c + d*x)*Cos[e + f*x]*(b*C
sc[e + f*x])^(n + 1))/(b*f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{x}{\sec^{\frac{5}{2}}(x)} - \frac{3x}{5\sqrt{\sec(x)}}\right) dx &= -\left(\frac{3}{5}\int \frac{x}{\sqrt{\sec(x)}}\,dx\right) + \int \frac{x}{\sec^{\frac{5}{2}}(x)}\,dx \\
&= \frac{4}{25\sec^{\frac{5}{2}}(x)} + \frac{2x\sin(x)}{5\sec^{\frac{3}{2}}(x)} + \frac{3}{5}\int \frac{x}{\sqrt{\sec(x)}}\,dx - \frac{1}{5}\left(3\sqrt{\cos(x)}\sqrt{\sec(x)}\right)\int x\sqrt{\cos(x)}\,dx \\
&= \frac{4}{25\sec^{\frac{5}{2}}(x)} + \frac{2x\sin(x)}{5\sec^{\frac{3}{2}}(x)}
\end{aligned}$$

Mathematica [A] time = 0.139029, size = 17, normalized size = 0.71

$$\frac{2(5x\tan(x)+2)}{25\sec^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sec[x]^(5/2) - (3*x)/(5*Sqrt[Sec[x]]),x]
```

```
[Out] (2*(2 + 5*x*Tan[x]))/(25*Sec[x]^(5/2))
```

Maple [F] time = 0.161, size = 0, normalized size = 0.

$$\int x(\sec(x))^{-\frac{5}{2}} - \frac{3x}{5}\frac{1}{\sqrt{\sec(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sec(x)^(5/2)-3/5*x/sec(x)^(1/2),x)
```

```
[Out] int(x/sec(x)^(5/2)-3/5*x/sec(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{3x}{5\sqrt{\sec(x)}} + \frac{x}{\sec(x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)^(5/2)-3/5*x/sec(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-3/5*x/sqrt(sec(x)) + x/sec(x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)^(5/2)-3/5*x/sec(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int -\frac{5x}{\sec^{\frac{5}{2}}(x)}\,dx + \int \frac{3x}{\sqrt{\sec(x)}}\,dx}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)**(5/2)-3/5*x/sec(x)**(1/2),x)
```

```
[Out] -(Integral(-5*x/sec(x)**(5/2), x) + Integral(3*x/sqrt(sec(x)), x))/5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{3\,x}{5\,\sqrt{\sec\left(x\right)}} + \frac{x}{\sec\left(x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)^(5/2)-3/5*x/sec(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-3/5*x/sqrt(sec(x)) + x/sec(x)^(5/2), x)
```

3.96 $\int \left(\frac{x}{\sec^{\frac{7}{2}}(x)}-\frac{5}{21}x\sqrt{\sec(x)}\right)dx$

Optimal. Leaf size=47

$$\frac{20}{63\sec^{\frac{3}{2}}(x)}+\frac{4}{49\sec^{\frac{7}{2}}(x)}+\frac{2x\sin(x)}{7\sec^{\frac{5}{2}}(x)}+\frac{10x\sin(x)}{21\sqrt{\sec(x)}}$$

```
[Out] 4/(49*Sec[x]^(7/2)) + 20/(63*Sec[x]^(3/2)) + (2*x*Sin[x])/(7*Sec[x]^(5/2))
+ (10*x*Sin[x])/(21*Sqrt[Sec[x]])
```

Rubi [A] time = 0.0943382, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4187, 4189}

$$\frac{20}{63\sec^{\frac{3}{2}}(x)}+\frac{4}{49\sec^{\frac{7}{2}}(x)}+\frac{2x\sin(x)}{7\sec^{\frac{5}{2}}(x)}+\frac{10x\sin(x)}{21\sqrt{\sec(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sec[x]^(7/2) - (5*x*Sqrt[Sec[x]])/21,x]
```

```
[Out] 4/(49*Sec[x]^(7/2)) + 20/(63*Sec[x]^(3/2)) + (2*x*Sin[x])/(7*Sec[x]^(5/2))
+ (10*x*Sin[x])/(21*Sqrt[Sec[x]])
```

Rule 4187

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(d*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[((c + d*x)*Cos[e + f*x]*(b*C
sc[e + f*x])^(n + 1))/(b*f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\sec^{\frac{7}{2}}(x)}-\frac{5}{21}x\sqrt{\sec(x)}\right)dx &= -\left(\frac{5}{21}\int x\sqrt{\sec(x)}\,dx\right)+\int \frac{x}{\sec^{\frac{7}{2}}(x)}\,dx\\ &= \frac{4}{49\sec^{\frac{7}{2}}(x)}+\frac{2x\sin(x)}{7\sec^{\frac{5}{2}}(x)}+\frac{5}{7}\int \frac{x}{\sec^{\frac{3}{2}}(x)}\,dx-\frac{1}{21}\left(5\sqrt{\cos(x)}\sqrt{\sec(x)}\right)\int \frac{x}{\sqrt{\cos(x)}}\,dx\\ &= \frac{4}{49\sec^{\frac{7}{2}}(x)}+\frac{20}{63\sec^{\frac{3}{2}}(x)}+\frac{2x\sin(x)}{7\sec^{\frac{5}{2}}(x)}+\frac{10x\sin(x)}{21\sqrt{\sec(x)}}+\frac{5}{21}\int x\sqrt{\sec(x)}\,dx-\frac{1}{21}\left(5\sqrt{\cos\right.\\ &= \frac{4}{49\sec^{\frac{7}{2}}(x)}+\frac{20}{63\sec^{\frac{3}{2}}(x)}+\frac{2x\sin(x)}{7\sec^{\frac{5}{2}}(x)}+\frac{10x\sin(x)}{21\sqrt{\sec(x)}}\end{aligned}$$

Mathematica [A] time = 0.1065, size = 45, normalized size = 0.96

$$\sqrt{\sec(x)}\left(\frac{13}{42}x\sin(2x)+\frac{1}{28}x\sin(4x)+\frac{88}{441}\cos(2x)+\frac{1}{98}\cos(4x)+\frac{167}{882}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sec[x]^(7/2) - (5*x*Sqrt[Sec[x]])/21,x]
```

```
[Out] Sqrt[Sec[x]]*(167/882 + (88*Cos[2*x])/441 + Cos[4*x]/98 + (13*x*Sin[2*x])/4
2 + (x*Sin[4*x])/28)
```

Maple [F] time = 0.167, size = 0, normalized size = 0.

$$\int x\left(\sec(x)\right)^{-\frac{7}{2}}-\frac{5x}{21}\sqrt{\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sec(x)^(7/2)-5/21*x*sec(x)^(1/2),x)
```

```
[Out] int(x/sec(x)^(7/2)-5/21*x*sec(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{5}{21}x\sqrt{\sec(x)}+\frac{x}{\sec(x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)^(7/2)-5/21*x*sec(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-5/21*x*sqrt(sec(x)) + x/sec(x)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)^(7/2)-5/21*x*sec(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)**(7/2)-5/21*x*sec(x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{5}{21}\,x\sqrt{\sec\left(x\right)} + \frac{x}{\sec\left(x\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sec(x)^(7/2)-5/21*x*sec(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-5/21*x*sqrt(sec(x)) + x/sec(x)^(7/2), x)
```

3.97 $\int \left(\frac{x^2}{\sec^{\frac{3}{2}}(x)}-\frac{1}{3}x^2\sqrt{\sec(x)}\right)dx$

Optimal. Leaf size=62

$$\frac{2x^2\sin(x)}{3\sqrt{\sec(x)}}+\frac{8x}{9\sec^{\frac{3}{2}}(x)}-\frac{16\sin(x)}{27\sqrt{\sec(x)}}-\frac{16}{27}\sqrt{\cos(x)}\sqrt{\sec(x)}F\left(\frac{x}{2}\middle|2\right)$$

```
[Out] (8*x)/(9*Sec[x]^(3/2)) - (16*Sqrt[Cos[x]]*EllipticF[x/2, 2]*Sqrt[Sec[x]])/2
7 - (16*Sin[x])/(27*Sqrt[Sec[x]]) + (2*x^2*Sin[x])/(3*Sqrt[Sec[x]])
```

Rubi [A] time = 0.153636, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4188, 4189, 3769, 3771, 2641}

$$\frac{2x^2\sin(x)}{3\sqrt{\sec(x)}}+\frac{8x}{9\sec^{\frac{3}{2}}(x)}-\frac{16\sin(x)}{27\sqrt{\sec(x)}}-\frac{16}{27}\sqrt{\cos(x)}\sqrt{\sec(x)}F\left(\frac{x}{2}\middle|2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sec[x]^(3/2) - (x^2*Sqrt[Sec[x]])/3,x]
```

```
[Out] (8*x)/(9*Sec[x]^(3/2)) - (16*Sqrt[Cos[x]]*EllipticF[x/2, 2]*Sqrt[Sec[x]])/2
7 - (16*Sin[x])/(27*Sqrt[Sec[x]]) + (2*x^2*Sin[x])/(3*Sqrt[Sec[x]])
```

Rule 4188

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(n + 1)/(b^2*n), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n + 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^n, x], x]
+ Simp[((c + d*x)^m*Cos[e + f*x]*(b*Csc[e + f*x])^(n + 1))/(b*f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
```

Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{x^2}{\sec^{\frac{3}{2}}(x)}-\frac{1}{3}x^2\sqrt{\sec(x)}\right)dx &= -\left(\frac{1}{3}\int x^2\sqrt{\sec(x)}\,dx\right)+\int \frac{x^2}{\sec^{\frac{3}{2}}(x)}\,dx\\
&= \frac{8x}{9\sec^{\frac{3}{2}}(x)}+\frac{2x^2\sin(x)}{3\sqrt{\sec(x)}}+\frac{1}{3}\int x^2\sqrt{\sec(x)}\,dx-\frac{8}{9}\int \frac{1}{\sec^{\frac{3}{2}}(x)}\,dx-\frac{1}{3}\left(\sqrt{\cos(x)}\sqrt{\sec(x)}\right)\\
&= \frac{8x}{9\sec^{\frac{3}{2}}(x)}-\frac{16\sin(x)}{27\sqrt{\sec(x)}}+\frac{2x^2\sin(x)}{3\sqrt{\sec(x)}}-\frac{8}{27}\int \sqrt{\sec(x)}\,dx\\
&= \frac{8x}{9\sec^{\frac{3}{2}}(x)}-\frac{16\sin(x)}{27\sqrt{\sec(x)}}+\frac{2x^2\sin(x)}{3\sqrt{\sec(x)}}-\frac{1}{27}\left(8\sqrt{\cos(x)}\sqrt{\sec(x)}\right)\int \frac{1}{\sqrt{\cos(x)}}\,dx\\
&= \frac{8x}{9\sec^{\frac{3}{2}}(x)}-\frac{16}{27}\sqrt{\cos(x)}F\left(\frac{x}{2}\middle|2\right)\sqrt{\sec(x)}-\frac{16\sin(x)}{27\sqrt{\sec(x)}}+\frac{2x^2\sin(x)}{3\sqrt{\sec(x)}}
\end{aligned}$$

Mathematica [A] time = 0.105178, size = 51, normalized size = 0.82

$$\frac{1}{27}\sqrt{\sec(x)}\left(9x^2\sin(2x)+12x-8\sin(2x)+12x\cos(2x)-16\sqrt{\cos(x)}F\left(\frac{x}{2}\middle|2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sec[x]^(3/2) - (x^2*Sqrt[Sec[x]])/3,x]
```

```
[Out] (Sqrt[Sec[x]]*(12*x + 12*x*Cos[2*x] - 16*Sqrt[Cos[x]]*EllipticF[x/2, 2] - 8
*Sin[2*x] + 9*x^2*Sin[2*x]))/27
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$\int x^2(\sec(x))^{-\frac{3}{2}}-\frac{x^2}{3}\sqrt{\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/sec(x)^(3/2)-1/3*x^2*sec(x)^(1/2),x)
```

```
[Out] int(x^2/sec(x)^(3/2)-1/3*x^2*sec(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{3}x^2\sqrt{\sec(x)}+\frac{x^2}{\sec(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sec(x)^(3/2)-1/3*x^2*sec(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-1/3*x^2*sqrt(sec(x)) + x^2/sec(x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sec(x)^(3/2)-1/3*x^2*sec(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int -\frac{3x^2}{\sec^{\frac{3}{2}}(x)}\,dx + \int x^2\sqrt{\sec(x)}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/sec(x)**(3/2)-1/3*x**2*sec(x)**(1/2),x)
```

```
[Out] -(Integral(-3*x**2/sec(x)**(3/2), x) + Integral(x**2*sqrt(sec(x)), x))/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{3}\,x^2\sqrt{\sec(x)} + \frac{x^2}{\sec(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sec(x)^(3/2)-1/3*x^2*sec(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/3*x^2*sqrt(sec(x)) + x^2/sec(x)^(3/2), x)
```

3.98 $\int (c+dx)^m(b\cos(e+fx))^n\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left((c+dx)^m(b\cos(e+fx))^n,x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*(b*Cos[e + f*x])^n, x]
```

Rubi [A] time = 0.041328, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m(b\cos(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*(b*Cos[e + f*x])^n,x]
```

```
[Out] Defer[Int][(c + d*x)^m*(b*Cos[e + f*x])^n, x]
```

Rubi steps

$$\int (c+dx)^m(b\cos(e+fx))^n\,dx=\int (c+dx)^m(b\cos(e+fx))^n\,dx$$

Mathematica [A] time = 0.726034, size = 0, normalized size = 0.

$$\int (c+dx)^m(b\cos(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(b*Cos[e + f*x])^n,x]
```

```
[Out] Integrate[(c + d*x)^m*(b*Cos[e + f*x])^n, x]
```

Maple [A] time = 0.559, size = 0, normalized size = 0.

$$\int (dx+c)^m\left(b\cos\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(b*cos(f*x+e))^n,x)
```

```
[Out] int((d*x+c)^m*(b*cos(f*x+e))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m\left(b\cos\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*cos(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*(b*cos(f*x + e))^n, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m\left(b\cos\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*cos(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*(b*cos(f*x + e))^n, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\left(b\cos\left(e+fx\right)\right)^n(c+dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(b*cos(f*x+e))**n,x)
```

```
[Out] Integral((b*cos(e + f*x))**n*(c + d*x)**m, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int(dx+c)^m\left(b\cos\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*cos(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*(b*cos(f*x + e))^n, x)
```

3.99 $\int (c+dx)^m \cos^3(a+bx)\,dx$

Optimal. Leaf size=275

$$-\frac{3ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}-\frac{i3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\right.}{8b}$$

```
[Out] (((-3*I)/8)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))
/d])/(b*(((-I)*b*(c + d*x))/d)^m) + (((3*I)/8)*(c + d*x)^m*Gamma[1 + m, (I*
b*(c + d*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) - ((I/8)*3^(
-1 - m)*E^((3*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x
))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/8)*3^(-1 - m)*(c + d*x)^m*Gamma[1
 + m, ((3*I)*b*(c + d*x))/d])/(b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d
)^m)
```

Rubi [A] time = 0.304003, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3312, 3307, 2181}

$$-\frac{3ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}-\frac{i3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^3,x]
```


```
[Out] (((-3*I)/8)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))
/d])/(b*(((-I)*b*(c + d*x))/d)^m) + (((3*I)/8)*(c + d*x)^m*Gamma[1 + m, (I*
b*(c + d*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) - ((I/8)*3^(
-1 - m)*E^((3*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x
))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/8)*3^(-1 - m)*(c + d*x)^m*Gamma[1
 + m, ((3*I)*b*(c + d*x))/d])/(b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d
)^m)
```


Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int (c+dx)^m \cos^3(a+bx)\,dx = \int \left(\frac{3}{4}(c+dx)^m \cos(a+bx) + \frac{1}{4}(c+dx)^m \cos(3a+3bx)\right) dx$$
$$= \frac{1}{4}\int (c+dx)^m \cos(3a+3bx)\,dx + \frac{3}{4}\int (c+dx)^m \cos(a+bx)\,dx$$
$$= \frac{1}{8}\int e^{-i(3a+3bx)}(c+dx)^m\,dx + \frac{1}{8}\int e^{i(3a+3bx)}(c+dx)^m\,dx + \frac{3}{8}\int e^{-i(a+bx)}(c+dx)^m\,dx +$$
$$= -\frac{3ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{8b} + \frac{3ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-}}{8b}$$

Mathematica [A] time = 0.182139, size = 253, normalized size = 0.92

$$\frac{i3^{-m-1}e^{-\frac{3i(ad+bc)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(3^{m+2}e^{2ia+\frac{4ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m \text{Gamma}\left(m+1,\frac{ib(c+dx)}{d}\right) - 3^{m+2}e^{2i\left(2a+\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^m\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^3,x]
```

```
[Out] ((I/8)*3^(-1 - m)*(c + d*x)^m*(-(3^(2 + m)*E^((2*I)*(2*a + (b*c)/d))*((I*b*
(c + d*x))/d)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d]) + 3^(2 + m)*E^((2*I)*a
+ ((4*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, (I*b*(c + d*x))/d] -
 E^((6*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d] + E
^(((6*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((3*I)*b*(c + d*x))/
d]))/(b*E^(((3*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m)
```

Maple [F] time = 0.342, size = 0, normalized size = 0.

$$\int (dx+c)^m (\cos(bx+a))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3, x)
```

Fricas [A] time = 1.84756, size = 470, normalized size = 1.71

$$\frac{ie^{\left(-\frac{dm\log\left(\frac{3ib}{d}\right)-3ibc+3iad}{d}\right)}\Gamma\left(m+1,\frac{3ibdx+3ibc}{d}\right)+9ie^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-ibc+iad}{d}\right)}\Gamma\left(m+1,\frac{ibdx+ibc}{d}\right)-9ie^{\left(-\frac{dm\log\left(-\frac{ib}{d}\right)+ibc-iad}{d}\right)}\Gamma\left(m+1,\frac{-ibd}{}\right.}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(I*e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*a*d)/d)*gamma(m + 1, (3*I*b*d
*x + 3*I*b*c)/d) + 9*I*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1,
 (I*b*d*x + I*b*c)/d) - 9*I*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma(
m + 1, (-I*b*d*x - I*b*c)/d) - I*e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*d
)/d)*gamma(m + 1, (-3*I*b*d*x - 3*I*b*c)/d))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^m \cos^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**m*cos(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3, x)
```

3.100 $\int (c+dx)^m \cos^2(a+bx)\,dx$

Optimal. Leaf size=162

$$-\frac{i2^{-m-3}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{i2^{-m-3}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}}{b}$$

```
[Out] (c + d*x)^(1 + m)/(2*d*(1 + m)) - (I*2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c
+ d*x)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)
 + (I*2^(-3 - m)*(c + d*x)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*
I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rubi [A] time = 0.213359, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3312, 3307, 2181}

$$-\frac{i2^{-m-3}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{i2^{-m-3}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(1 + m)/(2*d*(1 + m)) - (I*2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c
+ d*x)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)
 + (I*2^(-3 - m)*(c + d*x)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*
I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int (c+dx)^m \cos^2(a+bx)\,dx = \int \left(\frac{1}{2}(c+dx)^m + \frac{1}{2}(c+dx)^m \cos(2a+2bx)\right) dx$$
$$= \frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{1}{2}\int (c+dx)^m \cos(2a+2bx)\,dx$$
$$= \frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{1}{4}\int e^{-i(2a+2bx)}(c+dx)^m\,dx + \frac{1}{4}\int e^{i(2a+2bx)}(c+dx)^m\,dx$$
$$= \frac{(c+dx)^{1+m}}{2d(1+m)} - \frac{i2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} + \frac{i2^{-3-m}e^{-2i\left(a-\frac{b}{c}\right.}}{}$$

Mathematica [A] time = 0.198421, size = 150, normalized size = 0.93

$$\frac{1}{8}(c+dx)^m\left(-\frac{i2^{-m}e^{2i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b} + \frac{i2^{-m}e^{-2i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2ib(c+d}{d}\right.}{b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^2,x]
```

```
[Out] ((c + d*x)^m*((4*c + 4*d*x)/(d + d*m) - (I*E^((2*I)*(a - (b*c)/d))*Gamma[1
+ m, ((-2*I)*b*(c + d*x))/d])/(2^m*b*(((-I)*b*(c + d*x))/d)^m) + (I*Gamma[1
 + m, ((2*I)*b*(c + d*x))/d])/(2^m*b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x
))/d)^m)))/8
```

Maple [F] time = 0.287, size = 0, normalized size = 0.

$$\int (dx+c)^m (\cos(bx+a))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(dm+d)\int (dx+c)^m \cos(2\,bx+2\,a)\,dx + e^{(m\log(dx+c)+\log(dx+c))}}{2\,(dm+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((d*m + d)*integrate((d*x + c)^m*cos(2*b*x + 2*a), x) + e^(m*log(d*x +
c) + log(d*x + c)))/(d*m + d)
```

Fricas [A] time = 1.73254, size = 340, normalized size = 2.1

$$\frac{(i\,dm + i\,d)e^{\left(-\frac{dm\log\left(\frac{2i\,b}{d}\right) - 2i\,bc + 2i\,ad}{d}\right)}\Gamma\left(m+1, \frac{2i\,bdx + 2i\,bc}{d}\right) + (-i\,dm - i\,d)e^{\left(-\frac{dm\log\left(-\frac{2i\,b}{d}\right) + 2i\,bc - 2i\,ad}{d}\right)}\Gamma\left(m+1, \frac{-2i\,bdx - 2i\,bc}{d}\right) + 4\,(bdx}{8\,(bdm + bd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((I*d*m + I*d)*e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a*d)/d)*gamma(m +
1, (2*I*b*d*x + 2*I*b*c)/d) + (-I*d*m - I*d)*e^(-(d*m*log(-2*I*b/d) + 2*I*b
*c - 2*I*a*d)/d)*gamma(m + 1, (-2*I*b*d*x - 2*I*b*c)/d) + 4*(b*d*x + b*c)*(
d*x + c)^m)/(b*d*m + b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^m \cos^2(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*cos(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^2, x)
```

3.101 $\int (c+dx)^m \cos(a+bx)\,dx$

Optimal. Leaf size=131

$$\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}$$

```
[Out] ((-I/2)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])
/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/2)*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d
*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rubi [A] time = 0.0973537, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3307, 2181}

$$\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x],x]
```

```
[Out] ((-I/2)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])
/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/2)*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d
*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m\cos(a+bx)\,dx &= \frac{1}{2}\int e^{-i(a+bx)}(c+dx)^m\,dx+\frac{1}{2}\int e^{i(a+bx)}(c+dx)^m\,dx\\ &= -\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}+\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1\right.}{2b}\end{aligned}$$

Mathematica [A] time = 0.0484358, size = 122, normalized size = 0.93

$$-\frac{ie^{-\frac{i(ad+bc)}{d}}(c+dx)^m\left(e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)-e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{ib(c+dx)}{d}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x],x]
```

```
[Out] ((-I/2)*(c + d*x)^m*((E^((2*I)*a)*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(((-I
)*b*(c + d*x))/d)^m - (E^(((2*I)*b*c)/d)*Gamma[1 + m, (I*b*(c + d*x))/d])/(
(I*b*(c + d*x))/d)^m))/(b*E^((I*(b*c + a*d))/d))
```

Maple [F] time = 0.211, size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a),x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a), x)
```

Fricas [A] time = 1.76709, size = 223, normalized size = 1.7

$$\frac{ie^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-i\,bc+i\,ad}{d}\right)}\Gamma\left(m+1,\frac{i\,bdx+i\,bc}{d}\right)-ie^{\left(-\frac{dm\log\left(-\frac{ib}{d}\right)+i\,bc-i\,ad}{d}\right)}\Gamma\left(m+1,\frac{-i\,bdx-i\,bc}{d}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(I*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x + I*b*
c)/d) - I*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma(m + 1, (-I*b*d*x -
 I*b*c)/d))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^m \cos(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*cos(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a), x)
```

3.102 $\int (c+dx)^m \sec(a+bx)\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\sec(a+bx)(c+dx)^m,x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*Sec[a + b*x], x]
```

Rubi [A] time = 0.0188455, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \sec(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Sec[a + b*x],x]
```

```
[Out] Defer[Int][(c + d*x)^m*Sec[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \sec(a+bx)\,dx = \int (c+dx)^m \sec(a+bx)\,dx$$

Mathematica [A] time = 5.45638, size = 0, normalized size = 0.

$$\int (c+dx)^m \sec(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sec[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Sec[a + b*x], x]
```

Maple [A] time = 0.205, size = 0, normalized size = 0.

$$\int (dx+c)^m \sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a),x)
```

```
[Out] int((d*x+c)^m*sec(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m\sec(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*sec(b*x + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^m \sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*sec(a + b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a), x)
```

3.103 $\int (c+dx)^m \sec^2(a+bx)\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\sec^2(a+bx)(c+dx)^m,x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*Sec[a + b*x]^2, x]
```

Rubi [A] time = 0.0354351, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Sec[a + b*x]^2,x]

[Out] Defer[Int][(c + d*x)^m*Sec[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \sec^2(a+bx)\,dx = \int (c+dx)^m \sec^2(a+bx)\,dx$$

Mathematica [A] time = 0.825649, size = 0, normalized size = 0.

$$\int (c+dx)^m \sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sec[a + b*x]^2,x]

[Out] Integrate[(c + d*x)^m*Sec[a + b*x]^2, x]
```

Maple [A] time = 0.194, size = 0, normalized size = 0.

$$\int (dx+c)^m (\sec(bx+a))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)^2,x)

[Out] int((d*x+c)^m*sec(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sec(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m\sec(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*sec(b*x + a)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^m \sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*sec(a + b*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sec(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)^2, x)
```

3.104 $\int x^{3+m} \cos(a+bx)\,dx$

Optimal. Leaf size=75

$$-\frac{e^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+4,-ibx)}{2b^4}-\frac{e^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+4,ibx)}{2b^4}$$

```
[Out] -(E^(I*a)*x^m*Gamma[4 + m, (-I)*b*x])/(2*b^4*((-I)*b*x)^m) - (x^m*Gamma[4 +
 m, I*b*x])/(2*b^4*E^(I*a)*(I*b*x)^m)
```

Rubi [A] time = 0.0797813, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3307, 2181}

$$-\frac{e^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+4,-ibx)}{2b^4}-\frac{e^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+4,ibx)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 + m)*Cos[a + b*x],x]
```

```
[Out] -(E^(I*a)*x^m*Gamma[4 + m, (-I)*b*x])/(2*b^4*((-I)*b*x)^m) - (x^m*Gamma[4 +
 m, I*b*x])/(2*b^4*E^(I*a)*(I*b*x)^m)
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^{3+m}\cos(a+bx)\,dx &= \frac{1}{2}\int e^{-i(a+bx)}x^{3+m}\,dx+\frac{1}{2}\int e^{i(a+bx)}x^{3+m}\,dx\\ &= -\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(4+m,-ibx)}{2b^4}-\frac{e^{-ia}x^m(ibx)^{-m}\Gamma(4+m,ibx)}{2b^4}\end{aligned}$$

Mathematica [A] time = 0.018869, size = 75, normalized size = 1.

$$-\frac{e^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+4,-ibx)}{2b^4}-\frac{e^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+4,ibx)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 + m)*Cos[a + b*x],x]
```

```
[Out] -(E^(I*a)*x^m*Gamma[4 + m, (-I)*b*x])/(2*b^4*((-I)*b*x)^m) - (x^m*Gamma[4 +
 m, I*b*x])/(2*b^4*E^(I*a)*(I*b*x)^m)
```

Maple [C] time = 0.09, size = 455, normalized size = 6.1

$$\frac{2^{3+m}\sqrt{\pi}\cos(a)}{b^4}\left(b^2\right)^{-\frac{m}{2}}\left(3\,\frac{2^{-4-m}x^{3+m}b^3\left(b^2\right)^{m/2}(8/3+2/3\,m)\sin(bx)}{\sqrt{\pi}\,(4+m)}-\frac{2^{-3-m}x^{1+m}b\left(-m^2-7\,m-12\right)(\cos(bx)\,xb-\sin(}{\sqrt{\pi}\,(4+m)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3+m)*cos(b*x+a),x)
```

```
[Out] 2^(3+m)/b^4*(b^2)^(-1/2*m)*Pi^(1/2)*(3*2^(-4-m)/Pi^(1/2)/(4+m)*x^(3+m)*b^3*
(b^2)^(1/2*m)*(8/3+2/3*m)*sin(b*x)-2^(-3-m)/Pi^(1/2)/(4+m)*x^(1+m)*b*(b^2)^
(1/2*m)*(-m^2-7*m-12)*(cos(b*x)*x*b-sin(b*x))+2^(-3-m)/Pi^(1/2)/(4+m)*x^(2+
m)*b^2*(b^2)^(1/2*m)*(-m^3-8*m^2-19*m-12)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2
,b*x)*sin(b*x)-2^(-3-m)/Pi^(1/2)*x^(2+m)*b^2*(b^2)^(1/2*m)*(2+m)*(1+m)*(3+m
)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+1/2,1/2,b*x))*cos(a)-2^
(3+m)*b^(-4-m)*Pi^(1/2)*(2^(-3-m)/Pi^(1/2)/(5+m)*x^(2+m)*b^(2+m)*(m^2+7*m+1
0)*sin(b*x)-2^(-3-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(cos(b*x)*x*b-sin(b*x))-2^(-3
-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*m*(3+m)*(2+m)*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/
2,b*x)*sin(b*x)+2^(-3-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(3+m)*(2+m)*(b*x)^(-5/2-m
)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+3/2,1/2,b*x))*sin(a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+3}\cos(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 3)*cos(b*x + a), x)
```

Fricas [A] time = 1.70555, size = 153, normalized size = 2.04

$$\frac{i\,e^{(-(m+3)\log(i\,b)-i\,a)}\Gamma(m+4,i\,bx)-i\,e^{(-(m+3)\log(-i\,b)+i\,a)}\Gamma(m+4,-i\,bx)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(I*e^(-(m + 3)*log(I*b) - I*a)*gamma(m + 4, I*b*x) - I*e^(-(m + 3)*log(
-I*b) + I*a)*gamma(m + 4, -I*b*x))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3+m)*cos(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+3}\cos(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 3)*cos(b*x + a), x)
```

3.105 $\int x^{2+m}\cos(a+bx)\,dx$

Optimal. Leaf size=79

$$\frac{ie^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+3,-ibx)}{2b^3}-\frac{ie^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+3,ibx)}{2b^3}$$

```
[Out] ((I/2)*E^(I*a)*x^m*Gamma[3 + m, (-I)*b*x])/(b^3*((-I)*b*x)^m) - ((I/2)*x^m*
Gamma[3 + m, I*b*x])/(b^3*E^(I*a)*(I*b*x)^m)
```

Rubi [A] time = 0.0770637, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3307, 2181}

$$\frac{ie^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+3,-ibx)}{2b^3}-\frac{ie^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+3,ibx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 + m)*Cos[a + b*x],x]
```

```
[Out] ((I/2)*E^(I*a)*x^m*Gamma[3 + m, (-I)*b*x])/(b^3*((-I)*b*x)^m) - ((I/2)*x^m*
Gamma[3 + m, I*b*x])/(b^3*E^(I*a)*(I*b*x)^m)
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^{2+m}\cos(a+bx)\,dx &= \frac{1}{2}\int e^{-i(a+bx)}x^{2+m}\,dx+\frac{1}{2}\int e^{i(a+bx)}x^{2+m}\,dx\\ &= \frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(3+m,-ibx)}{2b^3}-\frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(3+m,ibx)}{2b^3}\end{aligned}$$

Mathematica [A] time = 0.0158999, size = 79, normalized size = 1.

$$\frac{ie^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+3,-ibx)}{2b^3}-\frac{ie^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+3,ibx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 + m)*Cos[a + b*x],x]
```

```
[Out] ((I/2)*E^(I*a)*x^m*Gamma[3 + m, (-I)*b*x])/(b^3*((-I)*b*x)^m) - ((I/2)*x^m*
Gamma[3 + m, I*b*x])/(b^3*E^(I*a)*(I*b*x)^m)
```

Maple [C] time = 0.078, size = 354, normalized size = 4.5

$$\frac{2^{2+m}\sqrt{\pi}\cos(a)}{b^2}\left(b^2\right)^{-\frac{1}{2}-\frac{m}{2}}\left(3\,\frac{2^{-3-m}x^{2+m}\left(b^2\right)^{3/2+m/2}(2+2/3\,m)\sin(bx)}{\sqrt{\pi}(3+m)\,b}-\frac{2^{-2-m}x^{2+m}(2+m)\,m\sin(bx)}{\sqrt{\pi}b}\left(b^2\right)^{\frac{3}{2}+\frac{m}{2}}(bx)^{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+m)*cos(b*x+a),x)
```

```
[Out] 2^(2+m)/b^2*(b^2)^(-1/2-1/2*m)*Pi^(1/2)*(3*2^(-3-m)/Pi^(1/2)/(3+m)*x^(2+m)*
(b^2)^(3/2+1/2*m)*(2+2/3*m)/b*sin(b*x)-2^(-2-m)/Pi^(1/2)*x^(2+m)*(b^2)^(3/2
+1/2*m)/b*(2+m)*m*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2,b*x)*sin(b*x)+2^(-2-m)/
Pi^(1/2)*x^(2+m)*(b^2)^(3/2+1/2*m)/b*(2+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin
(b*x))*LommelS1(m+3/2,1/2,b*x))*cos(a)-2^(2+m)*b^(-3-m)*Pi^(1/2)*(-2^(-2-m)
/Pi^(1/2)*x^(1+m)*b^(1+m)*(cos(b*x)*x*b-sin(b*x))+2^(-2-m)/Pi^(1/2)/(4+m)*x
^(2+m)*b^(2+m)*(m^2+5*m+4)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2,b*x)*sin(b*x)+
2^(-2-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(2+m)*(1+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-
sin(b*x))*LommelS1(m+1/2,1/2,b*x))*sin(a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+2}\cos(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 2)*cos(b*x + a), x)
```

Fricas [A] time = 1.68011, size = 153, normalized size = 1.94

$$\frac{i\,e^{(-(m+2)\log(i\,b)-i\,a)}\Gamma\left(m+3,i\,bx\right)-i\,e^{(-(m+2)\log(-i\,b)+i\,a)}\Gamma\left(m+3,-i\,bx\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(I*e^(-(m + 2)*log(I*b) - I*a)*gamma(m + 3, I*b*x) - I*e^(-(m + 2)*log(
-I*b) + I*a)*gamma(m + 3, -I*b*x))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+2}\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+m)*cos(b*x+a),x)
```

```
[Out] Integral(x**(m + 2)*cos(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+2}\cos(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 2)*cos(b*x + a), x)
```

3.106 $\int x^{1+m}\cos(a+bx)\,dx$

Optimal. Leaf size=75

$$\frac{e^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+2,-ibx)}{2b^2}+\frac{e^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+2,ibx)}{2b^2}$$

```
[Out] (E^(I*a)*x^m*Gamma[2 + m, (-I)*b*x])/(2*b^2*((-I)*b*x)^m) + (x^m*Gamma[2 +
m, I*b*x])/(2*b^2*E^(I*a)*(I*b*x)^m)
```

Rubi [A] time = 0.0766376, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3307, 2181}

$$\frac{e^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+2,-ibx)}{2b^2}+\frac{e^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+2,ibx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + m)*Cos[a + b*x],x]
```

```
[Out] (E^(I*a)*x^m*Gamma[2 + m, (-I)*b*x])/(2*b^2*((-I)*b*x)^m) + (x^m*Gamma[2 +
m, I*b*x])/(2*b^2*E^(I*a)*(I*b*x)^m)
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^{1+m}\cos(a+bx)\,dx&=\frac{1}{2}\int e^{-i(a+bx)}x^{1+m}\,dx+\frac{1}{2}\int e^{i(a+bx)}x^{1+m}\,dx\\&=\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(2+m,-ibx)}{2b^2}+\frac{e^{-ia}x^m(ibx)^{-m}\Gamma(2+m,ibx)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.0149496, size = 75, normalized size = 1.

$$\frac{e^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+2,-ibx)}{2b^2}+\frac{e^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+2,ibx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + m)*Cos[a + b*x],x]
```

```
[Out] (E^(I*a)*x^m*Gamma[2 + m, (-I)*b*x])/(2*b^2*((-I)*b*x)^m) + (x^m*Gamma[2 +
m, I*b*x])/(2*b^2*E^(I*a)*(I*b*x)^m)
```

Maple [C] time = 0.077, size = 291, normalized size = 3.9

$$\frac{2^{1+m}\sqrt{\pi}\cos(a)}{b^2}\left(b^2\right)^{-\frac{m}{2}}\left(\frac{2^{-1-m}x^{1+m}b\sin(bx)}{\sqrt{\pi}\,(2+m)}\left(b^2\right)^{\frac{m}{2}}+3\,\frac{2^{-2-m}x^{2+m}b^2\left(b^2\right)^{m/2}(2/3+2/3\,m)\,(bx)^{-3/2-m}\,\mathrm{LommelS1}\,(m+3}{\sqrt{\pi}\,(2+m)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+m)*cos(b*x+a),x)
```

```
[Out] 2^(1+m)/b^2*(b^2)^(-1/2*m)*Pi^(1/2)*(2^(-1-m)/Pi^(1/2)/(2+m)*x^(1+m)*b*(b^2
)^(1/2*m)*sin(b*x)+3*2^(-2-m)/Pi^(1/2)/(2+m)*x^(2+m)*b^2*(b^2)^(1/2*m)*(2/3
+2/3*m)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2,b*x)*sin(b*x)+2^(-1-m)/Pi^(1/2)*x
^(2+m)*b^2*(b^2)^(1/2*m)*(1+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*Lomme
lS1(m+1/2,1/2,b*x))*cos(a)-2^(1+m)*b^(-2-m)*Pi^(1/2)*(2^(-1-m)/Pi^(1/2)*x^(
2+m)*b^(2+m)*m*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2,b*x)*sin(b*x)-2^(-1-m)/Pi^
(1/2)*x^(2+m)*b^(2+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+3/2
,1/2,b*x))*sin(a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+1}\cos(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 1)*cos(b*x + a), x)
```

Fricas [A] time = 1.69074, size = 153, normalized size = 2.04

$$\frac{i\,e^{(-(m+1)\log(i\,b)-i\,a)}\Gamma\left(m+2,i\,bx\right)-i\,e^{(-(m+1)\log(-i\,b)+i\,a)}\Gamma\left(m+2,-i\,bx\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(I*e^(-(m + 1)*log(I*b) - I*a)*gamma(m + 2, I*b*x) - I*e^(-(m + 1)*log(
-I*b) + I*a)*gamma(m + 2, -I*b*x))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+1}\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)*cos(b*x+a),x)
```

```
[Out] Integral(x**(m + 1)*cos(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+1}\cos(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 1)*cos(b*x + a), x)
```

3.107 $\int x^m \cos(a+bx)\,dx$

Optimal. Leaf size=79

$$\frac{ie^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+1,ibx)}{2b}-\frac{ie^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+1,-ibx)}{2b}$$

```
[Out] ((-I/2)*E^(I*a)*x^m*Gamma[1 + m, (-I)*b*x])/(b*((-I)*b*x)^m) + ((I/2)*x^m*G
amma[1 + m, I*b*x])/(b*E^(I*a)*(I*b*x)^m)
```

Rubi [A] time = 0.0713828, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3307, 2181}

$$\frac{ie^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+1,ibx)}{2b}-\frac{ie^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+1,-ibx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cos[a + b*x],x]
```

```
[Out] ((-I/2)*E^(I*a)*x^m*Gamma[1 + m, (-I)*b*x])/(b*((-I)*b*x)^m) + ((I/2)*x^m*G
amma[1 + m, I*b*x])/(b*E^(I*a)*(I*b*x)^m)
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^m\cos(a+bx)\,dx &= \frac{1}{2}\int e^{-i(a+bx)}x^m\,dx+\frac{1}{2}\int e^{i(a+bx)}x^m\,dx\\ &= -\frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(1+m,-ibx)}{2b}+\frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(1+m,ibx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0137929, size = 79, normalized size = 1.

$$\frac{ie^{-ia}x^m(ibx)^{-m}\text{Gamma}(m+1,ibx)}{2b}-\frac{ie^{ia}x^m(-ibx)^{-m}\text{Gamma}(m+1,-ibx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cos[a + b*x],x]
```

```
[Out] ((-I/2)*E^(I*a)*x^m*Gamma[1 + m, (-I)*b*x])/(b*((-I)*b*x)^m) + ((I/2)*x^m*G
amma[1 + m, I*b*x])/(b*E^(I*a)*(I*b*x)^m)
```

Maple [C] time = 0.073, size = 379, normalized size = 4.8

$$2^{m}\left(b^{2}\right)^{-\frac{1}{2}-\frac{m}{2}}\sqrt{\pi}\left(3\,\frac{2^{-1-m}\left(b^{2}\right)^{1/2+m/2}x^{m}\left(6+2\,m\right)\sin\left(bx\right)}{\sqrt{\pi}\left(1+m\right)\left(9+3\,m\right)b}+\frac{x^{m}2^{-m}\left(\cos\left(bx\right)xb-\sin\left(bx\right)\right)}{\sqrt{\pi}\left(1+m\right)b}\left(b^{2}\right)^{\frac{1}{2}+\frac{m}{2}}+\frac{2^{-m}x^{2+m}bm\sin}{\sqrt{\pi}\left(1+m\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(b*x+a),x)
```

```
[Out] 2^m*(b^2)^(-1/2-1/2*m)*Pi^(1/2)*(3*2^(-1-m)/Pi^(1/2)/(1+m)*(b^2)^(1/2+1/2*m
)*x^m*(6+2*m)/(9+3*m)/b*sin(b*x)+1/Pi^(1/2)/(1+m)*(b^2)^(1/2+1/2*m)*x^m*2^(
-m)/b*(cos(b*x)*x*b-sin(b*x))+2^(-m)/Pi^(1/2)/(1+m)*x^(2+m)*(b^2)^(1/2+1/2*
m)*b*m*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2,b*x)*sin(b*x)-2^(-m)/Pi^(1/2)/(1+m
)*x^(2+m)*(b^2)^(1/2+1/2*m)*b*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*Lommel
S1(m+3/2,1/2,b*x))*cos(a)-2^m*b^(-1-m)*Pi^(1/2)*(1/Pi^(1/2)/(2+m)*x^(1+m)*b
^(1+m)*2^(-m)*sin(b*x)-2^(-m)/Pi^(1/2)/(2+m)*x^(2+m)*b^(2+m)*(b*x)^(-3/2-m)
*LommelS1(m+3/2,3/2,b*x)*sin(b*x)-3*2^(-1-m)/Pi^(1/2)/(2+m)*x^(2+m)*b^(2+m)
*(4/3+2/3*m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+1/2,1/2,b*x)
)*sin(a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m}\cos\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^m*cos(b*x + a), x)
```

Fricas [A] time = 1.69771, size = 136, normalized size = 1.72

$$\frac{i\,e^{\left(-m\log\left(i\,b\right)-i\,a\right)}\Gamma\left(m+1,i\,bx\right)-i\,e^{\left(-m\log\left(-i\,b\right)+i\,a\right)}\Gamma\left(m+1,-i\,bx\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(I*e^(-m*log(I*b) - I*a)*gamma(m + 1, I*b*x) - I*e^(-m*log(-I*b) + I*a)
*gamma(m + 1, -I*b*x))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m}\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cos(b*x+a),x)
```

```
[Out] Integral(x**m*cos(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \cos(bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*cos(b*x + a), x)
```

3.108 $\int x^{-1+m}\cos(a+bx)\,dx$

Optimal. Leaf size=65

$$-\frac{1}{2}e^{ia}x^m(-ibx)^{-m}\mathrm{Gamma}(m,-ibx)-\frac{1}{2}e^{-ia}x^m(ibx)^{-m}\mathrm{Gamma}(m,ibx)$$

```
[Out] -(E^(I*a)*x^m*Gamma[m, (-I)*b*x])/(2*((-I)*b*x)^m) - (x^m*Gamma[m, I*b*x])/
(2*E^(I*a)*(I*b*x)^m)
```

Rubi [A] time = 0.0731398, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3307, 2181}

$$-\frac{1}{2}e^{ia}x^m(-ibx)^{-m}\mathrm{Gamma}(m,-ibx)-\frac{1}{2}e^{-ia}x^m(ibx)^{-m}\mathrm{Gamma}(m,ibx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)*Cos[a + b*x],x]
```

```
[Out] -(E^(I*a)*x^m*Gamma[m, (-I)*b*x])/(2*((-I)*b*x)^m) - (x^m*Gamma[m, I*b*x])/
(2*E^(I*a)*(I*b*x)^m)
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^{-1+m}\cos(a+bx)\,dx &= \frac{1}{2}\int e^{-i(a+bx)}x^{-1+m}\,dx+\frac{1}{2}\int e^{i(a+bx)}x^{-1+m}\,dx\\ &= -\frac{1}{2}e^{ia}x^m(-ibx)^{-m}\Gamma(m,-ibx)-\frac{1}{2}e^{-ia}x^m(ibx)^{-m}\Gamma(m,ibx)\end{aligned}$$

Mathematica [A] time = 0.022577, size = 62, normalized size = 0.95

$$\frac{1}{2}e^{-ia}x^m\left(-e^{2ia}(-ibx)^{-m}\mathrm{Gamma}(m,-ibx)-(ibx)^{-m}\mathrm{Gamma}(m,ibx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)*Cos[a + b*x],x]
```

```
[Out] (x^m*(-((E^((2*I)*a)*Gamma[m, (-I)*b*x])/((-I)*b*x)^m) - Gamma[m, I*b*x]/(I
*b*x)^m))/(2*E^(I*a))
```

Maple [C] time = 0.078, size = 427, normalized size = 6.6

$$2^{-1+m}\left(b^2\right)^{-\frac{m}{2}}\sqrt{\pi}\left(3\,\frac{x^{-1+m}2^{-m}\left(b^2\right)^{m/2}\left(2\,x^2b^2+2\,m+4\right)\sin\left(bx\right)}{\sqrt{\pi}m\left(6+3\,m\right)b}+\frac{2^{1-m}x^{-1+m}\left(\cos\left(bx\right)xb-\sin\left(bx\right)\right)}{\sqrt{\pi}mb}\left(b^2\right)^{\frac{m}{2}}-3\,\frac{x^{2+m}2^{1}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)*cos(b*x+a),x)
```

```
[Out] 2^(-1+m)*(b^2)^(-1/2*m)*Pi^(1/2)*(3/Pi^(1/2)/m*x^(-1+m)*2^(-m)*(b^2)^(1/2*m
)*(2*b^2*x^2+2*m+4)/(6+3*m)/b*sin(b*x)+2^(1-m)/Pi^(1/2)/m*x^(-1+m)*(b^2)^(1
/2*m)/b*(cos(b*x)*x*b-sin(b*x))-3/Pi^(1/2)/m*x^(2+m)*2^(1-m)*(b^2)^(1/2*m)*
b^2/(6+3*m)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2,b*x)*sin(b*x)-1/Pi^(1/2)/m*x^
(2+m)*2^(1-m)*(b^2)^(1/2*m)*b^2*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*Lomm
elS1(m+1/2,1/2,b*x))*cos(a)-2^(-1+m)*b^(-m)*Pi^(1/2)*(2^(1-m)/Pi^(1/2)/(1+m
)*x^m*b^m*sin(b*x)-2^(1-m)/Pi^(1/2)/(1+m)*x^m*b^m/m*(cos(b*x)*x*b-sin(b*x))
-1/Pi^(1/2)/(1+m)*x^(2+m)*b^(2+m)*2^(1-m)*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2
,b*x)*sin(b*x)+1/Pi^(1/2)/(1+m)*x^(2+m)*b^(2+m)*2^(1-m)/m*(b*x)^(-5/2-m)*(c
os(b*x)*x*b-sin(b*x))*LommelS1(m+3/2,1/2,b*x))*sin(a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-1}\cos\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 1)*cos(b*x + a), x)
```

Fricas [A] time = 1.70119, size = 142, normalized size = 2.18

$$\frac{i\,e^{\left(-(m-1)\log(i\,b)-i\,a\right)}\Gamma\left(m,i\,bx\right)-i\,e^{\left(-(m-1)\log(-i\,b)+i\,a\right)}\Gamma\left(m,-i\,bx\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(I*e^(-(m - 1)*log(I*b) - I*a)*gamma(m, I*b*x) - I*e^(-(m - 1)*log(-I*b
) + I*a)*gamma(m, -I*b*x))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-1}\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*cos(b*x+a),x)
```

```
[Out] Integral(x**(m - 1)*cos(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-1}\cos\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 1)*cos(b*x + a), x)
```

3.109 $\int x^{-2+m}\cos(a+bx)\,dx$

Optimal. Leaf size=75

$$\frac{1}{2}ie^{ia}bx^m(-ibx)^{-m}\text{Gamma}(m-1,-ibx)-\frac{1}{2}ie^{-ia}bx^m(ibx)^{-m}\text{Gamma}(m-1,ibx)$$

```
[Out] ((I/2)*b*E^(I*a)*x^m*Gamma[-1 + m, (-I)*b*x])/((-I)*b*x)^m - ((I/2)*b*x^m*G
amma[-1 + m, I*b*x])/(E^(I*a)*(I*b*x)^m)
```

Rubi [A] time = 0.0742731, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3307, 2181}

$$\frac{1}{2}ie^{ia}bx^m(-ibx)^{-m}\text{Gamma}(m-1,-ibx)-\frac{1}{2}ie^{-ia}bx^m(ibx)^{-m}\text{Gamma}(m-1,ibx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + m)*Cos[a + b*x],x]
```

```
[Out] ((I/2)*b*E^(I*a)*x^m*Gamma[-1 + m, (-I)*b*x])/((-I)*b*x)^m - ((I/2)*b*x^m*G
amma[-1 + m, I*b*x])/(E^(I*a)*(I*b*x)^m)
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^{-2+m}\cos(a+bx)\,dx &= \frac{1}{2}\int e^{-i(a+bx)}x^{-2+m}\,dx+\frac{1}{2}\int e^{i(a+bx)}x^{-2+m}\,dx\\ &= \frac{1}{2}ibe^{ia}x^m(-ibx)^{-m}\Gamma(-1+m,-ibx)-\frac{1}{2}ibe^{-ia}x^m(ibx)^{-m}\Gamma(-1+m,ibx)\end{aligned}$$

Mathematica [A] time = 0.0154093, size = 75, normalized size = 1.

$$\frac{1}{2}ie^{ia}bx^m(-ibx)^{-m}\text{Gamma}(m-1,-ibx)-\frac{1}{2}ie^{-ia}bx^m(ibx)^{-m}\text{Gamma}(m-1,ibx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + m)*Cos[a + b*x],x]
```

```
[Out] ((I/2)*b*E^(I*a)*x^m*Gamma[-1 + m, (-I)*b*x])/((-I)*b*x)^m - ((I/2)*b*x^m*G
amma[-1 + m, I*b*x])/(E^(I*a)*(I*b*x)^m)
```

Maple [C] time = 0.082, size = 530, normalized size = 7.1

$$2^{m-2}\left(b^2\right)^{-\frac{1}{2}-\frac{m}{2}} b^2\sqrt{\pi}\left(3\,\frac{2^{1-m}x^{m-2}\left(b^2\right)^{-1/2+m/2}\left(2\,x^2b^2+2\,m+2\right)\sin\left(bx\right)}{\sqrt{\pi}\left(-1+m\right)\left(3+3\,m\right)b}-\frac{2^{2-m}x^{m-2}\left(x^2b^2-m^2-m\right)\left(\cos\left(bx\right)xb-\mathrm{s}\right.}{\sqrt{\pi}\left(-1+m\right)b\left(1+m\right)m}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m-2)*cos(b*x+a),x)
```

```
[Out] 2^(m-2)*(b^2)^(-1/2-1/2*m)*b^2*Pi^(1/2)*(3*2^(1-m)/Pi^(1/2)/(-1+m)*x^(m-2)*
(b^2)^(-1/2+1/2*m)*(2*b^2*x^2+2*m+2)/(3+3*m)/b*sin(b*x)-2^(2-m)/Pi^(1/2)/(-
1+m)*x^(m-2)*(b^2)^(-1/2+1/2*m)/b*(b^2*x^2-m^2-m)/(1+m)/m*(cos(b*x)*x*b-sin
(b*x))-3*2^(2-m)/Pi^(1/2)/(-1+m)*x^(2+m)*(b^2)^(-1/2+1/2*m)*b^3/(3+3*m)*(b*
x)^(-3/2-m)*LommelS1(m+1/2,3/2,b*x)*sin(b*x)+2^(2-m)/Pi^(1/2)/(-1+m)*x^(2+m
)*(b^2)^(-1/2+1/2*m)*b^3/(1+m)/m*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*Lom
melS1(m+3/2,1/2,b*x))*cos(a)-2^(m-2)*b^(1-m)*Pi^(1/2)*(2^(1-m)/Pi^(1/2)/m*x
^(-1+m)*b^(-1+m)*(-2*b^2*x^2+2*m^2+2*m-4)/(2+m)/(-1+m)*sin(b*x)-3*2^(2-m)/P
i^(1/2)/m*x^(-1+m)*b^(-1+m)/(-3+3*m)*(cos(b*x)*x*b-sin(b*x))+2^(2-m)/Pi^(1/
2)/m*x^(2+m)*b^(2+m)/(2+m)/(-1+m)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2,b*x)*si
n(b*x)+3*2^(2-m)/Pi^(1/2)/m*x^(2+m)*b^(2+m)/(-3+3*m)*(b*x)^(-5/2-m)*(cos(b*
x)*x*b-sin(b*x))*LommelS1(m+1/2,1/2,b*x))*sin(a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-2}\cos\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 2)*cos(b*x + a), x)
```

Fricas [A] time = 1.71395, size = 153, normalized size = 2.04

$$\frac{i\,e^{\left(-(m-2)\log(i\,b)-i\,a\right)}\Gamma\left(m-1,i\,bx\right)-i\,e^{\left(-(m-2)\log(-i\,b)+i\,a\right)}\Gamma\left(m-1,-i\,bx\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(I*e^(-(m - 2)*log(I*b) - I*a)*gamma(m - 1, I*b*x) - I*e^(-(m - 2)*log(
-I*b) + I*a)*gamma(m - 1, -I*b*x))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-2}\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+m)*cos(b*x+a),x)
```

```
[Out] Integral(x**(m - 2)*cos(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-2}\cos(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 2)*cos(b*x + a), x)
```

3.110 $\int x^{-3+m} \cos(a+bx)\, dx$

Optimal. Leaf size=75

$$\frac{1}{2}e^{ia}b^2x^m(-ibx)^{-m}\text{Gamma}(m-2,-ibx)+\frac{1}{2}e^{-ia}b^2x^m(ibx)^{-m}\text{Gamma}(m-2,ibx)$$

```
[Out] (b^2*E^(I*a)*x^m*Gamma[-2 + m, (-I)*b*x])/(2*((-I)*b*x)^m) + (b^2*x^m*Gamma
[-2 + m, I*b*x])/(2*E^(I*a)*(I*b*x)^m)
```

Rubi [A] time = 0.074068, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3307, 2181}

$$\frac{1}{2}e^{ia}b^2x^m(-ibx)^{-m}\text{Gamma}(m-2,-ibx)+\frac{1}{2}e^{-ia}b^2x^m(ibx)^{-m}\text{Gamma}(m-2,ibx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + m)*Cos[a + b*x],x]
```

```
[Out] (b^2*E^(I*a)*x^m*Gamma[-2 + m, (-I)*b*x])/(2*((-I)*b*x)^m) + (b^2*x^m*Gamma
[-2 + m, I*b*x])/(2*E^(I*a)*(I*b*x)^m)
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^{-3+m}\cos(a+bx)\,dx&=\frac{1}{2}\int e^{-i(a+bx)}x^{-3+m}\,dx+\frac{1}{2}\int e^{i(a+bx)}x^{-3+m}\,dx\\&=\frac{1}{2}b^2e^{ia}x^m(-ibx)^{-m}\Gamma(-2+m,-ibx)+\frac{1}{2}b^2e^{-ia}x^m(ibx)^{-m}\Gamma(-2+m,ibx)\end{aligned}$$

Mathematica [A] time = 0.0151217, size = 75, normalized size = 1.

$$\frac{1}{2}e^{ia}b^2x^m(-ibx)^{-m}\text{Gamma}(m-2,-ibx)+\frac{1}{2}e^{-ia}b^2x^m(ibx)^{-m}\text{Gamma}(m-2,ibx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + m)*Cos[a + b*x],x]
```

```
[Out] (b^2*E^(I*a)*x^m*Gamma[-2 + m, (-I)*b*x])/(2*((-I)*b*x)^m) + (b^2*x^m*Gamma
[-2 + m, I*b*x])/(2*E^(I*a)*(I*b*x)^m)
```

Maple [C] time = 0.086, size = 600, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m-3)*cos(b*x+a),x)
```

```
[Out] 2^(m-3)*(b^2)^(-1/2*m)*b^2*Pi^(1/2)*(2^(2-m)/Pi^(1/2)/(m-2)*x^(m-3)/b^3*(b^
2)^(1/2*m)*(-2*b^4*x^4+2*b^2*m^2*x^2+2*b^2*m*x^2-4*b^2*x^2+2*m^3+2*m^2-4*m)
/m/(2+m)/(-1+m)*sin(b*x)-2^(-m+3)/Pi^(1/2)/(m-2)*x^(m-3)/b^3*(b^2)^(1/2*m)*
(b^2*x^2-m^2+m)/m/(-1+m)*(cos(b*x)*x*b-sin(b*x))+2^(-m+3)/Pi^(1/2)/(m-2)*x^
(2+m)*b^2*(b^2)^(1/2*m)/m/(2+m)/(-1+m)*(b*x)^(-3/2-m)*LommelS1(m+3/2,3/2,b*
x)*sin(b*x)+2^(-m+3)/Pi^(1/2)/(m-2)*x^(2+m)*b^2*(b^2)^(1/2*m)/m/(-1+m)*(b*x
)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(m+1/2,1/2,b*x))*cos(a)-2^(m-3)*
b^(2-m)*Pi^(1/2)*(2^(2-m)/Pi^(1/2)/(-1+m)*x^(m-2)*b^(m-2)*(-2*b^2*x^2+2*m^2
-2*m-4)/(1+m)/(m-2)*sin(b*x)+2^(-m+3)/Pi^(1/2)/(-1+m)*x^(m-2)*b^(m-2)*(b^2*
x^2-m^2-m)/(1+m)/(m-2)/m*(cos(b*x)*x*b-sin(b*x))+2^(-m+3)/Pi^(1/2)/(-1+m)*x
^(2+m)*b^(2+m)/(1+m)/(m-2)*(b*x)^(-3/2-m)*LommelS1(m+1/2,3/2,b*x)*sin(b*x)-
2^(-m+3)/Pi^(1/2)/(-1+m)*x^(2+m)*b^(2+m)/(1+m)/(m-2)/m*(b*x)^(-5/2-m)*(cos(
b*x)*x*b-sin(b*x))*LommelS1(m+3/2,1/2,b*x))*sin(a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-3} \cos(bx + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 3)*cos(b*x + a), x)
```

Fricas [A] time = 1.65332, size = 153, normalized size = 2.04

$$\frac{i\, e^{(-(m-3)\log(i\,b) - i\,a)}\Gamma\left(m-2, i\,bx\right) - i\, e^{(-(m-3)\log(-i\,b) + i\,a)}\Gamma\left(m-2, -i\,bx\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(I*e^(-(m - 3)*log(I*b) - I*a)*gamma(m - 2, I*b*x) - I*e^(-(m - 3)*log(
-I*b) + I*a)*gamma(m - 2, -I*b*x))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+m)*cos(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-3} \cos(bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 3)*cos(b*x + a), x)
```

3.111 $\int x^{3+m}\cos^2(a+bx)\,dx$

Optimal. Leaf size=99

$$-\frac{e^{2ia}2^{-m-6}x^m(-ibx)^{-m}\text{Gamma}(m+4,-2ibx)}{b^4}-\frac{e^{-2ia}2^{-m-6}x^m(ibx)^{-m}\text{Gamma}(m+4,2ibx)}{b^4}+\frac{x^{m+4}}{2(m+4)}$$

```
[Out] x^(4 + m)/(2*(4 + m)) - (2^(-6 - m)*E^((2*I)*a)*x^m*Gamma[4 + m, (-2*I)*b*x
])/(b^4*((-I)*b*x)^m) - (2^(-6 - m)*x^m*Gamma[4 + m, (2*I)*b*x])/(b^4*E^((2
*I)*a)*(I*b*x)^m)
```

Rubi [A] time = 0.156664, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$-\frac{e^{2ia}2^{-m-6}x^m(-ibx)^{-m}\text{Gamma}(m+4,-2ibx)}{b^4}-\frac{e^{-2ia}2^{-m-6}x^m(ibx)^{-m}\text{Gamma}(m+4,2ibx)}{b^4}+\frac{x^{m+4}}{2(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 + m)*Cos[a + b*x]^2,x]
```

```
[Out] x^(4 + m)/(2*(4 + m)) - (2^(-6 - m)*E^((2*I)*a)*x^m*Gamma[4 + m, (-2*I)*b*x
])/(b^4*((-I)*b*x)^m) - (2^(-6 - m)*x^m*Gamma[4 + m, (2*I)*b*x])/(b^4*E^((2
*I)*a)*(I*b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int x^{3+m}\cos^2(a+bx)\,dx &= \int \left(\frac{x^{3+m}}{2}+\frac{1}{2}x^{3+m}\cos(2a+2bx)\right)dx \\
&= \frac{x^{4+m}}{2(4+m)}+\frac{1}{2}\int x^{3+m}\cos(2a+2bx)\,dx \\
&= \frac{x^{4+m}}{2(4+m)}+\frac{1}{4}\int e^{-i(2a+2bx)}x^{3+m}\,dx+\frac{1}{4}\int e^{i(2a+2bx)}x^{3+m}\,dx \\
&= \frac{x^{4+m}}{2(4+m)}-\frac{2^{-6-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(4+m,-2ibx)}{b^4}-\frac{2^{-6-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(4+m,2ibx)}{b^4}
\end{aligned}
$$

Mathematica [A] time = 0.111821, size = 92, normalized size = 0.93

$$
\frac{1}{64}x^m\left(-\frac{e^{2ia}2^{-m}(-ibx)^{-m}\text{Gamma}(m+4,-2ibx)}{b^4}-\frac{e^{-2ia}2^{-m}(ibx)^{-m}\text{Gamma}(m+4,2ibx)}{b^4}+\frac{32x^4}{m+4}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 + m)*Cos[a + b*x]^2,x]
```

```
[Out] (x^m*((32*x^4)/(4 + m) - (E^((2*I)*a)*Gamma[4 + m, (-2*I)*b*x])/(2^m*b^4*((
-I)*b*x)^m) - Gamma[4 + m, (2*I)*b*x]/(2^m*b^4*E^((2*I)*a)*(I*b*x)^m)))/64
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$
\int x^{3+m}\left(\cos\left(bx+a\right)\right)^2\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3+m)*cos(b*x+a)^2,x)
```

```
[Out] int(x^(3+m)*cos(b*x+a)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.69977, size = 235, normalized size = 2.37

$$
\frac{4\,bxx^{m+3}+(i\,m+4i)e^{(-(m+3)\log(2i\,b)-2i\,a)}\Gamma\left(m+4,2i\,bx\right)+(-i\,m-4i)e^{(-(m+3)\log(-2i\,b)+2i\,a)}\Gamma\left(m+4,-2i\,bx\right)}{8\left(bm+4\,b\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^(m + 3) + (I*m + 4*I)*e^(-(m + 3)*log(2*I*b) - 2*I*a)*gamma(m
+ 4, 2*I*b*x) + (-I*m - 4*I)*e^(-(m + 3)*log(-2*I*b) + 2*I*a)*gamma(m + 4,
-2*I*b*x))/(b*m + 4*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3+m)*cos(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+3} \cos(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 3)*cos(b*x + a)^2, x)
```

3.112 $\int x^{2+m} \cos^2(a+bx)\,dx$

Optimal. Leaf size=103

$$\frac{ie^{2ia}2^{-m-5}x^m(-ibx)^{-m}\text{Gamma}(m+3,-2ibx)}{b^3}-\frac{ie^{-2ia}2^{-m-5}x^m(ibx)^{-m}\text{Gamma}(m+3,2ibx)}{b^3}+\frac{x^{m+3}}{2(m+3)}$$

```
[Out] x^(3 + m)/(2*(3 + m)) + (I*2^(-5 - m)*E^((2*I)*a)*x^m*Gamma[3 + m, (-2*I)*b
*x])/(b^3*((-I)*b*x)^m) - (I*2^(-5 - m)*x^m*Gamma[3 + m, (2*I)*b*x])/(b^3*E
^((2*I)*a)*(I*b*x)^m)
```

Rubi [A] time = 0.143654, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$\frac{ie^{2ia}2^{-m-5}x^m(-ibx)^{-m}\text{Gamma}(m+3,-2ibx)}{b^3}-\frac{ie^{-2ia}2^{-m-5}x^m(ibx)^{-m}\text{Gamma}(m+3,2ibx)}{b^3}+\frac{x^{m+3}}{2(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 + m)*Cos[a + b*x]^2,x]
```

```
[Out] x^(3 + m)/(2*(3 + m)) + (I*2^(-5 - m)*E^((2*I)*a)*x^m*Gamma[3 + m, (-2*I)*b
*x])/(b^3*((-I)*b*x)^m) - (I*2^(-5 - m)*x^m*Gamma[3 + m, (2*I)*b*x])/(b^3*E
^((2*I)*a)*(I*b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^{2+m}\cos^2(a+bx)\,dx &= \int\left(\frac{x^{2+m}}{2}+\frac{1}{2}x^{2+m}\cos(2a+2bx)\right)dx\\ &= \frac{x^{3+m}}{2(3+m)}+\frac{1}{2}\int x^{2+m}\cos(2a+2bx)\,dx\\ &= \frac{x^{3+m}}{2(3+m)}+\frac{1}{4}\int e^{-i(2a+2bx)}x^{2+m}\,dx+\frac{1}{4}\int e^{i(2a+2bx)}x^{2+m}\,dx\\ &= \frac{x^{3+m}}{2(3+m)}+\frac{i2^{-5-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(3+m,-2ibx)}{b^3}-\frac{i2^{-5-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(3+m,2ibx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.103247, size = 96, normalized size = 0.93

$$\frac{1}{32}x^m\left(\frac{ie^{2ia}2^{-m}(-ibx)^{-m}\text{Gamma}(m+3,-2ibx)}{b^3}-\frac{ie^{-2ia}2^{-m}(ibx)^{-m}\text{Gamma}(m+3,2ibx)}{b^3}+\frac{16x^3}{m+3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 + m)*Cos[a + b*x]^2,x]
```

```
[Out] (x^m*((16*x^3)/(3 + m) + (I*E^((2*I)*a)*Gamma[3 + m, (-2*I)*b*x])/(2^m*b^3*
((-I)*b*x)^m) - (I*Gamma[3 + m, (2*I)*b*x])/(2^m*b^3*E^((2*I)*a)*(I*b*x)^m)
))/32
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int x^{2+m}(\cos(bx+a))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+m)*cos(b*x+a)^2,x)
```

```
[Out] int(x^(2+m)*cos(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(m+3)\int x^2x^m\cos(2\,bx+2\,a)\,dx+e^{(m\log(x)+3\log(x))}}{2\,(m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((m + 3)*integrate(x^2*x^m*cos(2*b*x + 2*a), x) + e^(m*log(x) + 3*log(x
)))/(m + 3)
```

Fricas [A] time = 1.67912, size = 235, normalized size = 2.28

$$\frac{4\,bxx^{m+2}+(i\,m+3i)e^{(-(m+2)\log(2i\,b)-2i\,a)}\Gamma(m+3,2i\,bx)+(-i\,m-3i)e^{(-(m+2)\log(-2i\,b)+2i\,a)}\Gamma(m+3,-2i\,bx)}{8\,(bm+3\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^(m + 2) + (I*m + 3*I)*e^(-(m + 2)*log(2*I*b) - 2*I*a)*gamma(m
+ 3, 2*I*b*x) + (-I*m - 3*I)*e^(-(m + 2)*log(-2*I*b) + 2*I*a)*gamma(m + 3,
-2*I*b*x))/(b*m + 3*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+m)*cos(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+2}\cos\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 2)*cos(b*x + a)^2, x)
```

3.113 $\int x^{1+m}\cos^2(a+bx)\,dx$

Optimal. Leaf size=97

$$\frac{e^{2ia}2^{-m-4}x^m(-ibx)^{-m}\text{Gamma}(m+2,-2ibx)}{b^2}+\frac{e^{-2ia}2^{-m-4}x^m(ibx)^{-m}\text{Gamma}(m+2,2ibx)}{b^2}+\frac{x^{m+2}}{2(m+2)}$$

```
[Out] x^(2 + m)/(2*(2 + m)) + (2^(-4 - m)*E^((2*I)*a)*x^m*Gamma[2 + m, (-2*I)*b*x
])/(b^2*((-I)*b*x)^m) + (2^(-4 - m)*x^m*Gamma[2 + m, (2*I)*b*x])/(b^2*E^((2
*I)*a)*(I*b*x)^m)
```

Rubi [A] time = 0.137478, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$\frac{e^{2ia}2^{-m-4}x^m(-ibx)^{-m}\text{Gamma}(m+2,-2ibx)}{b^2}+\frac{e^{-2ia}2^{-m-4}x^m(ibx)^{-m}\text{Gamma}(m+2,2ibx)}{b^2}+\frac{x^{m+2}}{2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + m)*Cos[a + b*x]^2,x]
```

```
[Out] x^(2 + m)/(2*(2 + m)) + (2^(-4 - m)*E^((2*I)*a)*x^m*Gamma[2 + m, (-2*I)*b*x
])/(b^2*((-I)*b*x)^m) + (2^(-4 - m)*x^m*Gamma[2 + m, (2*I)*b*x])/(b^2*E^((2
*I)*a)*(I*b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^{1+m}\cos^2(a+bx)\,dx &= \int\left(\frac{x^{1+m}}{2}+\frac{1}{2}x^{1+m}\cos(2a+2bx)\right)dx\\ &= \frac{x^{2+m}}{2(2+m)}+\frac{1}{2}\int x^{1+m}\cos(2a+2bx)\,dx\\ &= \frac{x^{2+m}}{2(2+m)}+\frac{1}{4}\int e^{-i(2a+2bx)}x^{1+m}\,dx+\frac{1}{4}\int e^{i(2a+2bx)}x^{1+m}\,dx\\ &= \frac{x^{2+m}}{2(2+m)}+\frac{2^{-4-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(2+m,-2ibx)}{b^2}+\frac{2^{-4-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(2+m,2ibx)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.0975459, size = 90, normalized size = 0.93

$$\frac{1}{16}x^m\left(\frac{e^{2ia}2^{-m}(-ibx)^{-m}\mathrm{Gamma}(m+2,-2ibx)}{b^2}+\frac{e^{-2ia}2^{-m}(ibx)^{-m}\mathrm{Gamma}(m+2,2ibx)}{b^2}+\frac{8x^2}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + m)*Cos[a + b*x]^2,x]
```

```
[Out] (x^m*((8*x^2)/(2 + m) + (E^((2*I)*a)*Gamma[2 + m, (-2*I)*b*x])/(2^m*b^2*((-
I)*b*x)^m) + Gamma[2 + m, (2*I)*b*x]/(2^m*b^2*E^((2*I)*a)*(I*b*x)^m)))/16
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int x^{1+m}\left(\cos\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+m)*cos(b*x+a)^2,x)
```

```
[Out] int(x^(1+m)*cos(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(m+2)\int xx^m\cos\left(2\,bx+2\,a\right)\,dx+e^{(m\log(x)+2\,\log(x))}}{2\,(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((m + 2)*integrate(x*x^m*cos(2*b*x + 2*a), x) + e^(m*log(x) + 2*log(x))
)/(m + 2)
```

Fricas [A] time = 2.1073, size = 235, normalized size = 2.42

$$\frac{4\,bxx^{m+1}+(i\,m+2i)e^{(-(m+1)\log(2i\,b)-2i\,a)}\Gamma\left(m+2,2i\,bx\right)+(-i\,m-2i)e^{(-(m+1)\log(-2i\,b)+2i\,a)}\Gamma\left(m+2,-2i\,bx\right)}{8\,(bm+2\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^(m + 1) + (I*m + 2*I)*e^(-(m + 1)*log(2*I*b) - 2*I*a)*gamma(m
+ 2, 2*I*b*x) + (-I*m - 2*I)*e^(-(m + 1)*log(-2*I*b) + 2*I*a)*gamma(m + 2,
-2*I*b*x))/(b*m + 2*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+1}\cos^2{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)*cos(b*x+a)**2,x)
```

```
[Out] Integral(x**(m + 1)*cos(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+1}\cos(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 1)*cos(b*x + a)^2, x)
```

3.114 $\int x^m \cos^2(a+bx)\,dx$

Optimal. Leaf size=103

$$-\frac{ie^{2ia}2^{-m-3}x^m(-ibx)^{-m}\mathrm{Gamma}(m+1,-2ibx)}{b}+\frac{ie^{-2ia}2^{-m-3}x^m(ibx)^{-m}\mathrm{Gamma}(m+1,2ibx)}{b}+\frac{x^{m+1}}{2(m+1)}$$

```
[Out] x^(1 + m)/(2*(1 + m)) - (I*2^(-3 - m)*E^((2*I)*a)*x^m*Gamma[1 + m, (-2*I)*b
*x])/(b*((-I)*b*x)^m) + (I*2^(-3 - m)*x^m*Gamma[1 + m, (2*I)*b*x])/(b*E^((2
*I)*a)*(I*b*x)^m)
```

Rubi [A] time = 0.132942, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3312, 3307, 2181}

$$-\frac{ie^{2ia}2^{-m-3}x^m(-ibx)^{-m}\mathrm{Gamma}(m+1,-2ibx)}{b}+\frac{ie^{-2ia}2^{-m-3}x^m(ibx)^{-m}\mathrm{Gamma}(m+1,2ibx)}{b}+\frac{x^{m+1}}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cos[a + b*x]^2,x]
```

```
[Out] x^(1 + m)/(2*(1 + m)) - (I*2^(-3 - m)*E^((2*I)*a)*x^m*Gamma[1 + m, (-2*I)*b
*x])/(b*((-I)*b*x)^m) + (I*2^(-3 - m)*x^m*Gamma[1 + m, (2*I)*b*x])/(b*E^((2
*I)*a)*(I*b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x^m \cos^2(a+bx)\,dx = \int \left(\frac{x^m}{2}+\frac{1}{2}x^m\cos(2a+2bx)\right)dx$$
$$= \frac{x^{1+m}}{2(1+m)}+\frac{1}{2}\int x^m\cos(2a+2bx)\,dx$$
$$= \frac{x^{1+m}}{2(1+m)}+\frac{1}{4}\int e^{-i(2a+2bx)}x^m\,dx+\frac{1}{4}\int e^{i(2a+2bx)}x^m\,dx$$
$$= \frac{x^{1+m}}{2(1+m)}-\frac{i2^{-3-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(1+m,-2ibx)}{b}+\frac{i2^{-3-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(1+m,2ibx)}{b}$$

Mathematica [A] time = 0.11927, size = 90, normalized size = 0.87

$$\frac{1}{8}x^m\left(e^{2ia}\left(-2^{-m}\right)x(-ibx)^{-m-1}\mathrm{Gamma}(m+1,-2ibx)-e^{-2ia}2^{-m}x(ibx)^{-m-1}\mathrm{Gamma}(m+1,2ibx)+\frac{4x}{m+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cos[a + b*x]^2,x]
```

```
[Out] (x^m*((4*x)/(1 + m) - (E^((2*I)*a)*x*((-I)*b*x)^(-1 - m)*Gamma[1 + m, (-2*I
)*b*x])/2^m - (x*(I*b*x)^(-1 - m)*Gamma[1 + m, (2*I)*b*x])/(2^m*E^((2*I)*a)
)))/8
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int x^m\left(\cos\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(b*x+a)^2,x)
```

```
[Out] int(x^m*cos(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(m+1)\int x^m\cos\left(2\,bx+2\,a\right)\,dx+e^{(m\log(x)+\log(x))}}{2\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((m + 1)*integrate(x^m*cos(2*b*x + 2*a), x) + e^(m*log(x) + log(x)))/(m
 + 1)
```

Fricas [A] time = 1.94255, size = 203, normalized size = 1.97

$$\frac{4\,bxx^m+(i\,m+i)e^{(-m\log(2i\,b)-2i\,a)}\Gamma\left(m+1,2i\,bx\right)+(-i\,m-i)e^{(-m\log(-2i\,b)+2i\,a)}\Gamma\left(m+1,-2i\,bx\right)}{8\,(bm+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^m + (I*m + I)*e^(-m*log(2*I*b) - 2*I*a)*gamma(m + 1, 2*I*b*x)
+ (-I*m - I)*e^(-m*log(-2*I*b) + 2*I*a)*gamma(m + 1, -2*I*b*x))/(b*m + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \cos^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cos(b*x+a)**2,x)
```

```
[Out] Integral(x**m*cos(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \cos(bx + a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*cos(b*x + a)^2, x)
```

3.115 $\int x^{-1+m}\cos^2(a+bx)\,dx$

Optimal. Leaf size=85

$$e^{2ia}\left(-2^{-m-2}\right)x^m(-ibx)^{-m}\text{Gamma}(m,-2ibx)-e^{-2ia}2^{-m-2}x^m(ibx)^{-m}\text{Gamma}(m,2ibx)+\frac{x^m}{2m}$$

```
[Out] x^m/(2*m) - (2^(-2 - m)*E^((2*I)*a)*x^m*Gamma[m, (-2*I)*b*x])/((-I)*b*x)^m
- (2^(-2 - m)*x^m*Gamma[m, (2*I)*b*x])/(E^((2*I)*a)*(I*b*x)^m)
```

Rubi [A] time = 0.126421, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$e^{2ia}\left(-2^{-m-2}\right)x^m(-ibx)^{-m}\text{Gamma}(m,-2ibx)-e^{-2ia}2^{-m-2}x^m(ibx)^{-m}\text{Gamma}(m,2ibx)+\frac{x^m}{2m}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)*Cos[a + b*x]^2,x]
```

```
[Out] x^m/(2*m) - (2^(-2 - m)*E^((2*I)*a)*x^m*Gamma[m, (-2*I)*b*x])/((-I)*b*x)^m
- (2^(-2 - m)*x^m*Gamma[m, (2*I)*b*x])/(E^((2*I)*a)*(I*b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^{-1+m}\cos^2(a+bx)\,dx &= \int\left(\frac{x^{-1+m}}{2}+\frac{1}{2}x^{-1+m}\cos(2a+2bx)\right)dx\\
&= \frac{x^m}{2m}+\frac{1}{2}\int x^{-1+m}\cos(2a+2bx)\,dx\\
&= \frac{x^m}{2m}+\frac{1}{4}\int e^{-i(2a+2bx)}x^{-1+m}\,dx+\frac{1}{4}\int e^{i(2a+2bx)}x^{-1+m}\,dx\\
&= \frac{x^m}{2m}-2^{-2-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(m,-2ibx)-2^{-2-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(m,2ibx)
\end{aligned}$$

Mathematica [A] time = 0.0583079, size = 77, normalized size = 0.91

$$\frac{1}{4}x^m\left(e^{2ia}\left(-2^{-m}\right)(-ibx)^{-m}\text{Gamma}(m,-2ibx)-e^{-2ia}2^{-m}(ibx)^{-m}\text{Gamma}(m,2ibx)+\frac{2}{m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)*Cos[a + b*x]^2,x]
```

```
[Out] (x^m*(2/m - (E^((2*I)*a)*Gamma[m, (-2*I)*b*x])/(2^m*((-I)*b*x)^m) - Gamma[m
, (2*I)*b*x]/(2^m*E^((2*I)*a)*(I*b*x)^m)))/4
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int x^{-1+m}\left(\cos\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)*cos(b*x+a)^2,x)
```

```
[Out] int(x^(-1+m)*cos(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{m\int\frac{x^m\cos(2\,bx+2\,a)}{x}\,dx+x^m}{2\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(m*integrate(x^m*cos(2*b*x + 2*a)/x, x) + x^m)/m
```

Fricas [A] time = 1.95532, size = 193, normalized size = 2.27

$$\frac{4\,bxx^{m-1}+i\,me^{(-(m-1)\log(2i\,b)-2i\,a)}\Gamma\left(m,2i\,bx\right)-i\,me^{(-(m-1)\log(-2i\,b)+2i\,a)}\Gamma\left(m,-2i\,bx\right)}{8\,bm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^(m - 1) + I*m*e^(-(m - 1)*log(2*I*b) - 2*I*a)*gamma(m, 2*I*b*x
) - I*m*e^(-(m - 1)*log(-2*I*b) + 2*I*a)*gamma(m, -2*I*b*x))/(b*m)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-1}\cos^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*cos(b*x+a)**2,x)
```

```
[Out] Integral(x**(m - 1)*cos(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-1} \cos(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 1)*cos(b*x + a)^2, x)
```

3.116 $\int x^{-2+m} \cos^2(a+bx)\,dx$

Optimal. Leaf size=101

$$ie^{2ia}b2^{-m-1}x^m(-ibx)^{-m}\text{Gamma}(m-1,-2ibx)-ie^{-2ia}b2^{-m-1}x^m(ibx)^{-m}\text{Gamma}(m-1,2ibx)-\frac{x^{m-1}}{2(1-m)}$$

```
[Out] -x^(-1 + m)/(2*(1 - m)) + (I*2^(-1 - m)*b*E^((2*I)*a)*x^m*Gamma[-1 + m, (-2
*I)*b*x])/((-I)*b*x)^m - (I*2^(-1 - m)*b*x^m*Gamma[-1 + m, (2*I)*b*x])/(E^(
(2*I)*a)*(I*b*x)^m)
```

Rubi [A] time = 0.137988, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$ie^{2ia}b2^{-m-1}x^m(-ibx)^{-m}\text{Gamma}(m-1,-2ibx)-ie^{-2ia}b2^{-m-1}x^m(ibx)^{-m}\text{Gamma}(m-1,2ibx)-\frac{x^{m-1}}{2(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + m)*Cos[a + b*x]^2,x]
```

```
[Out] -x^(-1 + m)/(2*(1 - m)) + (I*2^(-1 - m)*b*E^((2*I)*a)*x^m*Gamma[-1 + m, (-2
*I)*b*x])/((-I)*b*x)^m - (I*2^(-1 - m)*b*x^m*Gamma[-1 + m, (2*I)*b*x])/(E^(
(2*I)*a)*(I*b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\int x^{-2+m}\cos^2(a+bx)\,dx = \int\left(\frac{x^{-2+m}}{2}+\frac{1}{2}x^{-2+m}\cos(2a+2bx)\right)dx$$
$$= -\frac{x^{-1+m}}{2(1-m)}+\frac{1}{2}\int x^{-2+m}\cos(2a+2bx)\,dx$$
$$= -\frac{x^{-1+m}}{2(1-m)}+\frac{1}{4}\int e^{-i(2a+2bx)}x^{-2+m}\,dx+\frac{1}{4}\int e^{i(2a+2bx)}x^{-2+m}\,dx$$
$$= -\frac{x^{-1+m}}{2(1-m)}+i2^{-1-m}be^{2ia}x^m(-ibx)^{-m}\Gamma(-1+m,-2ibx)-i2^{-1-m}be^{-2ia}x^m(ibx)^{-m}\Gamma(-1+m,2ibx$$

Mathematica [A] time = 0.107953, size = 91, normalized size = 0.9

$$\frac{1}{2}x^m\left(ie^{2ia}b2^{-m}(-ibx)^{-m}\text{Gamma}(m-1,-2ibx)-ie^{-2ia}b2^{-m}(ibx)^{-m}\text{Gamma}(m-1,2ibx)+\frac{1}{(m-1)x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + m)*Cos[a + b*x]^2,x]
```

```
[Out] (x^m*(1/((-1 + m)*x) + (I*b*E^((2*I)*a)*Gamma[-1 + m, (-2*I)*b*x])/(2^m*((-
I)*b*x)^m) - (I*b*Gamma[-1 + m, (2*I)*b*x])/(2^m*E^((2*I)*a)*(I*b*x)^m)))/2
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int x^{m-2}\left(\cos(bx+a)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m-2)*cos(b*x+a)^2,x)
```

```
[Out] int(x^(m-2)*cos(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(m-1)x\int\frac{x^m\cos(2\,bx+2\,a)}{x^2}\,dx+x^m}{2\,(m-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((m - 1)*x*integrate(x^m*cos(2*b*x + 2*a)/x^2, x) + x^m)/((m - 1)*x)
```

Fricas [A] time = 2.035, size = 227, normalized size = 2.25

$$\frac{4\,bxx^{m-2}+(i\,m-i)e^{(-(m-2)\log(2i\,b)-2i\,a)}\Gamma(m-1,2i\,bx)+(-i\,m+i)e^{(-(m-2)\log(-2i\,b)+2i\,a)}\Gamma(m-1,-2i\,bx)}{8\,(bm-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^(m - 2) + (I*m - I)*e^(-(m - 2)*log(2*I*b) - 2*I*a)*gamma(m -
1, 2*I*b*x) + (-I*m + I)*e^(-(m - 2)*log(-2*I*b) + 2*I*a)*gamma(m - 1, -2*I
*b*x))/(b*m - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+m)*cos(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-2} \cos(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 2)*cos(b*x + a)^2, x)
```

3.117 $\int x^{-3+m}\cos^2(a+bx)\,dx$

Optimal. Leaf size=95

$$e^{2ia}b^2 2^{-m}x^m(-ibx)^{-m}\text{Gamma}(m-2,-2ibx)+e^{-2ia}b^2 2^{-m}x^m(ibx)^{-m}\text{Gamma}(m-2,2ibx)-\frac{x^{m-2}}{2(2-m)}$$

```
[Out] -x^(-2 + m)/(2*(2 - m)) + (b^2*E^((2*I)*a)*x^m*Gamma[-2 + m, (-2*I)*b*x])/(
2^m*((-I)*b*x)^m) + (b^2*x^m*Gamma[-2 + m, (2*I)*b*x])/(2^m*E^((2*I)*a)*(I*
b*x)^m)
```

Rubi [A] time = 0.141895, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$e^{2ia}b^2 2^{-m}x^m(-ibx)^{-m}\text{Gamma}(m-2,-2ibx)+e^{-2ia}b^2 2^{-m}x^m(ibx)^{-m}\text{Gamma}(m-2,2ibx)-\frac{x^{m-2}}{2(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + m)*Cos[a + b*x]^2,x]
```


```
[Out] -x^(-2 + m)/(2*(2 - m)) + (b^2*E^((2*I)*a)*x^m*Gamma[-2 + m, (-2*I)*b*x])/(
2^m*((-I)*b*x)^m) + (b^2*x^m*Gamma[-2 + m, (2*I)*b*x])/(2^m*E^((2*I)*a)*(I*
b*x)^m)
```


Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x^{-3+m}\cos^2(a+bx)\,dx=\int\left(\frac{x^{-3+m}}{2}+\frac{1}{2}x^{-3+m}\cos(2a+2bx)\right)dx$$
$$=-\frac{x^{-2+m}}{2(2-m)}+\frac{1}{2}\int x^{-3+m}\cos(2a+2bx)\,dx$$
$$=-\frac{x^{-2+m}}{2(2-m)}+\frac{1}{4}\int e^{-i(2a+2bx)}x^{-3+m}\,dx+\frac{1}{4}\int e^{i(2a+2bx)}x^{-3+m}\,dx$$
$$=-\frac{x^{-2+m}}{2(2-m)}+2^{-m}b^2e^{2ia}x^m(-ibx)^{-m}\Gamma(-2+m,-2ibx)+2^{-m}b^2e^{-2ia}x^m(ibx)^{-m}\Gamma(-2+m,2ibx)$$

Mathematica [A] time = 0.0920469, size = 95, normalized size = 1.

$$e^{2ia}b^2 2^{-m}x^m(-ibx)^{-m}\text{Gamma}(m-2,-2ibx)+e^{-2ia}b^2 2^{-m}x^m(ibx)^{-m}\text{Gamma}(m-2,2ibx)-\frac{x^{m-2}}{2(2-m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + m)*Cos[a + b*x]^2,x]
```

```
[Out] -x^(-2 + m)/(2*(2 - m)) + (b^2*E^((2*I)*a)*x^m*Gamma[-2 + m, (-2*I)*b*x])/(
2^m*((-I)*b*x)^m) + (b^2*x^m*Gamma[-2 + m, (2*I)*b*x])/(2^m*E^((2*I)*a)*(I*
b*x)^m)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int x^{m-3}\left(\cos\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m-3)*cos(b*x+a)^2,x)
```

```
[Out] int(x^(m-3)*cos(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(m-2)x^2\int\frac{x^m\cos(2\,bx+2\,a)}{x^3}\,dx+x^m}{2\,(m-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((m - 2)*x^2*integrate(x^m*cos(2*b*x + 2*a)/x^3, x) + x^m)/((m - 2)*x^2
)
```

Fricas [A] time = 1.83474, size = 235, normalized size = 2.47

$$\frac{4\,bxx^{m-3}+(i\,m-2i)e^{(-(m-3)\log(2i\,b)-2i\,a)}\Gamma\left(m-2,2i\,bx\right)+(-i\,m+2i)e^{(-(m-3)\log(-2i\,b)+2i\,a)}\Gamma\left(m-2,-2i\,bx\right)}{8\,(bm-2\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*x^(m - 3) + (I*m - 2*I)*e^(-(m - 3)*log(2*I*b) - 2*I*a)*gamma(m
- 2, 2*I*b*x) + (-I*m + 2*I)*e^(-(m - 3)*log(-2*I*b) + 2*I*a)*gamma(m - 2,
-2*I*b*x))/(b*m - 2*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+m)*cos(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-3} \cos(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 3)*cos(b*x + a)^2, x)
```

3.118 $\int (c+dx)^3(a+a\cos(e+fx))\,dx$

Optimal. Leaf size=89

$$-\frac{6ad^2(c+dx)\sin(e+fx)}{f^3}+\frac{3ad(c+dx)^2\cos(e+fx)}{f^2}+\frac{a(c+dx)^3\sin(e+fx)}{f}+\frac{a(c+dx)^4}{4d}-\frac{6ad^3\cos(e+fx)}{f^4}$$

```
[Out] (a*(c + d*x)^4)/(4*d) - (6*a*d^3*Cos[e + f*x])/f^4 + (3*a*d*(c + d*x)^2*Cos
[e + f*x])/f^2 - (6*a*d^2*(c + d*x)*Sin[e + f*x])/f^3 + (a*(c + d*x)^3*Sin[
e + f*x])/f
```

Rubi [A] time = 0.115207, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3317, 3296, 2638}

$$-\frac{6ad^2(c+dx)\sin(e+fx)}{f^3}+\frac{3ad(c+dx)^2\cos(e+fx)}{f^2}+\frac{a(c+dx)^3\sin(e+fx)}{f}+\frac{a(c+dx)^4}{4d}-\frac{6ad^3\cos(e+fx)}{f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + a*Cos[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^4)/(4*d) - (6*a*d^3*Cos[e + f*x])/f^4 + (3*a*d*(c + d*x)^2*Cos
[e + f*x])/f^2 - (6*a*d^2*(c + d*x)*Sin[e + f*x])/f^3 + (a*(c + d*x)^3*Sin[
e + f*x])/f
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3(a+a\cos(e+fx))\,dx &= \int \left(a(c+dx)^3+a(c+dx)^3\cos(e+fx)\right)\,dx\\ &= \frac{a(c+dx)^4}{4d}+a\int (c+dx)^3\cos(e+fx)\,dx\\ &= \frac{a(c+dx)^4}{4d}+\frac{a(c+dx)^3\sin(e+fx)}{f}-\frac{(3ad)\int (c+dx)^2\sin(e+fx)\,dx}{f}\\ &= \frac{a(c+dx)^4}{4d}+\frac{3ad(c+dx)^2\cos(e+fx)}{f^2}+\frac{a(c+dx)^3\sin(e+fx)}{f}-\frac{\left(6ad^2\right)\int (c+dx)\,c}{f^2}\\ &= \frac{a(c+dx)^4}{4d}+\frac{3ad(c+dx)^2\cos(e+fx)}{f^2}-\frac{6ad^2(c+dx)\sin(e+fx)}{f^3}+\frac{a(c+dx)^3\sin(e}{f}\\ &= \frac{a(c+dx)^4}{4d}-\frac{6ad^3\cos(e+fx)}{f^4}+\frac{3ad(c+dx)^2\cos(e+fx)}{f^2}-\frac{6ad^2(c+dx)\sin(e+fx}{f^3}\end{aligned}$$

Mathematica [A] time = 0.557306, size = 122, normalized size = 1.37

$$a\left(\frac{(c+dx)\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2-6\right)\right)\sin(e+fx)}{f^3}+\frac{3d\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2-2\right)\right)\cos(e+fx)}{f^4}+\frac{1}{4}x\left(6c^2dx+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + a*Cos[e + f*x]),x]
```

```
[Out] a*((x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 + (3*d*(c^2*f^2 + 2*c*
d*f^2*x + d^2*(-2 + f^2*x^2))*Cos[e + f*x])/f^4 + ((c + d*x)*(c^2*f^2 + 2*c
*d*f^2*x + d^2*(-6 + f^2*x^2))*Sin[e + f*x])/f^3)
```

Maple [B] time = 0.038, size = 476, normalized size = 5.4

$$\frac{1}{f}\left(\frac{ad^3\left(\left(fx+e\right)^3\sin\left(fx+e\right)+3\left(fx+e\right)^2\cos\left(fx+e\right)-6\cos\left(fx+e\right)-6\left(fx+e\right)\sin\left(fx+e\right)\right)}{f^3}+3\frac{acd^2\left(\left(fx+e\right)^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+a*cos(f*x+e)),x)
```

```
[Out] 1/f*(a/f^3*d^3*((f*x+e)^3*sin(f*x+e)+3*(f*x+e)^2*cos(f*x+e)-6*cos(f*x+e)-6*
(f*x+e)*sin(f*x+e))+3*a/f^2*c*d^2*((f*x+e)^2*sin(f*x+e)-2*sin(f*x+e)+2*(f*x
+e)*cos(f*x+e))-3*a/f^3*d^3*e*((f*x+e)^2*sin(f*x+e)-2*sin(f*x+e)+2*(f*x+e)*
cos(f*x+e))+3*a/f*c^2*d*(cos(f*x+e)+(f*x+e)*sin(f*x+e))-6*a/f^2*c*d^2*e*(co
s(f*x+e)+(f*x+e)*sin(f*x+e))+3*a/f^3*d^3*e^2*(cos(f*x+e)+(f*x+e)*sin(f*x+e)
)+a*c^3*sin(f*x+e)-3*a/f*c^2*d*e*sin(f*x+e)+3*a/f^2*c*d^2*e^2*sin(f*x+e)-a/
f^3*d^3*e^3*sin(f*x+e)+1/4*a/f^3*d^3*(f*x+e)^4+a/f^2*c*d^2*(f*x+e)^3-a/f^3*
d^3*e*(f*x+e)^3+3/2*a/f*c^2*d*(f*x+e)^2-3*a/f^2*c*d^2*e*(f*x+e)^2+3/2*a/f^3
*d^3*e^2*(f*x+e)^2+a*c^3*(f*x+e)-3*a/f*c^2*d*e*(f*x+e)+3*a/f^2*c*d^2*e^2*(f
*x+e)-a/f^3*d^3*e^3*(f*x+e))
```

Maxima [B] time = 1.31746, size = 616, normalized size = 6.92

$$4\left(fx+e\right)ac^3+\frac{\left(fx+e\right)^4ad^3}{f^3}-\frac{4\left(fx+e\right)^3ad^3e}{f^3}+\frac{6\left(fx+e\right)^2ad^3e^2}{f^3}-\frac{4\left(fx+e\right)ad^3e^3}{f^3}+\frac{4\left(fx+e\right)^3acd^2}{f^2}-\frac{12\left(fx+e\right)^2acd^2e}{f^2}+\frac{12\left(fx+e\right)acd^2e^2}{f^2}+\frac{6\left(fx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a*c^3 + (f*x + e)^4*a*d^3/f^3 - 4*(f*x + e)^3*a*d^3*e/f^3
+ 6*(f*x + e)^2*a*d^3*e^2/f^3 - 4*(f*x + e)*a*d^3*e^3/f^3 + 4*(f*x + e)^3*a
*c*d^2/f^2 - 12*(f*x + e)^2*a*c*d^2*e/f^2 + 12*(f*x + e)*a*c*d^2*e^2/f^2 +
6*(f*x + e)^2*a*c^2*d/f - 12*(f*x + e)*a*c^2*d*e/f + 4*a*c^3*sin(f*x + e) -
 4*a*d^3*e^3*sin(f*x + e)/f^3 + 12*a*c*d^2*e^2*sin(f*x + e)/f^2 - 12*a*c^2*
d*e*sin(f*x + e)/f + 12*((f*x + e)*sin(f*x + e) + cos(f*x + e))*a*d^3*e^2/f
^3 - 24*((f*x + e)*sin(f*x + e) + cos(f*x + e))*a*c*d^2*e/f^2 + 12*((f*x +
e)*sin(f*x + e) + cos(f*x + e))*a*c^2*d/f - 12*(2*(f*x + e)*cos(f*x + e) +
((f*x + e)^2 - 2)*sin(f*x + e))*a*d^3*e/f^3 + 12*(2*(f*x + e)*cos(f*x + e)
+ ((f*x + e)^2 - 2)*sin(f*x + e))*a*c*d^2/f^2 + 4*(3*((f*x + e)^2 - 2)*cos(
f*x + e) + ((f*x + e)^3 - 6*f*x - 6*e)*sin(f*x + e))*a*d^3/f^3)/f
```

Fricas [A] time = 1.68985, size = 362, normalized size = 4.07

$$\frac{ad^3f^4x^4 + 4\,acd^2f^4x^3 + 6\,ac^2df^4x^2 + 4\,ac^3f^4x + 12\left(ad^3f^2x^2 + 2\,acd^2f^2x + ac^2df^2 - 2\,ad^3\right)\cos\left(fx+e\right) + 4\left(ad^3f^3x\right.}{4\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4*x
+ 12*(a*d^3*f^2*x^2 + 2*a*c*d^2*f^2*x + a*c^2*d*f^2 - 2*a*d^3)*cos(f*x + e)
 + 4*(a*d^3*f^3*x^3 + 3*a*c*d^2*f^3*x^2 + a*c^3*f^3 - 6*a*c*d^2*f + 3*(a*c^
2*d*f^3 - 2*a*d^3*f)*x)*sin(f*x + e))/f^4
```

Sympy [A] time = 1.65896, size = 264, normalized size = 2.97

$$\begin{cases} ac^3x + \frac{ac^3\sin\left(e+fx\right)}{f} + \frac{3ac^2dx^2}{2} + \frac{3ac^2dx\sin\left(e+fx\right)}{f} + \frac{3ac^2d\cos\left(e+fx\right)}{f^2} + acd^2x^3 + \frac{3acd^2x^2\sin\left(e+fx\right)}{f} + \frac{6acd^2x\cos\left(e+fx\right)}{f^2} - \frac{6acd^2\sin\left(\right.}{f^3} \\ \left(a\cos\left(e\right)+a\right)\left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+a*cos(f*x+e)),x)
```

```
[Out] Piecewise((a*c**3*x + a*c**3*sin(e + f*x)/f + 3*a*c**2*d*x**2/2 + 3*a*c**2*
d*x*sin(e + f*x)/f + 3*a*c**2*d*cos(e + f*x)/f**2 + a*c*d**2*x**3 + 3*a*c*d
**2*x**2*sin(e + f*x)/f + 6*a*c*d**2*x*cos(e + f*x)/f**2 - 6*a*c*d**2*sin(e
 + f*x)/f**3 + a*d**3*x**4/4 + a*d**3*x**3*sin(e + f*x)/f + 3*a*d**3*x**2*c
os(e + f*x)/f**2 - 6*a*d**3*x*sin(e + f*x)/f**3 - 6*a*d**3*cos(e + f*x)/f**
4, Ne(f, 0)), ((a*cos(e) + a)*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**
3*x**4/4), True))
```

Giac [A] time = 1.15335, size = 211, normalized size = 2.37

$$\frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x + \frac{3\left(ad^3f^2x^2 + 2\,acd^2f^2x + ac^2df^2 - 2\,ad^3\right)\cos\left(fx+e\right)}{f^4} + \frac{\left(ad^3f^3x^3 + 3\,acd^2f^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 3*(a*d^3*f^2*x^2
+ 2*a*c*d^2*f^2*x + a*c^2*d*f^2 - 2*a*d^3)*cos(f*x + e)/f^4 + (a*d^3*f^3*x^
3 + 3*a*c*d^2*f^3*x^2 + 3*a*c^2*d*f^3*x + a*c^3*f^3 - 6*a*d^3*f*x - 6*a*c*d
^2*f)*sin(f*x + e)/f^4
```

3.119 $\int (c+dx)^2(a+a\cos(e+fx))\,dx$

Optimal. Leaf size=67

$$\frac{2ad(c+dx)\cos(e+fx)}{f^2}+\frac{a(c+dx)^2\sin(e+fx)}{f}+\frac{a(c+dx)^3}{3d}-\frac{2ad^2\sin(e+fx)}{f^3}$$

```
[Out] (a*(c + d*x)^3)/(3*d) + (2*a*d*(c + d*x)*Cos[e + f*x])/f^2 - (2*a*d^2*Sin[e
 + f*x])/f^3 + (a*(c + d*x)^2*Sin[e + f*x])/f
```

Rubi [A] time = 0.0845239, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3317, 3296, 2637}

$$\frac{2ad(c+dx)\cos(e+fx)}{f^2}+\frac{a(c+dx)^2\sin(e+fx)}{f}+\frac{a(c+dx)^3}{3d}-\frac{2ad^2\sin(e+fx)}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + a*Cos[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^3)/(3*d) + (2*a*d*(c + d*x)*Cos[e + f*x])/f^2 - (2*a*d^2*Sin[e
 + f*x])/f^3 + (a*(c + d*x)^2*Sin[e + f*x])/f
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2(a+a\cos(e+fx))\,dx &= \int \left(a(c+dx)^2+a(c+dx)^2\cos(e+fx)\right)dx\\
&= \frac{a(c+dx)^3}{3d}+a\int (c+dx)^2\cos(e+fx)\,dx\\
&= \frac{a(c+dx)^3}{3d}+\frac{a(c+dx)^2\sin(e+fx)}{f}-\frac{(2ad)\int (c+dx)\sin(e+fx)\,dx}{f}\\
&= \frac{a(c+dx)^3}{3d}+\frac{2ad(c+dx)\cos(e+fx)}{f^2}+\frac{a(c+dx)^2\sin(e+fx)}{f}-\frac{\left(2ad^2\right)\int \cos(e\cdots}{f^2}\\
&= \frac{a(c+dx)^3}{3d}+\frac{2ad(c+dx)\cos(e+fx)}{f^2}-\frac{2ad^2\sin(e+fx)}{f^3}+\frac{a(c+dx)^2\sin(e+fx)}{f}
\end{aligned}$$

Mathematica [A] time = 0.334301, size = 80, normalized size = 1.19

$$a\left(\frac{\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2-2\right)\right)\sin(e+fx)}{f^3}+c^2x+\frac{2d(c+dx)\cos(e+fx)}{f^2}+cdx^2+\frac{d^2x^3}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + a*Cos[e + f*x]),x]
```

```
[Out] a*(c^2*x + c*d*x^2 + (d^2*x^3)/3 + (2*d*(c + d*x)*Cos[e + f*x])/f^2 + ((c^2
*f^2 + 2*c*d*f^2*x + d^2*(-2 + f^2*x^2))*Sin[e + f*x])/f^3)
```

Maple [B] time = 0.04, size = 236, normalized size = 3.5

$$\frac{1}{f}\left(\frac{ad^2\left(\left(fx+e\right)^2\sin\left(fx+e\right)-2\sin\left(fx+e\right)+2\left(fx+e\right)\cos\left(fx+e\right)\right)}{f^2}+2\frac{acd\left(\cos\left(fx+e\right)+\left(fx+e\right)\sin\left(fx+e\right)\right)}{f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+a*cos(f*x+e)),x)
```

```
[Out] 1/f*(a/f^2*d^2*((f*x+e)^2*sin(f*x+e)-2*sin(f*x+e)+2*(f*x+e)*cos(f*x+e))+2*a
/f*c*d*(cos(f*x+e)+(f*x+e)*sin(f*x+e))-2*a/f^2*d^2*e*(cos(f*x+e)+(f*x+e)*si
n(f*x+e))+a*c^2*sin(f*x+e)-2*a/f*c*d*e*sin(f*x+e)+a/f^2*d^2*e^2*sin(f*x+e)+
1/3*a/f^2*d^2*(f*x+e)^3+a/f*c*d*(f*x+e)^2-a/f^2*d^2*e*(f*x+e)^2+a*c^2*(f*x+
e)-2*a/f*c*d*e*(f*x+e)+a/f^2*d^2*e^2*(f*x+e))
```

Maxima [B] time = 1.21301, size = 317, normalized size = 4.73

$$\frac{3\left(fx+e\right)ac^2+\frac{\left(fx+e\right)^3ad^2}{f^2}-\frac{3\left(fx+e\right)^2ad^2e}{f^2}+\frac{3\left(fx+e\right)ad^2e^2}{f^2}+\frac{3\left(fx+e\right)^2acd}{f}-\frac{6\left(fx+e\right)acde}{f}+3\,ac^2\sin\left(fx+e\right)+\frac{3\,ad^2e^2\sin\left(fx+e\right)}{f^2}-\cdots}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*a*c^2 + (f*x + e)^3*a*d^2/f^2 - 3*(f*x + e)^2*a*d^2*e/f^2
+ 3*(f*x + e)*a*d^2*e^2/f^2 + 3*(f*x + e)^2*a*c*d/f - 6*(f*x + e)*a*c*d*e/f
 + 3*a*c^2*sin(f*x + e) + 3*a*d^2*e^2*sin(f*x + e)/f^2 - 6*a*c*d*e*sin(f*x
+ e)/f - 6*((f*x + e)*sin(f*x + e) + cos(f*x + e))*a*d^2*e/f^2 + 6*((f*x +
e)*sin(f*x + e) + cos(f*x + e))*a*c*d/f + 3*(2*(f*x + e)*cos(f*x + e) + ((f
*x + e)^2 - 2)*sin(f*x + e))*a*d^2/f^2)/f
```

Fricas [A] time = 1.63074, size = 228, normalized size = 3.4

$$\frac{ad^2f^3x^3+3\,acdf^3x^2+3\,ac^2f^3x+6\left(ad^2fx+acdf\right)\cos\left(fx+e\right)+3\left(ad^2f^2x^2+2\,acdf^2x+ac^2f^2-2\,ad^2\right)\sin\left(fx+e\right)}{3\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/3*(a*d^2*f^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x + 6*(a*d^2*f*x + a*c*d
*f)*cos(f*x + e) + 3*(a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2 - 2*a*d^2)*
sin(f*x + e))/f^3
```

Sympy [A] time = 0.785503, size = 151, normalized size = 2.25

$$\begin{cases} ac^2x + \frac{ac^2\sin(e+fx)}{f} + acdx^2 + \frac{2acdx\sin(e+fx)}{f} + \frac{2acd\cos(e+fx)}{f^2} + \frac{ad^2x^3}{3} + \frac{ad^2x^2\sin(e+fx)}{f} + \frac{2ad^2x\cos(e+fx)}{f^2} - \frac{2ad^2\sin(e+fx)}{f^3} \\ (a\cos(e)+a)\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+a*cos(f*x+e)),x)
```

```
[Out] Piecewise((a*c**2*x + a*c**2*sin(e + f*x)/f + a*c*d*x**2 + 2*a*c*d*x*sin(e
+ f*x)/f + 2*a*c*d*cos(e + f*x)/f**2 + a*d**2*x**3/3 + a*d**2*x**2*sin(e +
f*x)/f + 2*a*d**2*x*cos(e + f*x)/f**2 - 2*a*d**2*sin(e + f*x)/f**3, Ne(f, 0
)), ((a*cos(e) + a)*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

Giac [A] time = 1.17491, size = 127, normalized size = 1.9

$$\frac{1}{3}ad^2x^3 + acdx^2 + ac^2x + \frac{2\left(ad^2fx + acdf\right)\cos\left(fx+e\right)}{f^3} + \frac{\left(ad^2f^2x^2 + 2\,acdf^2x + ac^2f^2 - 2\,ad^2\right)\sin\left(fx+e\right)}{f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + 2*(a*d^2*f*x + a*c*d*f)*cos(f*x + e)/
f^3 + (a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2 - 2*a*d^2)*sin(f*x + e)/f^
3
```

3.120 $\int (c+dx)(a+a\cos(e+fx))\,dx$

Optimal. Leaf size=44

$$\frac{a(c+dx)\sin(e+fx)}{f}+\frac{a(c+dx)^2}{2d}+\frac{ad\cos(e+fx)}{f^2}$$

```
[Out] (a*(c + d*x)^2)/(2*d) + (a*d*Cos[e + f*x])/f^2 + (a*(c + d*x)*Sin[e + f*x])
/f
```

Rubi [A] time = 0.0423506, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3317, 3296, 2638}

$$\frac{a(c+dx)\sin(e+fx)}{f}+\frac{a(c+dx)^2}{2d}+\frac{ad\cos(e+fx)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + a*Cos[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^2)/(2*d) + (a*d*Cos[e + f*x])/f^2 + (a*(c + d*x)*Sin[e + f*x])
/f
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)(a+a\cos(e+fx))\,dx &= \int (a(c+dx)+a(c+dx)\cos(e+fx))\,dx\\
&= \frac{a(c+dx)^2}{2d}+a\int (c+dx)\cos(e+fx)\,dx\\
&= \frac{a(c+dx)^2}{2d}+\frac{a(c+dx)\sin(e+fx)}{f}-\frac{(ad)\int \sin(e+fx)\,dx}{f}\\
&= \frac{a(c+dx)^2}{2d}+\frac{ad\cos(e+fx)}{f^2}+\frac{a(c+dx)\sin(e+fx)}{f}
\end{aligned}$$

Mathematica [A] time = 0.229933, size = 52, normalized size = 1.18

$$\frac{a(-2(e+fx)(-2cf+de-dfx)+4f(c+dx)\sin(e+fx)+4d\cos(e+fx))}{4f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + a*Cos[e + f*x]),x]
```

```
[Out] (a*(-2*(e + f*x)*(d*e - 2*c*f - d*f*x) + 4*d*Cos[e + f*x] + 4*f*(c + d*x)*S
in[e + f*x]))/(4*f^2)
```

Maple [B] time = 0.033, size = 89, normalized size = 2.

$$\frac{1}{f}\left(\frac{da\left(\cos\left(fx+e\right)+\left(fx+e\right)\sin\left(fx+e\right)\right)}{f}+ca\sin\left(fx+e\right)-\frac{ade\sin\left(fx+e\right)}{f}+\frac{da\left(fx+e\right)^{2}}{2f}+ca\left(fx+e\right)-\frac{ade\,(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+a*cos(f*x+e)),x)
```

```
[Out] 1/f*(a/f*d*(cos(f*x+e)+(f*x+e)*sin(f*x+e))+c*a*sin(f*x+e)-a/f*d*e*sin(f*x+e
)+1/2*a/f*d*(f*x+e)^2+c*a*(f*x+e)-a/f*d*e*(f*x+e))
```

Maxima [B] time = 1.16764, size = 123, normalized size = 2.8

$$\frac{2\left(fx+e\right)ac+\frac{\left(fx+e\right)^{2}ad}{f}-\frac{2\left(fx+e\right)ade}{f}+2\,ac\sin\left(fx+e\right)-\frac{2\,ade\sin\left(fx+e\right)}{f}+\frac{2\left(\left(fx+e\right)\sin\left(fx+e\right)+\cos\left(fx+e\right)\right)ad}{f}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(f*x + e)*a*c + (f*x + e)^2*a*d/f - 2*(f*x + e)*a*d*e/f + 2*a*c*sin(
f*x + e) - 2*a*d*e*sin(f*x + e)/f + 2*((f*x + e)*sin(f*x + e) + cos(f*x + e
))*a*d/f)/f
```

Fricas [A] time = 1.62376, size = 126, normalized size = 2.86

$$\frac{adf^2x^2+2\,acf^2x+2\,ad\cos\left(fx+e\right)+2\left(adfx+acf\right)\sin\left(fx+e\right)}{2\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*f^2*x^2 + 2*a*c*f^2*x + 2*a*d*cos(f*x + e) + 2*(a*d*f*x + a*c*f)*s
in(f*x + e))/f^2
```

Sympy [A] time = 0.31636, size = 68, normalized size = 1.55

$$\begin{cases} acx + \frac{ac\sin\left(e+fx\right)}{f} + \frac{adx^2}{2} + \frac{adx\sin\left(e+fx\right)}{f} + \frac{ad\cos\left(e+fx\right)}{f^2} & \text{for } f \neq 0 \\ \left(a\cos\left(e\right) + a\right)\left(cx + \frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x + a*c*sin(e + f*x)/f + a*d*x**2/2 + a*d*x*sin(e + f*x)/f +
 a*d*cos(e + f*x)/f**2, Ne(f, 0)), ((a*cos(e) + a)*(c*x + d*x**2/2), True))
```

Giac [A] time = 1.14328, size = 62, normalized size = 1.41

$$\frac{1}{2}adx^2 + acx + \frac{ad\cos\left(fx+e\right)}{f^2} + \frac{\left(adfx + acf\right)\sin\left(fx+e\right)}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a*d*x^2 + a*c*x + a*d*cos(f*x + e)/f^2 + (a*d*f*x + a*c*f)*sin(f*x + e)
/f^2
```

3.121 $\int \frac{a+a\cos(e+fx)}{c+dx}\,dx$

Optimal. Leaf size=65

$$\frac{a\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\cos\left(e-\frac{cf}{d}\right)}{d}-\frac{a\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d}+\frac{a\log(c+dx)}{d}$$

```
[Out] (a*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x])/d + (a*Log[c + d*x])/d - (a
*Sin[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d
```

Rubi [A] time = 0.149746, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3317, 3303, 3299, 3302}

$$\frac{a\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\cos\left(e-\frac{cf}{d}\right)}{d}-\frac{a\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d}+\frac{a\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[e + f*x])/(c + d*x),x]
```

```
[Out] (a*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x])/d + (a*Log[c + d*x])/d - (a
*Sin[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{a+a\cos(e+fx)}{c+dx}\,dx = \int \left(\frac{a}{c+dx}+\frac{a\cos(e+fx)}{c+dx}\right)dx$$
$$= \frac{a\log(c+dx)}{d}+a\int \frac{\cos(e+fx)}{c+dx}\,dx$$
$$= \frac{a\log(c+dx)}{d}+\left(a\cos\left(e-\frac{cf}{d}\right)\right)\int \frac{\cos\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx-\left(a\sin\left(e-\frac{cf}{d}\right)\right)\int \frac{\sin\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx$$
$$= \frac{a\cos\left(e-\frac{cf}{d}\right)\text{Ci}\left(\frac{cf}{d}+fx\right)}{d}+\frac{a\log(c+dx)}{d}-\frac{a\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d}$$

Mathematica [A] time = 0.12942, size = 55, normalized size = 0.85

$$\frac{a\left(\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\cos\left(e-\frac{cf}{d}\right)-\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)+\log(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[e + f*x])/(c + d*x),x]
```

```
[Out] (a*(Cos[e - (c*f)/d]*CosIntegral[f*(c/d + x)] + Log[c + d*x] - Sin[e - (c*f
)/d]*SinIntegral[f*(c/d + x)]))/d
```

Maple [A] time = 0.036, size = 95, normalized size = 1.5

$$\frac{a}{d}\text{Si}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)+\frac{a}{d}\text{Ci}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)+\frac{a\ln\left(\left(fx+e\right)d+cf-de\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(f*x+e))/(d*x+c),x)
```

```
[Out] a*Si(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d+a*Ci(f*x+e+(c*f-d*e)/d)*cos((c*f
-d*e)/d)/d+a*ln((f*x+e)*d+c*f-d*e)/d
```

Maxima [C] time = 1.42077, size = 232, normalized size = 3.57

$$\frac{\frac{2af\log\left(c+\frac{(fx+e)d}{f}-\frac{de}{f}\right)}{d}-\frac{\left(f\left(E_1\left(\frac{i(fx+e)d-ide+icf}{d}\right)+E_1\left(-\frac{i(fx+e)d-ide+icf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f\left(iE_1\left(\frac{i(fx+e)d-ide+icf}{d}\right)-iE_1\left(-\frac{i(fx+e)d-ide+icf}{d}\right)\right)\sin\left(-\frac{de-cf}{d}\right)\right)a}{d}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*f*log(c + (f*x + e)*d/f - d*e/f)/d - (f*(exp_integral_e(1, (I*(f*x
 + e)*d - I*d*e + I*c*f)/d) + exp_integral_e(1, -(I*(f*x + e)*d - I*d*e + I
*c*f)/d))*cos(-(d*e - c*f)/d) + f*(I*exp_integral_e(1, (I*(f*x + e)*d - I*d
*e + I*c*f)/d) - I*exp_integral_e(1, -(I*(f*x + e)*d - I*d*e + I*c*f)/d))*s
in(-(d*e - c*f)/d))*a/d)/f
```

Fricas [A] time = 1.67327, size = 234, normalized size = 3.6

$$\frac{2\,a\sin\left(-\frac{de-cf}{d}\right)\mathrm{Si}\left(\frac{dfx+cf}{d}\right)+\left(a\,\mathrm{Ci}\left(\frac{dfx+cf}{d}\right)+a\,\mathrm{Ci}\left(-\frac{dfx+cf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+2\,a\log\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*sin(-(d*e - c*f)/d)*sin_integral((d*f*x + c*f)/d) + (a*cos_integra
l((d*f*x + c*f)/d) + a*cos_integral(-(d*f*x + c*f)/d))*cos(-(d*e - c*f)/d)
+ 2*a*log(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\frac{\cos\left(e+fx\right)}{c+dx}\,dx+\int\frac{1}{c+dx}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c),x)
```

```
[Out] a*(Integral(cos(e + f*x)/(c + d*x), x) + Integral(1/(c + d*x), x))
```

Giac [C] time = 1.17065, size = 934, normalized size = 14.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*log(abs(d*x + c))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + a*real_part(cos_
integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + a*real_part(cos_inte
gral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*a*imag_part(cos_integ
ral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*a*imag_part(cos_integral(
-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e) + 4*a*sin_integral((d*f*x + c*f)
/d)*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*a*imag_part(cos_integral(f*x + c*f/d))*
tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*a*imag_part(cos_integral(-f*x - c*f/d))*tan
(1/2*c*f/d)*tan(1/2*e)^2 - 4*a*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)
*tan(1/2*e)^2 + 2*a*log(abs(d*x + c))*tan(1/2*c*f/d)^2 - a*real_part(cos_in
tegral(f*x + c*f/d))*tan(1/2*c*f/d)^2 - a*real_part(cos_integral(-f*x - c*f
/d))*tan(1/2*c*f/d)^2 + 4*a*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*
f/d)*tan(1/2*e) + 4*a*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*
tan(1/2*e) + 2*a*log(abs(d*x + c))*tan(1/2*e)^2 - a*real_part(cos_integral(
f*x + c*f/d))*tan(1/2*e)^2 - a*real_part(cos_integral(-f*x - c*f/d))*tan(1/
2*e)^2 + 2*a*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d) - 2*a*imag
_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d) + 4*a*sin_integral((d*f*x
+ c*f)/d)*tan(1/2*c*f/d) - 2*a*imag_part(cos_integral(f*x + c*f/d))*tan(1/2
*e) + 2*a*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*e) - 4*a*sin_integr
al((d*f*x + c*f)/d)*tan(1/2*e) + 2*a*log(abs(d*x + c)) + a*real_part(cos_in
tegral(f*x + c*f/d)) + a*real_part(cos_integral(-f*x - c*f/d)))/(d*tan(1/2*
c*f/d)^2*tan(1/2*e)^2 + d*tan(1/2*c*f/d)^2 + d*tan(1/2*e)^2 + d)
```

3.122 $\int \frac{a+a\cos(e+fx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=89

$$-\frac{af\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^2}-\frac{af\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{a\cos(e+fx)}{d(c+dx)}-\frac{a}{d(c+dx)}$$

```
[Out] -(a/(d*(c + d*x))) - (a*Cos[e + f*x])/(d*(c + d*x)) - (a*f*CosIntegral[(c*f
)/d + f*x]*Sin[e - (c*f)/d])/d^2 - (a*f*Cos[e - (c*f)/d]*SinIntegral[(c*f)/
d + f*x])/d^2
```

Rubi [A] time = 0.163309, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3317, 3297, 3303, 3299, 3302}

$$-\frac{af\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^2}-\frac{af\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{a\cos(e+fx)}{d(c+dx)}-\frac{a}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[e + f*x])/(c + d*x)^2,x]
```

```
[Out] -(a/(d*(c + d*x))) - (a*Cos[e + f*x])/(d*(c + d*x)) - (a*f*CosIntegral[(c*f
)/d + f*x]*Sin[e - (c*f)/d])/d^2 - (a*f*Cos[e - (c*f)/d]*SinIntegral[(c*f)/
d + f*x])/d^2
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{a+a\cos(e+fx)}{(c+dx)^2}\,dx = \int \left(\frac{a}{(c+dx)^2}+\frac{a\cos(e+fx)}{(c+dx)^2}\right)dx$$
$$= -\frac{a}{d(c+dx)} + a\int \frac{\cos(e+fx)}{(c+dx)^2}\,dx$$
$$= -\frac{a}{d(c+dx)} - \frac{a\cos(e+fx)}{d(c+dx)} - \frac{(af)\int \frac{\sin(e+fx)}{c+dx}\,dx}{d}$$
$$= -\frac{a}{d(c+dx)} - \frac{a\cos(e+fx)}{d(c+dx)} - \frac{\left(af\cos\left(e-\frac{cf}{d}\right)\right)\int \frac{\sin\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx}{d} - \frac{\left(af\sin\left(e-\frac{cf}{d}\right)\right)\int \frac{\cos\left(\frac{cf}{d}\right.}{c+d}}{d}$$
$$= -\frac{a}{d(c+dx)} - \frac{a\cos(e+fx)}{d(c+dx)} - \frac{af\text{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^2} - \frac{af\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d^2}$$

Mathematica [A] time = 0.319994, size = 78, normalized size = 0.88

$$-\frac{a\left(f(c+dx)\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+f(c+dx)\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)+d(\cos(e+fx)+1)\right)}{d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[e + f*x])/(c + d*x)^2,x]
```

```
[Out] -((a*(d*(1 + Cos[e + f*x]) + f*(c + d*x)*CosIntegral[f*(c/d + x)]*Sin[e - (
c*f)/d] + f*(c + d*x)*Cos[e - (c*f)/d]*SinIntegral[f*(c/d + x)]))/(d^2*(c +
 d*x)))
```

Maple [A] time = 0.036, size = 143, normalized size = 1.6

$$\frac{1}{f}\left(af^2\left(-\frac{\cos\left(fx+e\right)}{\left(\left(fx+e\right)d+cf-de\right)d}-\frac{1}{d}\left(\frac{1}{d}\text{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)-\frac{1}{d}\text{Ci}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(f*x+e))/(d*x+c)^2,x)
```

```
[Out] 1/f*(a*f^2*(-cos(f*x+e)/((f*x+e)*d+c*f-d*e)/d-(Si(f*x+e+(c*f-d*e)/d)*cos((c
*f-d*e)/d)/d-Ci(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d)/d)-a*f^2/((f*x+e)*d+
c*f-d*e)/d)
```

Maxima [C] time = 1.5252, size = 265, normalized size = 2.98

$$-\frac{\frac{16af^2}{(fx+e)d^2-d^2e+cdf}+\frac{\left(8f^2\left(E_2\left(\frac{i(fx+e)d-ide+icf}{d}\right)+E_2\left(-\frac{i(fx+e)d-ide+icf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f^2\left(8iE_2\left(\frac{i(fx+e)d-ide+icf}{d}\right)-8iE_2\left(-\frac{i(fx+e)d-ide+icf}{d}\right)\right)\sin\left(-\frac{de}{}\right.\right.}{(fx+e)d^2-d^2e+cdf}}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(16*a*f^2/((f*x + e)*d^2 - d^2*e + c*d*f) + (8*f^2*(exp_integral_e(2,
 (I*(f*x + e)*d - I*d*e + I*c*f)/d) + exp_integral_e(2, -(I*(f*x + e)*d - I
*d*e + I*c*f)/d))*cos(-(d*e - c*f)/d) + f^2*(8*I*exp_integral_e(2, (I*(f*x
+ e)*d - I*d*e + I*c*f)/d) - 8*I*exp_integral_e(2, -(I*(f*x + e)*d - I*d*e
+ I*c*f)/d))*sin(-(d*e - c*f)/d))*a/((f*x + e)*d^2 - d^2*e + c*d*f))/f
```

Fricas [A] time = 1.70656, size = 332, normalized size = 3.73

$$-\frac{2\,ad\cos\left(fx+e\right)+2\left(adfx+acf\right)\cos\left(-\frac{de-cf}{d}\right)\mathrm{Si}\left(\frac{dfx+cf}{d}\right)+2\,ad-\left(\left(adfx+acf\right)\mathrm{Ci}\left(\frac{dfx+cf}{d}\right)+\left(adfx+acf\right)\mathrm{Ci}\left(-\frac{d\cdots}{}\right.\right.}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*cos(f*x + e) + 2*(a*d*f*x + a*c*f)*cos(-(d*e - c*f)/d)*sin_inte
gral((d*f*x + c*f)/d) + 2*a*d - ((a*d*f*x + a*c*f)*cos_integral((d*f*x + c*
f)/d) + (a*d*f*x + a*c*f)*cos_integral(-(d*f*x + c*f)/d))*sin(-(d*e - c*f)/
d))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\frac{\cos\left(e+fx\right)}{c^2+2cdx+d^2x^2}\,dx+\int\frac{1}{c^2+2cdx+d^2x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c)**2,x)
```

```
[Out] a*(Integral(cos(e + f*x)/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(1/(c**
2 + 2*c*d*x + d**2*x**2), x))
```

Giac [C] time = 1.27412, size = 4419, normalized size = 49.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(d*f*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f
/d)^2*tan(1/2*e)^2 - d*f*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*
x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*d*f*x*sin_integral((d*f*x + c*f)/d)*
tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*d*f*x*real_part(cos_integr
al(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*d*f*x*real_
part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)
```

```
 + 2*d*f*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/
d)*tan(1/2*e)^2 + 2*d*f*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x
)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + c*f*imag_part(cos_integral(f*x + c*f/d))*
tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - c*f*imag_part(cos_integral(-
f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*c*f*sin_inte
gral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - d*f*x*
imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 + d*f*
x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 - 2
*d*f*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 + 4*d*
f*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(
1/2*e) - 4*d*f*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1
/2*c*f/d)*tan(1/2*e) + 8*d*f*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2
*tan(1/2*c*f/d)*tan(1/2*e) - 2*c*f*real_part(cos_integral(f*x + c*f/d))*tan
(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - 2*c*f*real_part(cos_integral(-f*x
 - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - d*f*x*imag_part(cos
_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 + d*f*x*imag_part(cos_i
ntegral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*d*f*x*sin_integral((
d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*c*f*real_part(cos_integral(
f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*c*f*real_part(
cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + d*
f*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - d*
f*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2
*d*f*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - 2*d*f*
x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d) - 2*d*
f*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d) - c
*f*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 + c
*f*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 -
2*c*f*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 + 2*d*f
*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*e) + 2*d*f*x
*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*e) + 4*c*f*im
ag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d)*tan(1/2*e)
 - 4*c*f*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d
)*tan(1/2*e) + 8*c*f*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*c
*f/d)*tan(1/2*e) - 2*d*f*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f
/d)^2*tan(1/2*e) - 2*d*f*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*
f/d)^2*tan(1/2*e) - c*f*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2
*tan(1/2*e)^2 + c*f*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*ta
n(1/2*e)^2 - 2*c*f*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2*tan(1/2*e)^
2 + 2*d*f*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)^
2 + 2*d*f*x*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)
^2 + c*f*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2
 - c*f*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2
+ 2*c*f*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*d*t
an(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + d*f*x*imag_part(cos_integral(
f*x + c*f/d))*tan(1/2*f*x)^2 - d*f*x*imag_part(cos_integral(-f*x - c*f/d))*
tan(1/2*f*x)^2 + 2*d*f*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2 - 2*c
*f*real_part(cos_integral(f*x + c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d) - 2*c
*f*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2*tan(1/2*c*f/d) - d*
f*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2 + d*f*x*imag_part
(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2 - 2*d*f*x*sin_integral((d*f*x
 + c*f)/d)*tan(1/2*c*f/d)^2 + 2*c*f*real_part(cos_integral(f*x + c*f/d))*ta
n(1/2*f*x)^2*tan(1/2*e) + 2*c*f*real_part(cos_integral(-f*x - c*f/d))*tan(1
/2*f*x)^2*tan(1/2*e) + 4*d*f*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2
*c*f/d)*tan(1/2*e) - 4*d*f*x*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*
c*f/d)*tan(1/2*e) + 8*d*f*x*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)*ta
n(1/2*e) - 2*c*f*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(
1/2*e) - 2*c*f*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1
/2*e) - d*f*x*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*e)^2 + d*f*x*ima
g_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2 - 2*d*f*x*sin_integral((d*f
```

```
*x + c*f)/d)*tan(1/2*e)^2 + 2*c*f*real_part(cos_integral(f*x + c*f/d))*tan(
1/2*c*f/d)*tan(1/2*e)^2 + 2*c*f*real_part(cos_integral(-f*x - c*f/d))*tan(1
/2*c*f/d)*tan(1/2*e)^2 + c*f*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*f
*x)^2 - c*f*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*f*x)^2 + 2*c*f*si
n_integral((d*f*x + c*f)/d)*tan(1/2*f*x)^2 - 2*d*f*x*real_part(cos_integral
(f*x + c*f/d))*tan(1/2*c*f/d) - 2*d*f*x*real_part(cos_integral(-f*x - c*f/d
))*tan(1/2*c*f/d) - c*f*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)
^2 + c*f*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2 - 2*c*f*sin
_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2 - 2*d*tan(1/2*f*x)^2*tan(1/2*c*
f/d)^2 + 2*d*f*x*real_part(cos_integral(f*x + c*f/d))*tan(1/2*e) + 2*d*f*x*
real_part(cos_integral(-f*x - c*f/d))*tan(1/2*e) + 4*c*f*imag_part(cos_inte
gral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e) - 4*c*f*imag_part(cos_integral
(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e) + 8*c*f*sin_integral((d*f*x + c*f
)/d)*tan(1/2*c*f/d)*tan(1/2*e) - 8*d*tan(1/2*f*x)*tan(1/2*c*f/d)^2*tan(1/2*
e) - c*f*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*e)^2 + c*f*imag_part(
cos_integral(-f*x - c*f/d))*tan(1/2*e)^2 - 2*c*f*sin_integral((d*f*x + c*f)
/d)*tan(1/2*e)^2 + 2*d*tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*d*tan(1/2*c*f/d)^2*t
an(1/2*e)^2 + d*f*x*imag_part(cos_integral(f*x + c*f/d)) - d*f*x*imag_part(
cos_integral(-f*x - c*f/d)) + 2*d*f*x*sin_integral((d*f*x + c*f)/d) - 2*c*f
*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d) - 2*c*f*real_part(cos_
integral(-f*x - c*f/d))*tan(1/2*c*f/d) + 2*c*f*real_part(cos_integral(f*x +
 c*f/d))*tan(1/2*e) + 2*c*f*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*e
) + c*f*imag_part(cos_integral(f*x + c*f/d)) - c*f*imag_part(cos_integral(-
f*x - c*f/d)) + 2*c*f*sin_integral((d*f*x + c*f)/d) - 2*d*tan(1/2*f*x)^2 +
2*d*tan(1/2*c*f/d)^2 - 8*d*tan(1/2*f*x)*tan(1/2*e) - 2*d*tan(1/2*e)^2 + 2*d
)*a/(d^3*x*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + c*d^2*tan(1/2*f*x
)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + d^3*x*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 +
 d^3*x*tan(1/2*f*x)^2*tan(1/2*e)^2 + d^3*x*tan(1/2*c*f/d)^2*tan(1/2*e)^2 +
c*d^2*tan(1/2*f*x)^2*tan(1/2*c*f/d)^2 + c*d^2*tan(1/2*f*x)^2*tan(1/2*e)^2 +
 c*d^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + d^3*x*tan(1/2*f*x)^2 + d^3*x*tan(1/2
*c*f/d)^2 + d^3*x*tan(1/2*e)^2 + c*d^2*tan(1/2*f*x)^2 + c*d^2*tan(1/2*c*f/d
)^2 + c*d^2*tan(1/2*e)^2 + d^3*x + c*d^2) - a/((d*x + c)*d)
```

3.123 $\int (c+dx)^3 (a+a\cos(e+fx))^2\,dx$

Optimal. Leaf size=237

$$-\frac{12a^2d^2(c+dx)\sin(e+fx)}{f^3}-\frac{3a^2d^2(c+dx)\sin(e+fx)\cos(e+fx)}{4f^3}-\frac{3a^2cd^2x}{4f^2}+\frac{3a^2d(c+dx)^2\cos^2(e+fx)}{4f^2}+\frac{6a^2d}{}$$

```
[Out] (-3*a^2*c*d^2*x)/(4*f^2) - (3*a^2*d^3*x^2)/(8*f^2) + (3*a^2*(c + d*x)^4)/(8
*d) - (12*a^2*d^3*Cos[e + f*x])/f^4 + (6*a^2*d*(c + d*x)^2*Cos[e + f*x])/f^
2 - (3*a^2*d^3*Cos[e + f*x]^2)/(8*f^4) + (3*a^2*d*(c + d*x)^2*Cos[e + f*x]^
2)/(4*f^2) - (12*a^2*d^2*(c + d*x)*Sin[e + f*x])/f^3 + (2*a^2*(c + d*x)^3*S
in[e + f*x])/f - (3*a^2*d^2*(c + d*x)*Cos[e + f*x]*Sin[e + f*x])/(4*f^3) +
(a^2*(c + d*x)^3*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rubi [A] time = 0.260717, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3317, 3296, 2638, 3311, 32, 3310}

$$-\frac{12a^2d^2(c+dx)\sin(e+fx)}{f^3}-\frac{3a^2d^2(c+dx)\sin(e+fx)\cos(e+fx)}{4f^3}-\frac{3a^2cd^2x}{4f^2}+\frac{3a^2d(c+dx)^2\cos^2(e+fx)}{4f^2}+\frac{6a^2d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + a*Cos[e + f*x])^2,x]
```


```
[Out] (-3*a^2*c*d^2*x)/(4*f^2) - (3*a^2*d^3*x^2)/(8*f^2) + (3*a^2*(c + d*x)^4)/(8
*d) - (12*a^2*d^3*Cos[e + f*x])/f^4 + (6*a^2*d*(c + d*x)^2*Cos[e + f*x])/f^
2 - (3*a^2*d^3*Cos[e + f*x]^2)/(8*f^4) + (3*a^2*d*(c + d*x)^2*Cos[e + f*x]^
2)/(4*f^2) - (12*a^2*d^2*(c + d*x)*Sin[e + f*x])/f^3 + (2*a^2*(c + d*x)^3*S
in[e + f*x])/f - (3*a^2*d^2*(c + d*x)*Cos[e + f*x]*Sin[e + f*x])/(4*f^3) +
(a^2*(c + d*x)^3*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^3(a+a\cos(e+fx))^2\,dx &= \int \left(a^2(c+dx)^3+2a^2(c+dx)^3\cos(e+fx)+a^2(c+dx)^3\cos^2(e+fx)\right)dx \\
&= \frac{a^2(c+dx)^4}{4d}+a^2\int (c+dx)^3\cos^2(e+fx)\,dx+\left(2a^2\right)\int (c+dx)^3\cos(e+fx)\,dx \\
&= \frac{a^2(c+dx)^4}{4d}+\frac{3a^2d(c+dx)^2\cos^2(e+fx)}{4f^2}+\frac{2a^2(c+dx)^3\sin(e+fx)}{f}+\frac{a^2(c+dx)^3 \cdots}{\cdots} \\
&= \frac{3a^2(c+dx)^4}{8d}+\frac{6a^2d(c+dx)^2\cos(e+fx)}{f^2}-\frac{3a^2d^3\cos^2(e+fx)}{8f^4}+\frac{3a^2d(c+dx)^2\cos \cdots}{4f^2} \\
&= -\frac{3a^2cd^2x}{4f^2}-\frac{3a^2d^3x^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d}+\frac{6a^2d(c+dx)^2\cos(e+fx)}{f^2}-\frac{3a^2d^3\cos^2(e+ \cdots}{8f^4} \\
&= -\frac{3a^2cd^2x}{4f^2}-\frac{3a^2d^3x^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d}-\frac{12a^2d^3\cos(e+fx)}{f^4}+\frac{6a^2d(c+dx)^2\cos(e+ \cdots}{f^2}
\end{aligned}$$

Mathematica [A] time = 1.43609, size = 217, normalized size = 0.92

$$\frac{a^2\left(2f\left(16(c+dx)\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2-6\right)\right)\sin(e+fx)+(c+dx)\left(2c^2f^2+4cdf^2x+d^2\left(2f^2x^2-3\right)\right)\sin(2(e+f \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + a*Cos[e + f*x])^2,x]
```

```
[Out] (a^2*(96*d*(c^2*f^2 + 2*c*d*f^2*x + d^2*(-2 + f^2*x^2))*Cos[e + f*x] + 3*d*
(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(-1 + 2*f^2*x^2))*Cos[2*(e + f*x)] + 2*f*(3*
f^3*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 16*(c + d*x)*(c^2*f^2 +
 2*c*d*f^2*x + d^2*(-6 + f^2*x^2))*Sin[e + f*x] + (c + d*x)*(2*c^2*f^2 + 4*
c*d*f^2*x + d^2*(-3 + 2*f^2*x^2))*Sin[2*(e + f*x)])))/(16*f^4)
```

Maple [B] time = 0.04, size = 1129, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+a*cos(f*x+e))^2,x)
```

```
[Out] 1/f*(-6*a^2/f*c^2*d*e*sin(f*x+e)+6*a^2/f^2*c*d^2*((f*x+e)^2*sin(f*x+e)-2*si
n(f*x+e)+2*(f*x+e)*cos(f*x+e))+a^2*c^3*(f*x+e)+2*a^2/f^3*d^3*((f*x+e)^3*sin
(f*x+e)+3*(f*x+e)^2*cos(f*x+e)-6*cos(f*x+e)-6*(f*x+e)*sin(f*x+e))+a^2/f^3*d
^3*((f*x+e)^3*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+3/4*(f*x+e)^2*cos(f
*x+e)^2-3/2*(f*x+e)*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+3/8*(f*x+e)^2
+3/8*sin(f*x+e)^2-3/8*(f*x+e)^4)+1/4*a^2/f^3*d^3*(f*x+e)^4+3*a^2/f^2*c*d^2*
((f*x+e)^2*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+1/2*(f*x+e)*cos(f*x+e)
^2-1/4*sin(f*x+e)*cos(f*x+e)-1/4*f*x-1/4*e-1/3*(f*x+e)^3)+a^2/f^2*c*d^2*(f*
x+e)^3-3*a^2/f^3*d^3*e*((f*x+e)^2*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)
+1/2*(f*x+e)*cos(f*x+e)^2-1/4*sin(f*x+e)*cos(f*x+e)-1/4*f*x-1/4*e-1/3*(f*x+
e)^3)+a^2*c^3*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2*a^2*c^3*sin(f*x+e
)-a^2/f^3*d^3*e^3*(f*x+e)+3*a^2/f*c^2*d*((f*x+e)*(1/2*sin(f*x+e)*cos(f*x+e)
+1/2*f*x+1/2*e)-1/4*(f*x+e)^2-1/4*sin(f*x+e)^2)+6*a^2/f*c^2*d*(cos(f*x+e)+(
f*x+e)*sin(f*x+e))-a^2/f^3*d^3*e*(f*x+e)^3+3/2*a^2/f*c^2*d*(f*x+e)^2-6*a^2/
f^3*d^3*e*((f*x+e)^2*sin(f*x+e)-2*sin(f*x+e)+2*(f*x+e)*cos(f*x+e))+3*a^2/f^
3*d^3*e^2*((f*x+e)*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2-
1/4*sin(f*x+e)^2)+6*a^2/f^3*d^3*e^2*(cos(f*x+e)+(f*x+e)*sin(f*x+e))+3*a^2/f
^2*c*d^2*e^2*(f*x+e)-12*a^2/f^2*c*d^2*e*(cos(f*x+e)+(f*x+e)*sin(f*x+e))+6*a
^2/f^2*c*d^2*e^2*sin(f*x+e)-6*a^2/f^2*c*d^2*e*((f*x+e)*(1/2*sin(f*x+e)*cos(
f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2-1/4*sin(f*x+e)^2)-3*a^2/f*c^2*d*e*(f*x+
e)+3*a^2/f^2*c*d^2*e^2*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-3*a^2/f^2*
c*d^2*e*(f*x+e)^2-3*a^2/f*c^2*d*e*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)
-a^2/f^3*d^3*e^3*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+3/2*a^2/f^3*d^3*
e^2*(f*x+e)^2-2*a^2/f^3*d^3*e^3*sin(f*x+e))
```

Maxima [B] time = 1.18899, size = 1281, normalized size = 5.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*c^3 + 16*(f*x + e)*a^2*c^3 + 4
*(f*x + e)^4*a^2*d^3/f^3 - 16*(f*x + e)^3*a^2*d^3*e/f^3 + 24*(f*x + e)^2*a^
2*d^3*e^2/f^3 - 4*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*d^3*e^3/f^3 - 16*(f*
x + e)*a^2*d^3*e^3/f^3 + 16*(f*x + e)^3*a^2*c*d^2/f^2 - 48*(f*x + e)^2*a^2*
c*d^2*e/f^2 + 12*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*c*d^2*e^2/f^2 + 48*(f
*x + e)*a^2*c*d^2*e^2/f^2 + 24*(f*x + e)^2*a^2*c^2*d/f - 12*(2*f*x + 2*e +
sin(2*f*x + 2*e))*a^2*c^2*d*e/f - 48*(f*x + e)*a^2*c^2*d*e/f + 32*a^2*c^3*s
in(f*x + e) - 32*a^2*d^3*e^3*sin(f*x + e)/f^3 + 96*a^2*c*d^2*e^2*sin(f*x +
e)/f^2 - 96*a^2*c^2*d*e*sin(f*x + e)/f + 6*(2*(f*x + e)^2 + 2*(f*x + e)*sin
(2*f*x + 2*e) + cos(2*f*x + 2*e))*a^2*d^3*e^2/f^3 + 96*((f*x + e)*sin(f*x +
 e) + cos(f*x + e))*a^2*d^3*e^2/f^3 - 12*(2*(f*x + e)^2 + 2*(f*x + e)*sin(2
*f*x + 2*e) + cos(2*f*x + 2*e))*a^2*c*d^2*e/f^2 - 192*((f*x + e)*sin(f*x +
e) + cos(f*x + e))*a^2*c*d^2*e/f^2 + 6*(2*(f*x + e)^2 + 2*(f*x + e)*sin(2*f
*x + 2*e) + cos(2*f*x + 2*e))*a^2*c^2*d/f + 96*((f*x + e)*sin(f*x + e) + co
s(f*x + e))*a^2*c^2*d/f - 2*(4*(f*x + e)^3 + 6*(f*x + e)*cos(2*f*x + 2*e) +
 3*(2*(f*x + e)^2 - 1)*sin(2*f*x + 2*e))*a^2*d^3*e/f^3 - 96*(2*(f*x + e)*co
s(f*x + e) + ((f*x + e)^2 - 2)*sin(f*x + e))*a^2*d^3*e/f^3 + 2*(4*(f*x + e)
^3 + 6*(f*x + e)*cos(2*f*x + 2*e) + 3*(2*(f*x + e)^2 - 1)*sin(2*f*x + 2*e))
*a^2*c*d^2/f^2 + 96*(2*(f*x + e)*cos(f*x + e) + ((f*x + e)^2 - 2)*sin(f*x +
 e))*a^2*c*d^2/f^2 + (2*(f*x + e)^4 + 3*(2*(f*x + e)^2 - 1)*cos(2*f*x + 2*e
) + 2*(2*(f*x + e)^3 - 3*f*x - 3*e)*sin(2*f*x + 2*e))*a^2*d^3/f^3 + 32*(3*(
(f*x + e)^2 - 2)*cos(f*x + e) + ((f*x + e)^3 - 6*f*x - 6*e)*sin(f*x + e))*a
^2*d^3/f^3)/f
```

Fricas [A] time = 1.78724, size = 751, normalized size = 3.17

$$\frac{3\,a^2d^3f^4x^4 + 12\,a^2cd^2f^4x^3 + 3\left(6\,a^2c^2df^4 - a^2d^3f^2\right)x^2 + 3\left(2\,a^2d^3f^2x^2 + 4\,a^2cd^2f^2x + 2\,a^2c^2df^2 - a^2d^3\right)\cos\left(fx+e\right)^2 +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(3*a^2*d^3*f^4*x^4 + 12*a^2*c*d^2*f^4*x^3 + 3*(6*a^2*c^2*d*f^4 - a^2*d^
3*f^2)*x^2 + 3*(2*a^2*d^3*f^2*x^2 + 4*a^2*c*d^2*f^2*x + 2*a^2*c^2*d*f^2 - a
^2*d^3)*cos(f*x + e)^2 + 6*(2*a^2*c^3*f^4 - a^2*c*d^2*f^2)*x + 48*(a^2*d^3*
f^2*x^2 + 2*a^2*c*d^2*f^2*x + a^2*c^2*d*f^2 - 2*a^2*d^3)*cos(f*x + e) + 2*(
8*a^2*d^3*f^3*x^3 + 24*a^2*c*d^2*f^3*x^2 + 8*a^2*c^3*f^3 - 48*a^2*c*d^2*f +
 24*(a^2*c^2*d*f^3 - 2*a^2*d^3*f)*x + (2*a^2*d^3*f^3*x^3 + 6*a^2*c*d^2*f^3*
x^2 + 2*a^2*c^3*f^3 - 3*a^2*c*d^2*f + 3*(2*a^2*c^2*d*f^3 - a^2*d^3*f)*x)*co
s(f*x + e))*sin(f*x + e))/f^4
```

Sympy [A] time = 4.30583, size = 779, normalized size = 3.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+a*cos(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**3*x*sin(e + f*x)**2/2 + a**2*c**3*x*cos(e + f*x)**2/2 +
a**2*c**3*x + a**2*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a**2*c**3*sin(e
 + f*x)/f + 3*a**2*c**2*d*x**2*sin(e + f*x)**2/4 + 3*a**2*c**2*d*x**2*cos(e
 + f*x)**2/4 + 3*a**2*c**2*d*x**2/2 + 3*a**2*c**2*d*x*sin(e + f*x)*cos(e +
f*x)/(2*f) + 6*a**2*c**2*d*x*sin(e + f*x)/f - 3*a**2*c**2*d*sin(e + f*x)**2
/(4*f**2) + 6*a**2*c**2*d*cos(e + f*x)/f**2 + a**2*c*d**2*x**3*sin(e + f*x)
**2/2 + a**2*c*d**2*x**3*cos(e + f*x)**2/2 + a**2*c*d**2*x**3 + 3*a**2*c*d*
*2*x**2*sin(e + f*x)*cos(e + f*x)/(2*f) + 6*a**2*c*d**2*x**2*sin(e + f*x)/f
 - 3*a**2*c*d**2*x*sin(e + f*x)**2/(4*f**2) + 3*a**2*c*d**2*x*cos(e + f*x)*
*2/(4*f**2) + 12*a**2*c*d**2*x*cos(e + f*x)/f**2 - 3*a**2*c*d**2*sin(e + f*
x)*cos(e + f*x)/(4*f**3) - 12*a**2*c*d**2*sin(e + f*x)/f**3 + a**2*d**3*x**
4*sin(e + f*x)**2/8 + a**2*d**3*x**4*cos(e + f*x)**2/8 + a**2*d**3*x**4/4 +
 a**2*d**3*x**3*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a**2*d**3*x**3*sin(e +
f*x)/f - 3*a**2*d**3*x**2*sin(e + f*x)**2/(8*f**2) + 3*a**2*d**3*x**2*cos(e
 + f*x)**2/(8*f**2) + 6*a**2*d**3*x**2*cos(e + f*x)/f**2 - 3*a**2*d**3*x*si
n(e + f*x)*cos(e + f*x)/(4*f**3) - 12*a**2*d**3*x*sin(e + f*x)/f**3 + 3*a**
2*d**3*sin(e + f*x)**2/(8*f**4) - 12*a**2*d**3*cos(e + f*x)/f**4, Ne(f, 0))
, ((a*cos(e) + a)**2*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)
, True))
```

Giac [A] time = 1.12357, size = 458, normalized size = 1.93

$$\frac{3}{8}\,a^2d^3x^4 + \frac{3}{2}\,a^2cd^2x^3 + \frac{9}{4}\,a^2c^2dx^2 + \frac{3}{2}\,a^2c^3x + \frac{3\left(2\,a^2d^3f^2x^2 + 4\,a^2cd^2f^2x + 2\,a^2c^2df^2 - a^2d^3\right)\cos\left(2\,fx+2\,e\right)}{16\,f^4} + \frac{6\left(a^2d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/8*a^2*d^3*x^4 + 3/2*a^2*c*d^2*x^3 + 9/4*a^2*c^2*d*x^2 + 3/2*a^2*c^3*x + 3
/16*(2*a^2*d^3*f^2*x^2 + 4*a^2*c*d^2*f^2*x + 2*a^2*c^2*d*f^2 - a^2*d^3)*cos
(2*f*x + 2*e)/f^4 + 6*(a^2*d^3*f^2*x^2 + 2*a^2*c*d^2*f^2*x + a^2*c^2*d*f^2
- 2*a^2*d^3)*cos(f*x + e)/f^4 + 1/8*(2*a^2*d^3*f^3*x^3 + 6*a^2*c*d^2*f^3*x^
2 + 6*a^2*c^2*d*f^3*x + 2*a^2*c^3*f^3 - 3*a^2*d^3*f*x - 3*a^2*c*d^2*f)*sin(
2*f*x + 2*e)/f^4 + 2*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f
^3*x + a^2*c^3*f^3 - 6*a^2*d^3*f*x - 6*a^2*c*d^2*f)*sin(f*x + e)/f^4
```

3.124 $\int (c+dx)^2(a+a\cos(e+fx))^2\,dx$

Optimal. Leaf size=168

$$\frac{a^2d(c+dx)\cos^2(e+fx)}{2f^2}+\frac{4a^2d(c+dx)\cos(e+fx)}{f^2}+\frac{2a^2(c+dx)^2\sin(e+fx)}{f}+\frac{a^2(c+dx)^2\sin(e+fx)\cos(e+fx)}{2f}+$$

```
[Out] -(a^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(2*d) + (4*a^2*d*(c + d*x)*Cos[e +
 f*x])/f^2 + (a^2*d*(c + d*x)*Cos[e + f*x]^2)/(2*f^2) - (4*a^2*d^2*Sin[e +
f*x])/f^3 + (2*a^2*(c + d*x)^2*Sin[e + f*x])/f - (a^2*d^2*Cos[e + f*x]*Sin[
e + f*x])/(4*f^3) + (a^2*(c + d*x)^2*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rubi [A] time = 0.179149, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {3317, 3296, 2637, 3311, 32, 2635, 8}

$$\frac{a^2d(c+dx)\cos^2(e+fx)}{2f^2}+\frac{4a^2d(c+dx)\cos(e+fx)}{f^2}+\frac{2a^2(c+dx)^2\sin(e+fx)}{f}+\frac{a^2(c+dx)^2\sin(e+fx)\cos(e+fx)}{2f}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + a*Cos[e + f*x])^2,x]
```


```
[Out] -(a^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(2*d) + (4*a^2*d*(c + d*x)*Cos[e +
 f*x])/f^2 + (a^2*d*(c + d*x)*Cos[e + f*x]^2)/(2*f^2) - (4*a^2*d^2*Sin[e +
f*x])/f^3 + (2*a^2*(c + d*x)^2*Sin[e + f*x])/f - (a^2*d^2*Cos[e + f*x]*Sin[
e + f*x])/(4*f^3) + (a^2*(c + d*x)^2*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2(a+a\cos(e+fx))^2\,dx &= \int \left(a^2(c+dx)^2+2a^2(c+dx)^2\cos(e+fx)+a^2(c+dx)^2\cos^2(e+fx)\right)dx\\ &= \frac{a^2(c+dx)^3}{3d}+a^2\int (c+dx)^2\cos^2(e+fx)\,dx+\left(2a^2\right)\int (c+dx)^2\cos(e+fx)\,dx\\ &= \frac{a^2(c+dx)^3}{3d}+\frac{a^2d(c+dx)\cos^2(e+fx)}{2f^2}+\frac{2a^2(c+dx)^2\sin(e+fx)}{f}+\frac{a^2(c+dx)^2}{}\\ &= \frac{a^2(c+dx)^3}{2d}+\frac{4a^2d(c+dx)\cos(e+fx)}{f^2}+\frac{a^2d(c+dx)\cos^2(e+fx)}{2f^2}+\frac{2a^2(c+dx}{}\\ &= -\frac{a^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{2d}+\frac{4a^2d(c+dx)\cos(e+fx)}{f^2}+\frac{a^2d(c+dx)\cos^2(e+fx)}{2f^2}-\end{aligned}$$

Mathematica [A] time = 0.655944, size = 193, normalized size = 1.15

$$\frac{a^2\left(16c^2f^2\sin(e+fx)+2c^2f^2\sin(2(e+fx))+12c^2f^3x+32cdf^2x\sin(e+fx)+4cdf^2x\sin(2(e+fx))+32df(c+d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + a*Cos[e + f*x])^2,x]
```

```
[Out] (a^2*(12*c^2*f^3*x + 12*c*d*f^3*x^2 + 4*d^2*f^3*x^3 + 32*d*f*(c + d*x)*Cos[
e + f*x] + 2*d*f*(c + d*x)*Cos[2*(e + f*x)] - 32*d^2*Sin[e + f*x] + 16*c^2*
f^2*Sin[e + f*x] + 32*c*d*f^2*x*Sin[e + f*x] + 16*d^2*f^2*x^2*Sin[e + f*x]
- d^2*Sin[2*(e + f*x)] + 2*c^2*f^2*Sin[2*(e + f*x)] + 4*c*d*f^2*x*Sin[2*(e
+ f*x)] + 2*d^2*f^2*x^2*Sin[2*(e + f*x)]))/(8*f^3)
```

Maple [B] time = 0.042, size = 564, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+a*cos(f*x+e))^2,x)
```

```
[Out] 1/f*(a^2/f^2*d^2*((f*x+e)^2*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+1/2*(
f*x+e)*cos(f*x+e)^2-1/4*sin(f*x+e)*cos(f*x+e)-1/4*f*x-1/4*e-1/3*(f*x+e)^3)+
```

```
2*a^2/f*c*d*((f*x+e)*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^
2-1/4*sin(f*x+e)^2)-2*a^2/f^2*d^2*e*((f*x+e)*(1/2*sin(f*x+e)*cos(f*x+e)+1/2
*f*x+1/2*e)-1/4*(f*x+e)^2-1/4*sin(f*x+e)^2)+c^2*a^2*(1/2*sin(f*x+e)*cos(f*x
+e)+1/2*f*x+1/2*e)-2*a^2/f*c*d*e*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+
a^2/f^2*d^2*e^2*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2*a^2/f^2*d^2*((f
*x+e)^2*sin(f*x+e)-2*sin(f*x+e)+2*(f*x+e)*cos(f*x+e))+4*a^2/f*c*d*(cos(f*x+
e)+(f*x+e)*sin(f*x+e))-4*a^2/f^2*d^2*e*(cos(f*x+e)+(f*x+e)*sin(f*x+e))+2*c^
2*a^2*sin(f*x+e)-4*a^2/f*c*d*e*sin(f*x+e)+2*a^2/f^2*d^2*e^2*sin(f*x+e)+1/3*
a^2/f^2*d^2*(f*x+e)^3+a^2/f*c*d*(f*x+e)^2-a^2/f^2*d^2*e*(f*x+e)^2+c^2*a^2*(
f*x+e)-2*a^2/f*c*d*e*(f*x+e)+a^2/f^2*d^2*e^2*(f*x+e))
```

Maxima [B] time = 1.19963, size = 667, normalized size = 3.97

$$\frac{6\left(2\,fx+2\,e+\sin\left(2\,fx+2\,e\right)\right)a^2c^2+24\left(fx+e\right)a^2c^2+\frac{8\left(fx+e\right)^3a^2d^2}{f^2}-\frac{24\left(fx+e\right)^2a^2d^2e}{f^2}+\frac{6\left(2\,fx+2\,e+\sin\left(2\,fx+2\,e\right)\right)a^2d^2e^2}{f^2}+\frac{24\left(fx\cdots\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/24*(6*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*c^2 + 24*(f*x + e)*a^2*c^2 + 8
*(f*x + e)^3*a^2*d^2/f^2 - 24*(f*x + e)^2*a^2*d^2*e/f^2 + 6*(2*f*x + 2*e +
sin(2*f*x + 2*e))*a^2*d^2*e^2/f^2 + 24*(f*x + e)*a^2*d^2*e^2/f^2 + 24*(f*x
+ e)^2*a^2*c*d/f - 12*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*c*d*e/f - 48*(f*
x + e)*a^2*c*d*e/f + 48*a^2*c^2*sin(f*x + e) + 48*a^2*d^2*e^2*sin(f*x + e)/
f^2 - 96*a^2*c*d*e*sin(f*x + e)/f - 6*(2*(f*x + e)^2 + 2*(f*x + e)*sin(2*f*
x + 2*e) + cos(2*f*x + 2*e))*a^2*d^2*e/f^2 - 96*((f*x + e)*sin(f*x + e) + c
os(f*x + e))*a^2*d^2*e/f^2 + 6*(2*(f*x + e)^2 + 2*(f*x + e)*sin(2*f*x + 2*e
) + cos(2*f*x + 2*e))*a^2*c*d/f + 96*((f*x + e)*sin(f*x + e) + cos(f*x + e)
)*a^2*c*d/f + (4*(f*x + e)^3 + 6*(f*x + e)*cos(2*f*x + 2*e) + 3*(2*(f*x + e
)^2 - 1)*sin(2*f*x + 2*e))*a^2*d^2/f^2 + 48*(2*(f*x + e)*cos(f*x + e) + ((f
*x + e)^2 - 2)*sin(f*x + e))*a^2*d^2/f^2)/f
```

Fricas [A] time = 1.66156, size = 448, normalized size = 2.67

$$\frac{2\,a^2d^2f^3x^3+6\,a^2cdf^3x^2+2\left(a^2d^2fx+a^2cdf\right)\cos\left(fx+e\right)^2+\left(6\,a^2c^2f^3-a^2d^2f\right)x+16\left(a^2d^2fx+a^2cdf\right)\cos\left(fx+e\right)+\cdots}{4\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*d^2*f^3*x^3 + 6*a^2*c*d*f^3*x^2 + 2*(a^2*d^2*f*x + a^2*c*d*f)*co
s(f*x + e)^2 + (6*a^2*c^2*f^3 - a^2*d^2*f)*x + 16*(a^2*d^2*f*x + a^2*c*d*f)
*cos(f*x + e) + (8*a^2*d^2*f^2*x^2 + 16*a^2*c*d*f^2*x + 8*a^2*c^2*f^2 - 16*
a^2*d^2 + (2*a^2*d^2*f^2*x^2 + 4*a^2*c*d*f^2*x + 2*a^2*c^2*f^2 - a^2*d^2)*c
os(f*x + e))*sin(f*x + e))/f^3
```

Sympy [A] time = 1.96152, size = 456, normalized size = 2.71

$$\begin{cases}\frac{a^2c^2x\sin^2\left(e+fx\right)}{2}+\frac{a^2c^2x\cos^2\left(e+fx\right)}{2}+a^2c^2x+\frac{a^2c^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}+\frac{2a^2c^2\sin\left(e+fx\right)}{f}+\frac{a^2cdx^2\sin^2\left(e+fx\right)}{2}+\frac{a^2cdx^2\cos^2\left(e+fx\right)}{2}+a^2c\cdots \\ \left(a\cos\left(e\right)+a\right)^2\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+a*cos(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**2*x*sin(e + f*x)**2/2 + a**2*c**2*x*cos(e + f*x)**2/2 +
a**2*c**2*x + a**2*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a**2*c**2*sin(e
 + f*x)/f + a**2*c*d*x**2*sin(e + f*x)**2/2 + a**2*c*d*x**2*cos(e + f*x)**2
/2 + a**2*c*d*x**2 + a**2*c*d*x*sin(e + f*x)*cos(e + f*x)/f + 4*a**2*c*d*x*
sin(e + f*x)/f + a**2*c*d*cos(e + f*x)**2/(2*f**2) + 4*a**2*c*d*cos(e + f*x
)/f**2 + a**2*d**2*x**3*sin(e + f*x)**2/6 + a**2*d**2*x**3*cos(e + f*x)**2/
6 + a**2*d**2*x**3/3 + a**2*d**2*x**2*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a
**2*d**2*x**2*sin(e + f*x)/f - a**2*d**2*x*sin(e + f*x)**2/(4*f**2) + a**2*
d**2*x*cos(e + f*x)**2/(4*f**2) + 4*a**2*d**2*x*cos(e + f*x)/f**2 - a**2*d*
*2*sin(e + f*x)*cos(e + f*x)/(4*f**3) - 4*a**2*d**2*sin(e + f*x)/f**3, Ne(f
, 0)), ((a*cos(e) + a)**2*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

Giac [A] time = 1.15298, size = 279, normalized size = 1.66

$$\frac{1}{2}a^2d^2x^3 + \frac{3}{2}a^2cdx^2 + \frac{3}{2}a^2c^2x + \frac{\left(a^2d^2fx + a^2cdf\right)\cos\left(2fx + 2e\right)}{4f^3} + \frac{4\left(a^2d^2fx + a^2cdf\right)\cos\left(fx + e\right)}{f^3} + \frac{\left(2a^2d^2f^2x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*d^2*x^3 + 3/2*a^2*c*d*x^2 + 3/2*a^2*c^2*x + 1/4*(a^2*d^2*f*x + a^2*
c*d*f)*cos(2*f*x + 2*e)/f^3 + 4*(a^2*d^2*f*x + a^2*c*d*f)*cos(f*x + e)/f^3
+ 1/8*(2*a^2*d^2*f^2*x^2 + 4*a^2*c*d*f^2*x + 2*a^2*c^2*f^2 - a^2*d^2)*sin(2
*f*x + 2*e)/f^3 + 2*(a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2 - 2*a^
2*d^2)*sin(f*x + e)/f^3
```

3.125 $\int (c+dx)(a+a\cos(e+fx))^2\,dx$

Optimal. Leaf size=118

$$\frac{2a^2(c+dx)\sin(e+fx)}{f}+\frac{a^2(c+dx)\sin(e+fx)\cos(e+fx)}{2f}+\frac{a^2(c+dx)^2}{2d}+\frac{1}{2}a^2cx+\frac{a^2d\cos^2(e+fx)}{4f^2}+\frac{2a^2d\cos(e+fx)}{f^2}$$

```
[Out] (a^2*c*x)/2 + (a^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) + (2*a^2*d*Cos[e + f*
x])/f^2 + (a^2*d*Cos[e + f*x]^2)/(4*f^2) + (2*a^2*(c + d*x)*Sin[e + f*x])/f
 + (a^2*(c + d*x)*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rubi [A] time = 0.0973405, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3317, 3296, 2638, 3310}

$$\frac{2a^2(c+dx)\sin(e+fx)}{f}+\frac{a^2(c+dx)\sin(e+fx)\cos(e+fx)}{2f}+\frac{a^2(c+dx)^2}{2d}+\frac{1}{2}a^2cx+\frac{a^2d\cos^2(e+fx)}{4f^2}+\frac{2a^2d\cos(e+fx)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + a*Cos[e + f*x])^2,x]
```


```
[Out] (a^2*c*x)/2 + (a^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) + (2*a^2*d*Cos[e + f*
x])/f^2 + (a^2*d*Cos[e + f*x]^2)/(4*f^2) + (2*a^2*(c + d*x)*Sin[e + f*x])/f
 + (a^2*(c + d*x)*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)(a+a\cos(e+fx))^2\,dx = \int \left(a^2(c+dx)+2a^2(c+dx)\cos(e+fx)+a^2(c+dx)\cos^2(e+fx)\right)dx$$
$$= \frac{a^2(c+dx)^2}{2d}+a^2\int (c+dx)\cos^2(e+fx)\,dx+\left(2a^2\right)\int (c+dx)\cos(e+fx)\,dx$$
$$= \frac{a^2(c+dx)^2}{2d}+\frac{a^2d\cos^2(e+fx)}{4f^2}+\frac{2a^2(c+dx)\sin(e+fx)}{f}+\frac{a^2(c+dx)\cos(e+f}{2f}$$
$$= \frac{1}{2}a^2cx+\frac{1}{4}a^2dx^2+\frac{a^2(c+dx)^2}{2d}+\frac{2a^2d\cos(e+fx)}{f^2}+\frac{a^2d\cos^2(e+fx)}{4f^2}+\frac{2a^2(c+}{}$$

Mathematica [A] time = 0.48289, size = 80, normalized size = 0.68

$$\frac{a^2(-6(e+fx)(d(e-fx)-2cf)+16f(c+dx)\sin(e+fx)+2f(c+dx)\sin(2(e+fx))+16d\cos(e+fx)+d\cos(2(e+}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + a*Cos[e + f*x])^2,x]
```

```
[Out] (a^2*(-6*(e + f*x)*(-2*c*f + d*(e - f*x)) + 16*d*Cos[e + f*x] + d*Cos[2*(e
+ f*x)] + 16*f*(c + d*x)*Sin[e + f*x] + 2*f*(c + d*x)*Sin[2*(e + f*x)]))/(8
*f^2)
```

Maple [B] time = 0.037, size = 218, normalized size = 1.9

$$\frac{1}{f}\left(\frac{a^2d}{f}\left(\left(fx+e\right)\left(\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{\left(fx+e\right)^2}{4}-\frac{\left(\sin\left(fx+e\right)\right)^2}{4}\right)+a^2c\left(\frac{\sin\left(fx+e\right)\cos\left(fx+\right.}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+a*cos(f*x+e))^2,x)
```

```
[Out] 1/f*(a^2/f*d*((f*x+e)*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)
^2-1/4*sin(f*x+e)^2)+a^2*c*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-a^2/f*
d*e*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2*a^2/f*d*(cos(f*x+e)+(f*x+e)
*sin(f*x+e))+2*a^2*c*sin(f*x+e)-2*a^2/f*d*e*sin(f*x+e)+1/2*a^2/f*d*(f*x+e)^
2+a^2*c*(f*x+e)-a^2/f*d*e*(f*x+e))
```

Maxima [A] time = 1.2145, size = 266, normalized size = 2.25

$$\frac{2\left(2fx+2e+\sin\left(2fx+2e\right)\right)a^2c+8\left(fx+e\right)a^2c+\frac{4\left(fx+e\right)^2a^2d}{f}-\frac{2\left(2fx+2e+\sin\left(2fx+2e\right)\right)a^2de}{f}-\frac{8\left(fx+e\right)a^2de}{f}+16a^2c\sin\left(\right.}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*c + 8*(f*x + e)*a^2*c + 4*(f*x
+ e)^2*a^2*d/f - 2*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*d*e/f - 8*(f*x + e)
*a^2*d*e/f + 16*a^2*c*sin(f*x + e) - 16*a^2*d*e*sin(f*x + e)/f + (2*(f*x +
```

```
e)^2 + 2*(f*x + e)*sin(2*f*x + 2*e) + cos(2*f*x + 2*e))*a^2*d/f + 16*((f*x
+ e)*sin(f*x + e) + cos(f*x + e))*a^2*d/f)/f
```

Fricas [A] time = 1.62719, size = 228, normalized size = 1.93

$$\frac{3\,a^2df^2x^2+6\,a^2cf^2x+a^2d\cos\left(fx+e\right)^2+8\,a^2d\cos\left(fx+e\right)+2\left(4\,a^2dfx+4\,a^2cf+\left(a^2dfx+a^2cf\right)\cos\left(fx+e\right)\right)\sin\left(\right.}{4\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^2*d*f^2*x^2 + 6*a^2*c*f^2*x + a^2*d*cos(f*x + e)^2 + 8*a^2*d*cos(f
*x + e) + 2*(4*a^2*d*f*x + 4*a^2*c*f + (a^2*d*f*x + a^2*c*f)*cos(f*x + e))*
sin(f*x + e))/f^2
```

Sympy [A] time = 0.821017, size = 219, normalized size = 1.86

$$\begin{cases}\frac{a^2cx\sin^2\left(e+fx\right)}{2}+\frac{a^2cx\cos^2\left(e+fx\right)}{2}+a^2cx+\frac{a^2c\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}+\frac{2a^2c\sin\left(e+fx\right)}{f}+\frac{a^2dx^2\sin^2\left(e+fx\right)}{4}+\frac{a^2dx^2\cos^2\left(e+fx\right)}{4}+\frac{a^2dx^2}{2}+ \\ \left(a\cos\left(e\right)+a\right)^2\left(cx+\frac{dx^2}{2}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c*x*sin(e + f*x)**2/2 + a**2*c*x*cos(e + f*x)**2/2 + a**2*c
*x + a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a**2*c*sin(e + f*x)/f + a**
2*d*x**2*sin(e + f*x)**2/4 + a**2*d*x**2*cos(e + f*x)**2/4 + a**2*d*x**2/2
+ a**2*d*x*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a**2*d*x*sin(e + f*x)/f + a*
*2*d*cos(e + f*x)**2/(4*f**2) + 2*a**2*d*cos(e + f*x)/f**2, Ne(f, 0)), ((a*
cos(e) + a)**2*(c*x + d*x**2/2), True))
```

Giac [A] time = 1.12837, size = 144, normalized size = 1.22

$$\frac{3}{4}\,a^2dx^2+\frac{3}{2}\,a^2cx+\frac{a^2d\cos\left(2\,fx+2\,e\right)}{8\,f^2}+\frac{2\,a^2d\cos\left(fx+e\right)}{f^2}+\frac{\left(a^2dfx+a^2cf\right)\sin\left(2\,fx+2\,e\right)}{4\,f^2}+\frac{2\left(a^2dfx+a^2cf\right)\sin\left(\right.}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/4*a^2*d*x^2 + 3/2*a^2*c*x + 1/8*a^2*d*cos(2*f*x + 2*e)/f^2 + 2*a^2*d*cos(
f*x + e)/f^2 + 1/4*(a^2*d*f*x + a^2*c*f)*sin(2*f*x + 2*e)/f^2 + 2*(a^2*d*f*
x + a^2*c*f)*sin(f*x + e)/f^2
```

3.126 $\int \frac{(a+a\cos(e+fx))^2}{c+dx}\,dx$

Optimal. Leaf size=145

$$\frac{2a^2\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\cos\left(e-\frac{cf}{d}\right)}{d}+\frac{a^2\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{2d}-\frac{2a^2\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d}$$

```
[Out] (2*a^2*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x])/d + (a^2*Cos[2*e - (2*c
*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(2*d) + (3*a^2*Log[c + d*x])/(2*d) -
 (2*a^2*Sin[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d - (a^2*Sin[2*e - (2*
c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(2*d)
```

Rubi [A] time = 0.337966, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3318, 3312, 3303, 3299, 3302}

$$\frac{2a^2\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\cos\left(e-\frac{cf}{d}\right)}{d}+\frac{a^2\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{2d}-\frac{2a^2\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[e + f*x])^2/(c + d*x),x]
```

```
[Out] (2*a^2*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x])/d + (a^2*Cos[2*e - (2*c
*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(2*d) + (3*a^2*Log[c + d*x])/(2*d) -
 (2*a^2*Sin[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d - (a^2*Sin[2*e - (2*
c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(2*d)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(e+fx))^2}{c+dx}\,dx = \left(4a^2\right)\int \frac{\sin^4\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)}{c+dx}\,dx$$
$$= \left(4a^2\right)\int \left(\frac{3}{8(c+dx)}+\frac{\cos(e+fx)}{2(c+dx)}+\frac{\cos(2e+2fx)}{8(c+dx)}\right)dx$$
$$= \frac{3a^2\log(c+dx)}{2d}+\frac{1}{2}a^2\int \frac{\cos(2e+2fx)}{c+dx}\,dx+\left(2a^2\right)\int \frac{\cos(e+fx)}{c+dx}\,dx$$
$$= \frac{3a^2\log(c+dx)}{2d}+\frac{1}{2}\left(a^2\cos\left(2e-\frac{2cf}{d}\right)\right)\int \frac{\cos\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx+\left(2a^2\cos\left(e-\frac{cf}{d}\right)\right)\int \frac{\cos}{}$$
$$= \frac{2a^2\cos\left(e-\frac{cf}{d}\right)\text{Ci}\left(\frac{cf}{d}+fx\right)}{d}+\frac{a^2\cos\left(2e-\frac{2cf}{d}\right)\text{Ci}\left(\frac{2cf}{d}+2fx\right)}{2d}+\frac{3a^2\log(c+dx)}{2d}-\frac{2a^2\,\text{s}}{}$$

Mathematica [A] time = 0.231772, size = 114, normalized size = 0.79

$$\frac{a^2\left(4\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\cos\left(e-\frac{cf}{d}\right)+\text{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)\cos\left(2e-\frac{2cf}{d}\right)-4\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)-\sin\left(\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[e + f*x])^2/(c + d*x),x]
```

```
[Out] (a^2*(4*Cos[e - (c*f)/d]*CosIntegral[f*(c/d + x)] + Cos[2*e - (2*c*f)/d]*Co
sIntegral[(2*f*(c + d*x))/d] + 3*Log[c + d*x] - 4*Sin[e - (c*f)/d]*SinInteg
ral[f*(c/d + x)] - Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d]))/(2
*d)
```

Maple [A] time = 0.04, size = 192, normalized size = 1.3

$$\frac{a^2}{2d}\text{Si}\left(2fx+2e+2\frac{cf-de}{d}\right)\sin\left(2\frac{cf-de}{d}\right)+\frac{a^2}{2d}\text{Ci}\left(2fx+2e+2\frac{cf-de}{d}\right)\cos\left(2\frac{cf-de}{d}\right)+\frac{3a^2\ln\left(\left(fx+e\right)d+c\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(f*x+e))^2/(d*x+c),x)
```

```
[Out] 1/2*a^2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d+1/2*a^2*Ci(2*f*x+2
*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d+3/2*a^2*ln((f*x+e)*d+c*f-d*e)/d+2*a^
2*Si(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d+2*a^2*Ci(f*x+e+(c*f-d*e)/d)*cos(
(c*f-d*e)/d)/d
```

Maxima [C] time = 1.52674, size = 455, normalized size = 3.14

$$\frac{4a^2f\log\left(c+\frac{(fx+e)d}{f}-\frac{de}{f}\right)}{d}-\frac{4\left(f\left(E_1\left(\frac{i(fx+e)d-ide+icf}{d}\right)+E_1\left(-\frac{i(fx+e)d-ide+icf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f\left(iE_1\left(\frac{i(fx+e)d-ide+icf}{d}\right)-iE_1\left(-\frac{i(fx+e)d-ide+icf}{d}\right)\right)\sin\left(-\frac{de-cf}{d}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a^2*f*log(c + (f*x + e)*d/f - d*e/f)/d - 4*(f*(exp_integral_e(1, (I*
(f*x + e)*d - I*d*e + I*c*f)/d) + exp_integral_e(1, -(I*(f*x + e)*d - I*d*e
 + I*c*f)/d))*cos(-(d*e - c*f)/d) + f*(I*exp_integral_e(1, (I*(f*x + e)*d -
 I*d*e + I*c*f)/d) - I*exp_integral_e(1, -(I*(f*x + e)*d - I*d*e + I*c*f)/d
))*sin(-(d*e - c*f)/d))*a^2/d - (f*(exp_integral_e(1, (2*I*(f*x + e)*d - 2*
I*d*e + 2*I*c*f)/d) + exp_integral_e(1, -(2*I*(f*x + e)*d - 2*I*d*e + 2*I*c
*f)/d))*cos(-2*(d*e - c*f)/d) - f*(-I*exp_integral_e(1, (2*I*(f*x + e)*d -
2*I*d*e + 2*I*c*f)/d) + I*exp_integral_e(1, -(2*I*(f*x + e)*d - 2*I*d*e + 2
*I*c*f)/d))*sin(-2*(d*e - c*f)/d) - 2*f*log((f*x + e)*d - d*e + c*f))*a^2/d
)/f
```

Fricas [A] time = 1.66132, size = 467, normalized size = 3.22

$$\frac{2\,a^2\sin\left(-\frac{2\,(de-cf)}{d}\right)\mathrm{Si}\left(\frac{2\,(dfx+cf)}{d}\right)+8\,a^2\sin\left(-\frac{de-cf}{d}\right)\mathrm{Si}\left(\frac{dfx+cf}{d}\right)+6\,a^2\log\left(dx+c\right)+4\left(a^2\,\mathrm{Ci}\left(\frac{dfx+cf}{d}\right)+a^2\,\mathrm{Ci}\left(-\frac{dfx\cdots}{d}\right.\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*sin(-2*(d*e - c*f)/d)*sin_integral(2*(d*f*x + c*f)/d) + 8*a^2*si
n(-(d*e - c*f)/d)*sin_integral((d*f*x + c*f)/d) + 6*a^2*log(d*x + c) + 4*(a
^2*cos_integral((d*f*x + c*f)/d) + a^2*cos_integral(-(d*f*x + c*f)/d))*cos(
-(d*e - c*f)/d) + (a^2*cos_integral(2*(d*f*x + c*f)/d) + a^2*cos_integral(-
2*(d*f*x + c*f)/d))*cos(-2*(d*e - c*f)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\frac{2\cos\left(e+fx\right)}{c+dx}\,dx+\int\frac{\cos^2\left(e+fx\right)}{c+dx}\,dx+\int\frac{1}{c+dx}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))**2/(d*x+c),x)
```

```
[Out] a**2*(Integral(2*cos(e + f*x)/(c + d*x), x) + Integral(cos(e + f*x)**2/(c +
 d*x), x) + Integral(1/(c + d*x), x))
```

Giac [C] time = 1.6169, size = 9360, normalized size = 64.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(6*a^2*log(abs(d*x + c))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan
(e)^2 + a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c
*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 4*a^2*real_part(cos_integral(f*x + c*f/d))*
tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 4*a^2*real_part(cos_i
ntegral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2
+ a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)
^2*tan(1/2*e)^2*tan(e)^2 + 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*t
an(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) - 2*a^2*imag_part(cos_inte
gral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) +
 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/
2*e)^2*tan(e) + 8*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan
(1/2*c*f/d)^2*tan(1/2*e)*tan(e)^2 - 8*a^2*imag_part(cos_integral(-f*x - c*f
/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)*tan(e)^2 + 16*a^2*sin_integra
l((d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)*tan(e)^2 - 8*a^
2*imag_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*
e)^2*tan(e)^2 + 8*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*ta
n(1/2*c*f/d)*tan(1/2*e)^2*tan(e)^2 - 16*a^2*sin_integral((d*f*x + c*f)/d)*t
an(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)^2*tan(e)^2 - 2*a^2*imag_part(cos_inte
gral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 +
2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2
*tan(1/2*e)^2*tan(e)^2 - 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)*t
an(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 6*a^2*log(abs(d*x + c))*tan(c*f/d)^
2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 - a^2*real_part(cos_integral(2*f*x + 2*c*f/
d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 4*a^2*real_part(cos_integr
al(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 4*a^2*real_pa
rt(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 -
 a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^
2*tan(1/2*e)^2 + 4*a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*
tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) + 4*a^2*real_part(cos_integral(-2*f*x
- 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) + 6*a^2*log(abs
(d*x + c))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 + a^2*real_part(cos_integ
ral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 - 4*a^2*real_p
art(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e)^2 - 4*a
^2*real_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(
e)^2 + a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c
*f/d)^2*tan(e)^2 + 16*a^2*real_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2
*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 + 16*a^2*real_part(cos_integral(-f*x -
c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 + 6*a^2*log(abs(d*x
 + c))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + a^2*real_part(cos_integral(2*f*
x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 4*a^2*real_part(cos_inte
gral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - 4*a^2*real_part(cos
_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + a^2*real_part
(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 6*a^2
*log(abs(d*x + c))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 - a^2*real_part(c
os_integral(2*f*x + 2*c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + 4*a^
2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)
^2 + 4*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e
)^2*tan(e)^2 - a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*c*f/d)
^2*tan(1/2*e)^2*tan(e)^2 + 8*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(c
*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - 8*a^2*imag_part(cos_integral(-f*x - c
*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e) + 16*a^2*sin_integral((d*f*
x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e) - 8*a^2*imag_part(cos_
integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)^2 + 8*a^2*ima
g_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e)^2
 - 16*a^2*sin_integral((d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2
*e)^2 + 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*c
*f/d)^2*tan(1/2*e)^2 - 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(
c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 4*a^2*sin_integral(2*(d*f*x + c*f)/d
)*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 2*a^2*imag_part(cos_integral(2
```

*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e) - 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e) + 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(e) + 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e) - 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2*tan(e) + 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*e)^2*tan(e) - 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) + 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) - 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e) + 8*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(e)^2 - 8*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(e)^2 + 16*a^2*sin_integral((d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(e)^2 - 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(e)^2 + 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(e)^2 - 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(e)^2 - 8*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*e)*tan(e)^2 + 8*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*e)*tan(e)^2 - 16*a^2*sin_integral((d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*e)*tan(e)^2 + 8*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)*tan(e)^2 - 8*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)*tan(e)^2 + 16*a^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)^2*tan(1/2*e)*tan(e)^2 - 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*e)^2*tan(e)^2 + 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*e)^2*tan(e)^2 - 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)*tan(1/2*e)^2*tan(e)^2 - 8*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)^2*tan(e)^2 + 8*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)^2*tan(e)^2 - 16*a^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)*tan(1/2*e)^2*tan(e)^2 + 6*a^2*log(abs(d*x + c))*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - 4*a^2*real_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - 4*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 16*a^2*real_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e) + 16*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)*tan(1/2*e) + 6*a^2*log(abs(d*x + c))*tan(c*f/d)^2*tan(1/2*e)^2 - a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2 - 4*a^2*real_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2 - 4*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2 - a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*e)^2 + 6*a^2*log(abs(d*x + c))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 4*a^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 4*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + 4*a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(e) + 4*a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2*tan(e) + 4*a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*e)^2*tan(e) + 4*a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*e)^2*tan(e) + 6*a^2*log(abs(d*x + c))*tan(c*f/d)^2*tan(e)^2 + a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(e)^2 + 4*a^2*real_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(e)^2 + 4*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(e)^2 + a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(e)^2 + 6*a^2*log(abs(d*x + c))*tan(1/2*c*f/d)^2*tan(e)^2 - a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*c*f/d)^2*tan(e)^2 - 4*a^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2*tan(e)^2 - 4*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2*tan(e)^2 - a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*c*f/d)^2*tan(e)^2 + 16*a^2*real_part(cos_integral(f*x + c*f/d))*ta

```
n(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 + 16*a^2*real_part(cos_integral(-f*x - c*f
/d))*tan(1/2*c*f/d)*tan(1/2*e)*tan(e)^2 + 6*a^2*log(abs(d*x + c))*tan(1/2*e
)^2*tan(e)^2 - a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*e)^2*ta
n(e)^2 - 4*a^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*e)^2*tan(e)^2 -
 4*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2*tan(e)^2 - a^2*re
al_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*e)^2*tan(e)^2 + 8*a^2*imag_
part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d) - 8*a^2*imag_pa
rt(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d) + 16*a^2*sin_int
egral((d*f*x + c*f)/d)*tan(c*f/d)^2*tan(1/2*c*f/d) + 2*a^2*imag_part(cos_in
tegral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2 - 2*a^2*imag_part(cos_
integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2 + 4*a^2*sin_integra
l(2*(d*f*x + c*f)/d)*tan(c*f/d)*tan(1/2*c*f/d)^2 - 8*a^2*imag_part(cos_inte
gral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*e) + 8*a^2*imag_part(cos_integral(-
f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*e) - 16*a^2*sin_integral((d*f*x + c*f)/d
)*tan(c*f/d)^2*tan(1/2*e) + 8*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(
1/2*c*f/d)^2*tan(1/2*e) - 8*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1
/2*c*f/d)^2*tan(1/2*e) + 16*a^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d
)^2*tan(1/2*e) + 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*
tan(1/2*e)^2 - 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*t
an(1/2*e)^2 + 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)*tan(1/2*e)^2
 - 8*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)^2 +
 8*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)*tan(1/2*e)^2 -
16*a^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)*tan(1/2*e)^2 + 2*a^2*im
ag_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(e) - 2*a^2*imag_par
t(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(e) + 4*a^2*sin_integral(
2*(d*f*x + c*f)/d)*tan(c*f/d)^2*tan(e) - 2*a^2*imag_part(cos_integral(2*f*x
 + 2*c*f/d))*tan(1/2*c*f/d)^2*tan(e) + 2*a^2*imag_part(cos_integral(-2*f*x
- 2*c*f/d))*tan(1/2*c*f/d)^2*tan(e) - 4*a^2*sin_integral(2*(d*f*x + c*f)/d)
*tan(1/2*c*f/d)^2*tan(e) - 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*t
an(1/2*e)^2*tan(e) + 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/
2*e)^2*tan(e) - 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(1/2*e)^2*tan(e) -
 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(e)^2 + 2*a^2
*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(e)^2 - 4*a^2*sin_
integral(2*(d*f*x + c*f)/d)*tan(c*f/d)*tan(e)^2 + 8*a^2*imag_part(cos_integ
ral(f*x + c*f/d))*tan(1/2*c*f/d)*tan(e)^2 - 8*a^2*imag_part(cos_integral(-f
*x - c*f/d))*tan(1/2*c*f/d)*tan(e)^2 + 16*a^2*sin_integral((d*f*x + c*f)/d)
*tan(1/2*c*f/d)*tan(e)^2 - 8*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(1
/2*e)*tan(e)^2 + 8*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)*tan
(e)^2 - 16*a^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*e)*tan(e)^2 + 6*a^2*lo
g(abs(d*x + c))*tan(c*f/d)^2 - a^2*real_part(cos_integral(2*f*x + 2*c*f/d))
*tan(c*f/d)^2 + 4*a^2*real_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2 + 4
*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2 - a^2*real_part(cos
_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2 + 6*a^2*log(abs(d*x + c))*tan(1/2
*c*f/d)^2 + a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*c*f/d)^2 -
 4*a^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2 - 4*a^2*real_p
art(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d)^2 + a^2*real_part(cos_integr
al(-2*f*x - 2*c*f/d))*tan(1/2*c*f/d)^2 + 16*a^2*real_part(cos_integral(f*x
+ c*f/d))*tan(1/2*c*f/d)*tan(1/2*e) + 16*a^2*real_part(cos_integral(-f*x -
c*f/d))*tan(1/2*c*f/d)*tan(1/2*e) + 6*a^2*log(abs(d*x + c))*tan(1/2*e)^2 +
a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*e)^2 - 4*a^2*real_part
(cos_integral(f*x + c*f/d))*tan(1/2*e)^2 - 4*a^2*real_part(cos_integral(-f*
x - c*f/d))*tan(1/2*e)^2 + a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*ta
n(1/2*e)^2 + 4*a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(
e) + 4*a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(e) + 6*
a^2*log(abs(d*x + c))*tan(e)^2 - a^2*real_part(cos_integral(2*f*x + 2*c*f/d
))*tan(e)^2 + 4*a^2*real_part(cos_integral(f*x + c*f/d))*tan(e)^2 + 4*a^2*r
eal_part(cos_integral(-f*x - c*f/d))*tan(e)^2 - a^2*real_part(cos_integral(
-2*f*x - 2*c*f/d))*tan(e)^2 + 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d)
)*tan(c*f/d) - 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d) +
```

```
 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d) + 8*a^2*imag_part(cos_int
egral(f*x + c*f/d))*tan(1/2*c*f/d) - 8*a^2*imag_part(cos_integral(-f*x - c*
f/d))*tan(1/2*c*f/d) + 16*a^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d)
- 8*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*e) + 8*a^2*imag_part(c
os_integral(-f*x - c*f/d))*tan(1/2*e) - 16*a^2*sin_integral((d*f*x + c*f)/d
)*tan(1/2*e) - 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(e) + 2*a^
2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(e) - 4*a^2*sin_integral(2*(
d*f*x + c*f)/d)*tan(e) + 6*a^2*log(abs(d*x + c)) + a^2*real_part(cos_integr
al(2*f*x + 2*c*f/d)) + 4*a^2*real_part(cos_integral(f*x + c*f/d)) + 4*a^2*r
eal_part(cos_integral(-f*x - c*f/d)) + a^2*real_part(cos_integral(-2*f*x -
2*c*f/d)))/(d*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + d*tan(c
*f/d)^2*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + d*tan(c*f/d)^2*tan(1/2*c*f/d)^2*tan
(e)^2 + d*tan(c*f/d)^2*tan(1/2*e)^2*tan(e)^2 + d*tan(1/2*c*f/d)^2*tan(1/2*e
)^2*tan(e)^2 + d*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + d*tan(c*f/d)^2*tan(1/2*e)^
2 + d*tan(1/2*c*f/d)^2*tan(1/2*e)^2 + d*tan(c*f/d)^2*tan(e)^2 + d*tan(1/2*c
*f/d)^2*tan(e)^2 + d*tan(1/2*e)^2*tan(e)^2 + d*tan(c*f/d)^2 + d*tan(1/2*c*f
/d)^2 + d*tan(1/2*e)^2 + d*tan(e)^2 + d)
```

3.127 $\int \frac{(a+a\cos(e+fx))^2}{(c+dx)^2}\,dx$

Optimal. Leaf size=159

$$-\frac{a^2f\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{d^2}-\frac{2a^2f\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^2}-\frac{2a^2f\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right.}{d^2}$$

```
[Out] (-4*a^2*Cos[e/2 + (f*x)/2]^4)/(d*(c + d*x)) - (a^2*f*CosIntegral[(2*c*f)/d
+ 2*f*x]*Sin[2*e - (2*c*f)/d])/d^2 - (2*a^2*f*CosIntegral[(c*f)/d + f*x]*Si
n[e - (c*f)/d])/d^2 - (2*a^2*f*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])
/d^2 - (a^2*f*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/d^2
```

Rubi [A] time = 0.316648, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3318, 3313, 3303, 3299, 3302}

$$-\frac{a^2f\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{d^2}-\frac{2a^2f\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^2}-\frac{2a^2f\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right.}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] (-4*a^2*Cos[e/2 + (f*x)/2]^4)/(d*(c + d*x)) - (a^2*f*CosIntegral[(2*c*f)/d
+ 2*f*x]*Sin[2*e - (2*c*f)/d])/d^2 - (2*a^2*f*CosIntegral[(c*f)/d + f*x]*Si
n[e - (c*f)/d])/d^2 - (2*a^2*f*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])
/d^2 - (a^2*f*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/d^2
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(e+fx))^2}{(c+dx)^2}\,dx = \left(4a^2\right)\int \frac{\sin^4\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)}{(c+dx)^2}\,dx$$
$$= -\frac{4a^2\cos^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)} + \frac{\left(8a^2f\right)\int\left(-\frac{\sin(e+fx)}{4(c+dx)}-\frac{\sin(2e+2fx)}{8(c+dx)}\right)dx}{d}$$
$$= -\frac{4a^2\cos^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)} - \frac{\left(a^2f\right)\int\frac{\sin(2e+2fx)}{c+dx}\,dx}{d} - \frac{\left(2a^2f\right)\int\frac{\sin(e+fx)}{c+dx}\,dx}{d}$$
$$= -\frac{4a^2\cos^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)} - \frac{\left(a^2f\cos\left(2e-\frac{2cf}{d}\right)\right)\int\frac{\sin\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx}{d} - \frac{\left(2a^2f\cos\left(e-\frac{cf}{d}\right)\right)\int\frac{\sin(}{c}}{d}$$
$$= -\frac{4a^2\cos^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)} - \frac{a^2f\operatorname{Ci}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{d^2} - \frac{2a^2f\operatorname{Ci}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^2}$$

Mathematica [A] time = 0.497997, size = 206, normalized size = 1.3

$$-\frac{a^2\left(2f(c+dx)\text{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)\sin\left(2e-\frac{2cf}{d}\right)+4f(c+dx)\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+4dfx\cos\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] -(a^2*(3*d + 4*d*Cos[e + f*x] + d*Cos[2*(e + f*x)] + 2*f*(c + d*x)*CosInteg
ral[(2*f*(c + d*x))/d]*Sin[2*e - (2*c*f)/d] + 4*f*(c + d*x)*CosIntegral[f*(
c/d + x)]*Sin[e - (c*f)/d] + 4*c*f*Cos[e - (c*f)/d]*SinIntegral[f*(c/d + x)
] + 4*d*f*x*Cos[e - (c*f)/d]*SinIntegral[f*(c/d + x)] + 2*c*f*Cos[2*e - (2*
c*f)/d]*SinIntegral[(2*f*(c + d*x))/d] + 2*d*f*x*Cos[2*e - (2*c*f)/d]*SinIn
tegral[(2*f*(c + d*x))/d]))/(2*d^2*(c + d*x))
```

Maple [A] time = 0.046, size = 276, normalized size = 1.7

$$\frac{1}{f}\left(\frac{a^2f^2}{4}\left(-2\frac{\cos\left(2fx+2e\right)}{\left(\left(fx+e\right)d+cf-de\right)d}-2\frac{1}{d}\left(2\frac{1}{d}\operatorname{Si}\left(2fx+2e+2\frac{cf-de}{d}\right)\cos\left(2\frac{cf-de}{d}\right)-2\frac{1}{d}\operatorname{Ci}\left(2fx+2e+2\frac{c}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(f*x+e))^2/(d*x+c)^2,x)
```

```
[Out] 1/f*(1/4*a^2*f^2*(-2*cos(2*f*x+2*e)/((f*x+e)*d+c*f-d*e)/d-2*(2*Si(2*f*x+2*e
+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d-2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c
*f-d*e)/d)/d)/d)-3/2*a^2*f^2/((f*x+e)*d+c*f-d*e)/d+2*a^2*f^2*(-cos(f*x+e)/(
(f*x+e)*d+c*f-d*e)/d-(Si(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d-Ci(f*x+e+(c*
```

```
f-d*e)/d)*sin((c*f-d*e)/d)/d)/d))
```

Maxima [C] time = 1.82966, size = 500, normalized size = 3.14

$$-\frac{\frac{64\,a^2f^2}{(fx+e)d^2-d^2e+cdf}+\frac{8\left(8f^2\left(E_2\left(\frac{i(fx+e)d-ide+icf}{d}\right)+E_2\left(-\frac{i(fx+e)d-ide+icf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f^2\left(8i\,E_2\left(\frac{i(fx+e)d-ide+icf}{d}\right)-8i\,E_2\left(-\frac{i(fx+e)d-ide+icf}{d}\right)\right)\sin\left(-\frac{de-cf}{d}\right)\right.}{(fx+e)d^2-d^2e+cdf}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/64*(64*a^2*f^2/((f*x + e)*d^2 - d^2*e + c*d*f) + 8*(8*f^2*(exp_integral_
e(2, (I*(f*x + e)*d - I*d*e + I*c*f)/d) + exp_integral_e(2, -(I*(f*x + e)*d
 - I*d*e + I*c*f)/d))*cos(-(d*e - c*f)/d) + f^2*(8*I*exp_integral_e(2, (I*(
f*x + e)*d - I*d*e + I*c*f)/d) - 8*I*exp_integral_e(2, -(I*(f*x + e)*d - I*
d*e + I*c*f)/d))*sin(-(d*e - c*f)/d))*a^2/((f*x + e)*d^2 - d^2*e + c*d*f) +
 (16*f^2*(exp_integral_e(2, (2*I*(f*x + e)*d - 2*I*d*e + 2*I*c*f)/d) + exp_
integral_e(2, -(2*I*(f*x + e)*d - 2*I*d*e + 2*I*c*f)/d))*cos(-2*(d*e - c*f)
/d) + f^2*(16*I*exp_integral_e(2, (2*I*(f*x + e)*d - 2*I*d*e + 2*I*c*f)/d)
- 16*I*exp_integral_e(2, -(2*I*(f*x + e)*d - 2*I*d*e + 2*I*c*f)/d))*sin(-2*
(d*e - c*f)/d) + 32*f^2)*a^2/((f*x + e)*d^2 - d^2*e + c*d*f))/f
```

Fricas [A] time = 1.83768, size = 683, normalized size = 4.3

$$-\frac{2\,a^2d\cos\left(fx+e\right)^2+4\,a^2d\cos\left(fx+e\right)+2\,a^2d+2\left(a^2dfx+a^2cf\right)\cos\left(-\frac{2(de-cf)}{d}\right)\mathrm{Si}\left(\frac{2(dfx+cf)}{d}\right)+4\left(a^2dfx+a^2cf\right)\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*d*cos(f*x + e)^2 + 4*a^2*d*cos(f*x + e) + 2*a^2*d + 2*(a^2*d*f*
x + a^2*c*f)*cos(-2*(d*e - c*f)/d)*sin_integral(2*(d*f*x + c*f)/d) + 4*(a^2
*d*f*x + a^2*c*f)*cos(-(d*e - c*f)/d)*sin_integral((d*f*x + c*f)/d) - 2*((a
^2*d*f*x + a^2*c*f)*cos_integral((d*f*x + c*f)/d) + (a^2*d*f*x + a^2*c*f)*c
os_integral(-(d*f*x + c*f)/d))*sin(-(d*e - c*f)/d) - ((a^2*d*f*x + a^2*c*f)
*cos_integral(2*(d*f*x + c*f)/d) + (a^2*d*f*x + a^2*c*f)*cos_integral(-2*(d
*f*x + c*f)/d))*sin(-2*(d*e - c*f)/d))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\frac{2\cos\left(e+fx\right)}{c^2+2cdx+d^2x^2}\,dx+\int\frac{\cos^2\left(e+fx\right)}{c^2+2cdx+d^2x^2}\,dx+\int\frac{1}{c^2+2cdx+d^2x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))**2/(d*x+c)**2,x)
```

```
[Out] a**2*(Integral(2*cos(e + f*x)/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(c
os(e + f*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(1/(c**2 + 2*c*d*
x + d**2*x**2), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.128 $\int \frac{(c+dx)^3}{a+a\cos(e+fx)}\,dx$

Optimal. Leaf size=134

$$-\frac{12id^2(c+dx)\text{Li}_2\left(-e^{i(e+fx)}\right)}{af^3}+\frac{6d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{af^2}+\frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{i(c+dx)^3}{af}+\frac{12d^3\text{Li}_3\left(-e^{i(e+fx)}\right)}{af^4}$$

```
[Out] ((-I)*(c + d*x)^3)/(a*f) + (6*d*(c + d*x)^2*Log[1 + E^(I*(e + f*x))])/(a*f^
2) - ((12*I)*d^2*(c + d*x)*PolyLog[2, -E^(I*(e + f*x))])/(a*f^3) + (12*d^3*
PolyLog[3, -E^(I*(e + f*x))])/(a*f^4) + ((c + d*x)^3*Tan[e/2 + (f*x)/2])/(a
*f)
```

Rubi [A] time = 0.281878, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {3318, 4184, 3719, 2190, 2531, 2282, 6589}

$$-\frac{12id^2(c+dx)\text{Li}_2\left(-e^{i(e+fx)}\right)}{af^3}+\frac{6d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{af^2}+\frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{i(c+dx)^3}{af}+\frac{12d^3\text{Li}_3\left(-e^{i(e+fx)}\right)}{af^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + a*Cos[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)^3)/(a*f) + (6*d*(c + d*x)^2*Log[1 + E^(I*(e + f*x))])/(a*f^
2) - ((12*I)*d^2*(c + d*x)*PolyLog[2, -E^(I*(e + f*x))])/(a*f^3) + (12*d^3*
PolyLog[3, -E^(I*(e + f*x))])/(a*f^4) + ((c + d*x)^3*Tan[e/2 + (f*x)/2])/(a
*f)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a+a\cos(e+fx)}\,dx = \frac{\int (c+dx)^3\csc^2\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)dx}{2a}$$

$$= \frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{(3d)\int (c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{af}$$

$$= -\frac{i(c+dx)^3}{af} + \frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{(6id)\int \frac{e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)^2}{1+e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}}dx}{af}$$

$$= -\frac{i(c+dx)^3}{af} + \frac{6d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{af^2} + \frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{\left(12d^2\right)\int (c+dx)\log ...}{a ...}$$

$$= -\frac{i(c+dx)^3}{af} + \frac{6d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\text{Li}_2\left(-e^{i(e+fx)}\right)}{af^3} + \frac{(c+dx)^3\tan ...}{af}$$

$$= -\frac{i(c+dx)^3}{af} + \frac{6d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\text{Li}_2\left(-e^{i(e+fx)}\right)}{af^3} + \frac{(c+dx)^3\tan ...}{af}$$

$$= -\frac{i(c+dx)^3}{af} + \frac{6d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\text{Li}_2\left(-e^{i(e+fx)}\right)}{af^3} + \frac{12d^3\text{Li}_3\left(-e^{i(e+ ...}\right.}{af^4}$$

Mathematica [A] time = 0.304826, size = 151, normalized size = 1.13

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left((c+dx)^3\sin\left(\frac{1}{2}(e+fx)\right) - \frac{i\cos\left(\frac{1}{2}(e+fx)\right)\left(12d^2f(c+dx)\text{Li}_2\left(-e^{i(e+fx)}\right)+f^2(c+dx)^2\left(f(c+dx)+6id\log\left(1+e^{i(e+fx)}\right)\right)+12id^3\text{Li} ...\right.}{f^3}\right.}{af(\cos(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + a*Cos[e + f*x]),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(((-I)*Cos[(e + f*x)/2]*(f^2*(c + d*x)^2*(f*(c + d*x) +
 (6*I)*d*Log[1 + E^(I*(e + f*x))]) + 12*d^2*f*(c + d*x)*PolyLog[2, -E^(I*(e
 + f*x))] + (12*I)*d^3*PolyLog[3, -E^(I*(e + f*x))]))/f^3 + (c + d*x)^3*Sin
[(e + f*x)/2]))/(a*f*(1 + Cos[e + f*x]))
```

Maple [B] time = 0.453, size = 364, normalized size = 2.7

$$\frac{-12\,id^3\mathrm{polylog}\left(2,-\mathrm{e}^{i(fx+e)}\right)x}{af^3}+6\,\frac{c^2d\ln\left(\mathrm{e}^{i(fx+e)}+1\right)}{af^2}-6\,\frac{c^2d\ln\left(\mathrm{e}^{i(fx+e)}\right)}{af^2}-6\,\frac{d^3e^2\ln\left(\mathrm{e}^{i(fx+e)}\right)}{f^4a}+12\,\frac{cd^2\ln\left(\mathrm{e}^{i(fx+e)}+1\right.}{af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+a*cos(f*x+e)),x)
```

```
[Out] -12*I*d^3/f^3/a*polylog(2,-exp(I*(f*x+e)))*x+6*d/f^2/a*c^2*ln(exp(I*(f*x+e)
)+1)-6*d/f^2/a*c^2*ln(exp(I*(f*x+e)))-6*d^3/f^4/a*e^2*ln(exp(I*(f*x+e)))+12
*d^2/f^2/a*c*ln(exp(I*(f*x+e))+1)*x-12*I*d^2/f^2/a*c*e*x-12*I*d^2/f^3/a*c*p
olylog(2,-exp(I*(f*x+e)))-6*I*d^2/f^3/a*c*e^2+6*d^3/f^2/a*ln(exp(I*(f*x+e))
+1)*x^2+6*I*d^3/f^3/a*e^2*x+12*d^3*polylog(3,-exp(I*(f*x+e)))/a/f^4+12*d^2/
f^3/a*c*e*ln(exp(I*(f*x+e)))+4*I*d^3/f^4/a*e^3-6*I*d^2/f/a*c*x^2+2*I*(d^3*x
^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/f/a/(exp(I*(f*x+e))+1)-2*I*d^3/f/a*x^3
```

Maxima [B] time = 1.72115, size = 1264, normalized size = 9.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] -(6*((cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1)*log(cos(f*x + e
)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + 2*(f*x + e)*sin(f*x + e))*c*d^
2*e/(a*f^2*cos(f*x + e)^2 + a*f^2*sin(f*x + e)^2 + 2*a*f^2*cos(f*x + e) + a
*f^2) - 3*((cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1)*log(cos(f
*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + 2*(f*x + e)*sin(f*x + e)
)*c^2*d/(a*f*cos(f*x + e)^2 + a*f*sin(f*x + e)^2 + 2*a*f*cos(f*x + e) + a*f
) - c^3*sin(f*x + e)/(a*(cos(f*x + e) + 1)) - 3*c*d^2*e^2*sin(f*x + e)/(a*f
^2*(cos(f*x + e) + 1)) + 3*c^2*d*e*sin(f*x + e)/(a*f*(cos(f*x + e) + 1)) +
(2*d^3*e^3 - (6*(f*x + e)^2*d^3 + 6*d^3*e^2 - 12*(d^3*e - c*d^2*f)*(f*x + e
) + 6*((f*x + e)^2*d^3 + d^3*e^2 - 2*(d^3*e - c*d^2*f)*(f*x + e))*cos(f*x +
 e) + (6*I*(f*x + e)^2*d^3 + 6*I*d^3*e^2 + (-12*I*d^3*e + 12*I*c*d^2*f)*(f*
x + e))*sin(f*x + e))*arctan2(sin(f*x + e), cos(f*x + e) + 1) + 2*((f*x + e
)^3*d^3 + 3*(f*x + e)*d^3*e^2 - 3*(d^3*e - c*d^2*f)*(f*x + e)^2)*cos(f*x +
e) + (12*(f*x + e)*d^3 - 12*d^3*e + 12*c*d^2*f + 12*((f*x + e)*d^3 - d^3*e
+ c*d^2*f)*cos(f*x + e) - (-12*I*(f*x + e)*d^3 + 12*I*d^3*e - 12*I*c*d^2*f)
*sin(f*x + e))*dilog(-e^(I*f*x + I*e)) - (-3*I*(f*x + e)^2*d^3 - 3*I*d^3*e^
2 + (6*I*d^3*e - 6*I*c*d^2*f)*(f*x + e) + (-3*I*(f*x + e)^2*d^3 - 3*I*d^3*e
^2 + (6*I*d^3*e - 6*I*c*d^2*f)*(f*x + e))*cos(f*x + e) + 3*((f*x + e)^2*d^3
 + d^3*e^2 - 2*(d^3*e - c*d^2*f)*(f*x + e))*sin(f*x + e))*log(cos(f*x + e)^
2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) - (-12*I*d^3*cos(f*x + e) + 12*d^3
*sin(f*x + e) - 12*I*d^3)*polylog(3, -e^(I*f*x + I*e)) - (-2*I*(f*x + e)^3*
```

```
d^3 - 6*I*(f*x + e)*d^3*e^2 + (6*I*d^3*e - 6*I*c*d^2*f)*(f*x + e)^2)*sin(f*
x + e))/(-I*a*f^3*cos(f*x + e) + a*f^3*sin(f*x + e) - I*a*f^3))/f
```

Fricas [C] time = 1.76965, size = 1019, normalized size = 7.6

$$\frac{\left(6i\,d^3fx+6i\,cd^2f+\left(6i\,d^3fx+6i\,cd^2f\right)\cos\left(fx+e\right)\right)\mathrm{Li}_2\left(-\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)+\left(-6i\,d^3fx-6i\,cd^2f+\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] ((6*I*d^3*f*x + 6*I*c*d^2*f + (6*I*d^3*f*x + 6*I*c*d^2*f)*cos(f*x + e))*dil
og(-cos(f*x + e) + I*sin(f*x + e)) + (-6*I*d^3*f*x - 6*I*c*d^2*f + (-6*I*d^
3*f*x - 6*I*c*d^2*f)*cos(f*x + e))*dilog(-cos(f*x + e) - I*sin(f*x + e)) +
3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x +
 c^2*d*f^2)*cos(f*x + e))*log(cos(f*x + e) + I*sin(f*x + e) + 1) + 3*(d^3*f
^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f
^2)*cos(f*x + e))*log(cos(f*x + e) - I*sin(f*x + e) + 1) + 6*(d^3*cos(f*x +
 e) + d^3)*polylog(3, -cos(f*x + e) + I*sin(f*x + e)) + 6*(d^3*cos(f*x + e)
 + d^3)*polylog(3, -cos(f*x + e) - I*sin(f*x + e)) + (d^3*f^3*x^3 + 3*c*d^2
*f^3*x^2 + 3*c^2*d*f^3*x + c^3*f^3)*sin(f*x + e))/(a*f^4*cos(f*x + e) + a*f
^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c^3}{\cos\left(e+fx\right)+1}\,dx+\int \frac{d^3x^3}{\cos\left(e+fx\right)+1}\,dx+\int \frac{3cd^2x^2}{\cos\left(e+fx\right)+1}\,dx+\int \frac{3c^2dx}{\cos\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+a*cos(f*x+e)),x)
```

```
[Out] (Integral(c**3/(cos(e + f*x) + 1), x) + Integral(d**3*x**3/(cos(e + f*x) +
1), x) + Integral(3*c*d**2*x**2/(cos(e + f*x) + 1), x) + Integral(3*c**2*d*
x/(cos(e + f*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^3}{a\cos\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(a*cos(f*x + e) + a), x)
```

3.129 $\int \frac{(c+dx)^2}{a+a\cos(e+fx)}\,dx$

Optimal. Leaf size=101

$$\frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{af^2}+\frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{i(c+dx)^2}{af}-\frac{4id^2\text{Li}_2\left(-e^{i(e+fx)}\right)}{af^3}$$

```
[Out] ((-I)*(c + d*x)^2)/(a*f) + (4*d*(c + d*x)*Log[1 + E^(I*(e + f*x))])/(a*f^2)
 - ((4*I)*d^2*PolyLog[2, -E^(I*(e + f*x))])/(a*f^3) + ((c + d*x)^2*Tan[e/2
+ (f*x)/2])/(a*f)
```

Rubi [A] time = 0.19869, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3318, 4184, 3719, 2190, 2279, 2391}

$$\frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{af^2}+\frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{i(c+dx)^2}{af}-\frac{4id^2\text{Li}_2\left(-e^{i(e+fx)}\right)}{af^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + a*Cos[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)^2)/(a*f) + (4*d*(c + d*x)*Log[1 + E^(I*(e + f*x))])/(a*f^2)
 - ((4*I)*d^2*PolyLog[2, -E^(I*(e + f*x))])/(a*f^3) + ((c + d*x)^2*Tan[e/2
+ (f*x)/2])/(a*f)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(c+dx)^2}{a+a\cos(e+fx)}\,dx = \frac{\int (c+dx)^2\csc^2\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)dx}{2a}$$
$$= \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{(2d)\int (c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{af}$$
$$= -\frac{i(c+dx)^2}{af} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{(4id)\int \frac{e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)}{1+e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}}dx}{af}$$
$$= -\frac{i(c+dx)^2}{af} + \frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{af^2} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{(4d^2)\int \log\left(1+e^{2i(\cdots}\right.}{af^2}$$
$$= -\frac{i(c+dx)^2}{af} + \frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{af^2} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{(4id^2)\,\mathrm{Subst}\left(\int \frac{\log(\cdots}{\cdots}\right.}{a\cdots}$$
$$= -\frac{i(c+dx)^2}{af} + \frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{af^2} - \frac{4id^2\mathrm{Li}_2\left(-e^{i(e+fx)}\right)}{af^3} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}$$

Mathematica [A] time = 0.329933, size = 125, normalized size = 1.24

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(f(c+dx)\left(f(c+dx)\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\left(4d\log\left(1+e^{i(e+fx)}\right)-if(c+dx)\right)\right)-4id^2\mathrm{Li}\cdots}{af^3(\cos(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + a*Cos[e + f*x]),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*((-4*I)*d^2*Cos[(e + f*x)/2]*PolyLog[2, -E^(I*(e + f*x)
)] + f*(c + d*x)*(Cos[(e + f*x)/2]*((-I)*f*(c + d*x) + 4*d*Log[1 + E^(I*(e
+ f*x))]) + f*(c + d*x)*Sin[(e + f*x)/2])))/(a*f^3*(1 + Cos[e + f*x]))
```

Maple [B] time = 0.381, size = 197, normalized size = 2.

$$\frac{2i\left(d^2x^2+2cdx+c^2\right)}{af\left(e^{i(fx+e)}+1\right)}+4\frac{cd\ln\left(e^{i(fx+e)}+1\right)}{af^2}-4\frac{cd\ln\left(e^{i(fx+e)}\right)}{af^2}-\frac{2id^2x^2}{af}-\frac{4id^2ex}{af^2}-\frac{2id^2e^2}{f^3a}+4\frac{d^2\ln\left(e^{i(fx+e)}+1\right)\cdots}{af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+a*cos(f*x+e)),x)
```

```
[Out] 2*I*(d^2*x^2+2*c*d*x+c^2)/f/a/(exp(I*(f*x+e))+1)+4*d/f^2/a*c*ln(exp(I*(f*x+
e))+1)-4*d/f^2/a*c*ln(exp(I*(f*x+e)))-2*I*d^2/f/a*x^2-4*I*d^2/f^2/a*e*x-2*I
*d^2/f^3/a*e^2+4*d^2/f^2/a*ln(exp(I*(f*x+e))+1)*x-4*I*d^2*polylog(2,-exp(I*
(f*x+e)))/a/f^3+4*d^2/f^3/a*e*ln(exp(I*(f*x+e)))
```

Maxima [B] time = 1.60632, size = 382, normalized size = 3.78

$$\frac{2\,c^2f^2 + \left(4\,d^2fx + 4\,cdf + 4\left(d^2fx + cdf\right)\cos\left(fx + e\right) + \left(4i\,d^2fx + 4i\,cdf\right)\sin\left(fx + e\right)\right)\arctan\left(\sin\left(fx + e\right), \cos\left(fx \cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] (2*c^2*f^2 + (4*d^2*f*x + 4*c*d*f + 4*(d^2*f*x + c*d*f)*cos(f*x + e) + (4*I
*d^2*f*x + 4*I*c*d*f)*sin(f*x + e))*arctan2(sin(f*x + e), cos(f*x + e) + 1)
 - 2*(d^2*f^2*x^2 + 2*c*d*f^2*x)*cos(f*x + e) - 4*(d^2*cos(f*x + e) + I*d^2
*sin(f*x + e) + d^2)*dilog(-e^(I*f*x + I*e)) + (-2*I*d^2*f*x - 2*I*c*d*f +
(-2*I*d^2*f*x - 2*I*c*d*f)*cos(f*x + e) + 2*(d^2*f*x + c*d*f)*sin(f*x + e))
*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + (-2*I*d^2*f^2*
x^2 - 4*I*c*d*f^2*x)*sin(f*x + e))/(-I*a*f^3*cos(f*x + e) + a*f^3*sin(f*x +
 e) - I*a*f^3)
```

Fricas [B] time = 1.68263, size = 564, normalized size = 5.58

$$\frac{\left(2i\,d^2\cos\left(fx + e\right) + 2i\,d^2\right)\mathrm{Li}_2\left(-\cos\left(fx + e\right) + i\,\sin\left(fx + e\right)\right) + \left(-2i\,d^2\cos\left(fx + e\right) - 2i\,d^2\right)\mathrm{Li}_2\left(-\cos\left(fx + e\right) - i\,\sin \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] ((2*I*d^2*cos(f*x + e) + 2*I*d^2)*dilog(-cos(f*x + e) + I*sin(f*x + e)) + (
-2*I*d^2*cos(f*x + e) - 2*I*d^2)*dilog(-cos(f*x + e) - I*sin(f*x + e)) + 2*
(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(f*x + e))*log(cos(f*x + e) + I*sin
(f*x + e) + 1) + 2*(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(f*x + e))*log(c
os(f*x + e) - I*sin(f*x + e) + 1) + (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*s
in(f*x + e))/(a*f^3*cos(f*x + e) + a*f^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c^2}{\cos(e+fx)+1}\,dx + \int \frac{d^2x^2}{\cos(e+fx)+1}\,dx + \int \frac{2cdx}{\cos(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+a*cos(f*x+e)),x)
```

```
[Out] (Integral(c**2/(cos(e + f*x) + 1), x) + Integral(d**2*x**2/(cos(e + f*x) +
1), x) + Integral(2*c*d*x/(cos(e + f*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^2}{a\cos\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(a*cos(f*x + e) + a), x)
```

3.130 $\int \frac{c+dx}{a+a\cos(e+fx)}\,dx$

Optimal. Leaf size=49

$$\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2}$$

```
[Out] (2*d*Log[Cos[e/2 + (f*x)/2]])/(a*f^2) + ((c + d*x)*Tan[e/2 + (f*x)/2])/(a*f
)
```

Rubi [A] time = 0.0641462, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3318, 4184, 3475}

$$\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + a*Cos[e + f*x]),x]
```

```
[Out] (2*d*Log[Cos[e/2 + (f*x)/2]])/(a*f^2) + ((c + d*x)*Tan[e/2 + (f*x)/2])/(a*f
)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{a+a\cos(e+fx)}\,dx &= \frac{\int (c+dx)\csc^2\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)dx}{2a}\\ &= \frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{d\int \tan\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{af}\\ &= \frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2}+\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}\end{aligned}$$

Mathematica [A] time = 0.0763534, size = 70, normalized size = 1.43

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(f(c+dx)\sin\left(\frac{1}{2}(e+fx)\right)+2d\cos\left(\frac{1}{2}(e+fx)\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)}{af^2(\cos(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + a*Cos[e + f*x]),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(2*d*Cos[(e + f*x)/2]*Log[Cos[(e + f*x)/2]] + f*(c + d*
x)*Sin[(e + f*x)/2]))/(a*f^2*(1 + Cos[e + f*x]))
```

Maple [A] time = 0.054, size = 60, normalized size = 1.2

$$\frac{c}{af}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{dx}{af}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{d}{af^2}\ln\left(1+\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+a*cos(f*x+e)),x)
```

```
[Out] 1/a*c/f*tan(1/2*f*x+1/2*e)+1/a*d*x/f*tan(1/2*f*x+1/2*e)-1/a*d/f^2*ln(1+tan(
1/2*f*x+1/2*e)^2)
```

Maxima [B] time = 1.20137, size = 216, normalized size = 4.41

$$\frac{\frac{\left(\left(\cos\left(fx+e\right)^2+\sin\left(fx+e\right)^2+2\cos\left(fx+e\right)+1\right)\log\left(\cos\left(fx+e\right)^2+\sin\left(fx+e\right)^2+2\cos\left(fx+e\right)+1\right)+2\left(fx+e\right)\sin\left(fx+e\right)\right)d}{af\cos\left(fx+e\right)^2+af\sin\left(fx+e\right)^2+2af\cos\left(fx+e\right)+af}+\frac{c\sin\left(fx+e\right)}{a\left(\cos\left(fx+e\right)+1\right)}-\frac{de\sin\left(fx+e\right)}{af\left(\cos\left(fx+e\right)+1\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] (((cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1)*log(cos(f*x + e)^2
 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + 2*(f*x + e)*sin(f*x + e))*d/(a*f*
cos(f*x + e)^2 + a*f*sin(f*x + e)^2 + 2*a*f*cos(f*x + e) + a*f) + c*sin(f*x
 + e)/(a*(cos(f*x + e) + 1)) - d*e*sin(f*x + e)/(a*f*(cos(f*x + e) + 1)))/f
```

Fricas [A] time = 1.66931, size = 149, normalized size = 3.04

$$\frac{\left(d\cos\left(fx+e\right)+d\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+\left(dfx+cf\right)\sin\left(fx+e\right)}{af^2\cos\left(fx+e\right)+af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] ((d*cos(f*x + e) + d)*log(1/2*cos(f*x + e) + 1/2) + (d*f*x + c*f)*sin(f*x +
 e))/(a*f^2*cos(f*x + e) + a*f^2)
```

Sympy [A] time = 0.749595, size = 70, normalized size = 1.43

$$\begin{cases} \frac{c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{dx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{d\log\left(\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{af^{2}} & \text{for } f \neq 0 \\ \frac{cx+\frac{dx^{2}}{2}}{a\cos\left(e\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e)),x)
```

```
[Out] Piecewise((c*tan(e/2 + f*x/2)/(a*f) + d*x*tan(e/2 + f*x/2)/(a*f) - d*log(ta
n(e/2 + f*x/2)**2 + 1)/(a*f**2), Ne(f, 0)), ((c*x + d*x**2/2)/(a*cos(e) + a
), True))
```

Giac [B] time = 1.14482, size = 316, normalized size = 6.45

$$-\frac{dfx\tan\left(\frac{1}{2}fx\right) + dfx\tan\left(\frac{1}{2}e\right) - d\log\Bigg(\frac{4\left(\tan\left(\frac{1}{2}e\right)^{2}+1\right)}{\tan\left(\frac{1}{2}fx\right)^{4}\tan\left(\frac{1}{2}e\right)^{2} - 2\tan\left(\frac{1}{2}fx\right)^{3}\tan\left(\frac{1}{2}e\right) + \tan\left(\frac{1}{2}fx\right)^{2}\tan\left(\frac{1}{2}e\right)^{2} + \tan\left(\frac{1}{2}fx\right)^{2} - 2\tan\left(\frac{1}{2}fx\right)\tan\left(\ldots\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] -(d*f*x*tan(1/2*f*x) + d*f*x*tan(1/2*e) - d*log(4*(tan(1/2*e)^2 + 1)/(tan(1
/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^3*tan(1/2*e) + tan(1/2*f*x)^2*tan(1
/2*e)^2 + tan(1/2*f*x)^2 - 2*tan(1/2*f*x)*tan(1/2*e) + 1))*tan(1/2*f*x)*tan
(1/2*e) + c*f*tan(1/2*f*x) + c*f*tan(1/2*e) + d*log(4*(tan(1/2*e)^2 + 1)/(t
an(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^3*tan(1/2*e) + tan(1/2*f*x)^2*t
an(1/2*e)^2 + tan(1/2*f*x)^2 - 2*tan(1/2*f*x)*tan(1/2*e) + 1)))/(a*f^2*tan(
1/2*f*x)*tan(1/2*e) - a*f^2)
```

3.131 $\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)(a\cos(e+fx)+a)},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)*(a + a*Cos[e + f*x])), x]
```

Rubi [A] time = 0.0596785, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + a*Cos[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a + a*Cos[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx=\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx$$

Mathematica [A] time = 2.74819, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + a*Cos[e + f*x])),x]
```

```
[Out] Integrate[1/((c + d*x)*(a + a*Cos[e + f*x])), x]
```

Maple [A] time = 0.293, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(a+a\cos\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+a*cos(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)/(a+a*cos(f*x+e)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{adx + ac + (adx + ac)\cos\left(fx + e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d*x + a*c + (a*d*x + a*c)*cos(f*x + e)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{c\cos\left(e+fx\right)+c+dx\cos\left(e+fx\right)+dx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e)),x)
```

```
[Out] Integral(1/(c*cos(e + f*x) + c + d*x*cos(e + f*x) + d*x), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx + c)\left(a\cos\left(fx + e\right) + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(a*cos(f*x + e) + a)), x)
```

3.132 $\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)^2(a\cos(e+fx)+a)},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)^2*(a + a*Cos[e + f*x])), x]
```

Rubi [A] time = 0.0565067, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + a*Cos[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + a*Cos[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx=\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx$$

Mathematica [A] time = 2.72868, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + a*Cos[e + f*x])),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + a*Cos[e + f*x])), x]
```

Maple [A] time = 0.335, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2\left(a+a\cos\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+a*cos(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)^2/(a+a*cos(f*x+e)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{ad^2x^2 + 2\,acdx + ac^2 + \left(ad^2x^2 + 2\,acdx + ac^2\right)\cos\left(fx + e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (a*d^2*x^2 + 2*a*c*d*x + a*c^2)
*cos(f*x + e)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{c^2\cos\left(e+fx\right)+c^2+2cdx\cos\left(e+fx\right)+2cdx+d^2x^2\cos\left(e+fx\right)+d^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+a*cos(f*x+e)),x)
```

```
[Out] Integral(1/(c**2*cos(e + f*x) + c**2 + 2*c*d*x*cos(e + f*x) + 2*c*d*x + d**
2*x**2*cos(e + f*x) + d**2*x**2), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx + c)^2\left(a\cos\left(fx + e\right) + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(a*cos(f*x + e) + a)), x)
```

3.133 $\int \frac{(c+dx)^3}{(a+a\cos(e+fx))^2}\,dx$

Optimal. Leaf size=271

$$-\frac{4id^2(c+dx)\text{Li}_2\left(-e^{i(e+fx)}\right)}{a^2f^3}+\frac{2d^2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{a^2f^3}+\frac{2d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{a^2f^2}-\frac{d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2}+$$

```
[Out] ((-I/3)*(c + d*x)^3)/(a^2*f) + (2*d*(c + d*x)^2*Log[1 + E^(I*(e + f*x))])/(
a^2*f^2) + (4*d^3*Log[Cos[e/2 + (f*x)/2]])/(a^2*f^4) - ((4*I)*d^2*(c + d*x)
*PolyLog[2, -E^(I*(e + f*x))])/(a^2*f^3) + (4*d^3*PolyLog[3, -E^(I*(e + f*x
))])/(a^2*f^4) - (d*(c + d*x)^2*Sec[e/2 + (f*x)/2]^2)/(2*a^2*f^2) + (2*d^2*
(c + d*x)*Tan[e/2 + (f*x)/2])/(a^2*f^3) + ((c + d*x)^3*Tan[e/2 + (f*x)/2])/
(3*a^2*f) + ((c + d*x)^3*Sec[e/2 + (f*x)/2]^2*Tan[e/2 + (f*x)/2])/(6*a^2*f)
```

Rubi [A] time = 0.366719, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {3318, 4186, 4184, 3475, 3719, 2190, 2531, 2282, 6589}

$$-\frac{4id^2(c+dx)\text{Li}_2\left(-e^{i(e+fx)}\right)}{a^2f^3}+\frac{2d^2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{a^2f^3}+\frac{2d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{a^2f^2}-\frac{d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + a*Cos[e + f*x])^2,x]
```


```
[Out] ((-I/3)*(c + d*x)^3)/(a^2*f) + (2*d*(c + d*x)^2*Log[1 + E^(I*(e + f*x))])/(
a^2*f^2) + (4*d^3*Log[Cos[e/2 + (f*x)/2]])/(a^2*f^4) - ((4*I)*d^2*(c + d*x)
*PolyLog[2, -E^(I*(e + f*x))])/(a^2*f^3) + (4*d^3*PolyLog[3, -E^(I*(e + f*x
))])/(a^2*f^4) - (d*(c + d*x)^2*Sec[e/2 + (f*x)/2]^2)/(2*a^2*f^2) + (2*d^2*
(c + d*x)*Tan[e/2 + (f*x)/2])/(a^2*f^3) + ((c + d*x)^3*Tan[e/2 + (f*x)/2])/
(3*a^2*f) + ((c + d*x)^3*Sec[e/2 + (f*x)/2]^2*Tan[e/2 + (f*x)/2])/(6*a^2*f)
```


Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+a\cos(e+fx))^2}\,dx = \frac{\int (c+dx)^3\csc^4\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)dx}{4a^2}$$

$$= -\frac{d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2} + \frac{(c+dx)^3\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} + \frac{\int (c+dx)^3\sec^2\left(\frac{e}{2}\right.}{6a^2}$$

$$= -\frac{d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2} + \frac{2d^2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{a^2f^3} + \frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+}{}$$

$$= -\frac{i(c+dx)^3}{3a^2f} + \frac{4d^3\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} - \frac{d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2} + \frac{2d^2(c+dx)\tan\left(\frac{e}{2}+\right.}{a^2f^3}$$

$$= -\frac{i(c+dx)^3}{3a^2f} + \frac{2d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{a^2f^2} + \frac{4d^3\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} - \frac{d(c+dx)^2\sec^2\left(\right.}{2a^2f^2}$$

$$= -\frac{i(c+dx)^3}{3a^2f} + \frac{2d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{a^2f^2} + \frac{4d^3\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} - \frac{4id^2(c+dx)\text{Li}_2\left(\right.}{a^2f^3}$$

$$= -\frac{i(c+dx)^3}{3a^2f} + \frac{2d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{a^2f^2} + \frac{4d^3\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} - \frac{4id^2(c+dx)\text{Li}_2\left(\right.}{a^2f^3}$$

$$= -\frac{i(c+dx)^3}{3a^2f} + \frac{2d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{a^2f^2} + \frac{4d^3\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} - \frac{4id^2(c+dx)\text{Li}_2\left(\right.}{a^2f^3}$$

Mathematica [A] time = 1.01766, size = 250, normalized size = 0.92

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(-2\cos^3\left(\frac{1}{2}(e+fx)\right)\left(-6d\left(f^2(c+dx)^2\log\left(1+e^{i(e+fx)}\right)-2idf(c+dx)\text{Li}_2\left(-e^{i(e+fx)}\right)+2d^2\text{Li}_3\left(-e^{i(\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + a*Cos[e + f*x])^2,x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(-3*d*f^2*(c + d*x)^2*Cos[(e + f*x)/2] + f^3*(c + d*x)^
3*Sin[(e + f*x)/2] + 12*d^2*Cos[(e + f*x)/2]^3*(2*d*Log[Cos[(e + f*x)/2]] +
 f*(c + d*x)*Tan[(e + f*x)/2]) - 2*Cos[(e + f*x)/2]^3*(I*f^3*(c + d*x)^3 -
6*d*(f^2*(c + d*x)^2*Log[1 + E^(I*(e + f*x))] - (2*I)*d*f*(c + d*x)*PolyLog
[2, -E^(I*(e + f*x))] + 2*d^2*PolyLog[3, -E^(I*(e + f*x))]) - f^3*(c + d*x)
^3*Tan[(e + f*x)/2])))/(3*a^2*f^4*(1 + Cos[e + f*x])^2)
```

Maple [B] time = 0.628, size = 678, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+a*cos(f*x+e))^2,x)
```

```
[Out] -4*I/f^2/a^2*c*d^2*e*x+4*d^3*polylog(3,-exp(I*(f*x+e)))/a^2/f^4+4*d^3/f^4/a
^2*ln(exp(I*(f*x+e))+1)-4/f^4/a^2*ln(exp(I*(f*x+e)))*d^3-4*I*d^3/f^3/a^2*po
```

```
lylog(2,-exp(I*(f*x+e)))*x+2*d/f^2/a^2*c^2*ln(exp(I*(f*x+e))+1)-2/f^2/a^2*l
n(exp(I*(f*x+e)))*c^2*d-2/f^4/a^2*ln(exp(I*(f*x+e)))*d^3*e^2+4/3*I/f^4/a^2*
d^3*e^3+4/f^3/a^2*ln(exp(I*(f*x+e)))*c*d^2*e-2*I/f^3/a^2*c*d^2*e^2-2*I/f/a^
2*c*d^2*x^2+2*I/f^3/a^2*d^3*e^2*x+2/3*I*(6*I*f*c*d^2*x*exp(I*(f*x+e))+3*d^3
*f^2*x^3*exp(I*(f*x+e))+3*I*f*c^2*d*exp(I*(f*x+e))+3*I*f*d^3*x^2*exp(I*(f*x
+e))+9*c*d^2*f^2*x^2*exp(I*(f*x+e))+d^3*f^2*x^3+3*I*d^3*f*x^2*exp(2*I*(f*x+
e))+6*I*c*d^2*f*x*exp(2*I*(f*x+e))+9*c^2*d*f^2*x*exp(I*(f*x+e))+3*c*d^2*f^2
*x^2+3*I*c^2*d*f*exp(2*I*(f*x+e))+3*c^3*f^2*exp(I*(f*x+e))+3*c^2*d*f^2*x+6*
d^3*x*exp(2*I*(f*x+e))+c^3*f^2+6*c*d^2*exp(2*I*(f*x+e))+12*d^3*x*exp(I*(f*x
+e))+12*c*d^2*exp(I*(f*x+e))+6*d^3*x+6*c*d^2)/f^3/a^2/(exp(I*(f*x+e))+1)^3-
4*I*d^2/f^3/a^2*c*polylog(2,-exp(I*(f*x+e)))+2*d^3/f^2/a^2*ln(exp(I*(f*x+e)
)+1)*x^2-2/3*I/f/a^2*d^3*x^3+4*d^2/f^2/a^2*ln(exp(I*(f*x+e))+1)*c*x
```

Maxima [B] time = 4.91657, size = 4420, normalized size = 16.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(12*(2*(3*(f*x + e)*sin(f*x + e) + cos(2*f*x + 2*e) + cos(f*x + e))*cos
(3*f*x + 3*e) + 2*(9*(f*x + e)*sin(f*x + e) + 6*cos(f*x + e) + 1)*cos(2*f*x
 + 2*e) + 6*cos(2*f*x + 2*e)^2 + 6*cos(f*x + e)^2 - (2*(3*cos(2*f*x + 2*e)
+ 3*cos(f*x + e) + 1)*cos(3*f*x + 3*e) + cos(3*f*x + 3*e)^2 + 6*(3*cos(f*x
+ e) + 1)*cos(2*f*x + 2*e) + 9*cos(2*f*x + 2*e)^2 + 9*cos(f*x + e)^2 + 6*(s
in(2*f*x + 2*e) + sin(f*x + e))*sin(3*f*x + 3*e) + sin(3*f*x + 3*e)^2 + 9*s
in(2*f*x + 2*e)^2 + 18*sin(2*f*x + 2*e)*sin(f*x + e) + 9*sin(f*x + e)^2 + 6
*cos(f*x + e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1
) - 2*(f*x + 3*(f*x + e)*cos(f*x + e) + e - sin(2*f*x + 2*e) - sin(f*x + e)
)*sin(3*f*x + 3*e) - 6*(f*x + 3*(f*x + e)*cos(f*x + e) + e - 2*sin(f*x + e)
)*sin(2*f*x + 2*e) + 6*sin(2*f*x + 2*e)^2 + 6*sin(f*x + e)^2 + 2*cos(f*x +
e))*c*d^2*e/(a^2*f^2*cos(3*f*x + 3*e)^2 + 9*a^2*f^2*cos(2*f*x + 2*e)^2 + 9*
a^2*f^2*cos(f*x + e)^2 + a^2*f^2*sin(3*f*x + 3*e)^2 + 9*a^2*f^2*sin(2*f*x +
 2*e)^2 + 18*a^2*f^2*sin(2*f*x + 2*e)*sin(f*x + e) + 9*a^2*f^2*sin(f*x + e)
^2 + 6*a^2*f^2*cos(f*x + e) + a^2*f^2 + 2*(3*a^2*f^2*cos(2*f*x + 2*e) + 3*a
^2*f^2*cos(f*x + e) + a^2*f^2)*cos(3*f*x + 3*e) + 6*(3*a^2*f^2*cos(f*x + e)
 + a^2*f^2)*cos(2*f*x + 2*e) + 6*(a^2*f^2*sin(2*f*x + 2*e) + a^2*f^2*sin(f*
x + e))*sin(3*f*x + 3*e)) - 6*(2*(3*(f*x + e)*sin(f*x + e) + cos(2*f*x + 2*
e) + cos(f*x + e))*cos(3*f*x + 3*e) + 2*(9*(f*x + e)*sin(f*x + e) + 6*cos(f
*x + e) + 1)*cos(2*f*x + 2*e) + 6*cos(2*f*x + 2*e)^2 + 6*cos(f*x + e)^2 - (
2*(3*cos(2*f*x + 2*e) + 3*cos(f*x + e) + 1)*cos(3*f*x + 3*e) + cos(3*f*x +
3*e)^2 + 6*(3*cos(f*x + e) + 1)*cos(2*f*x + 2*e) + 9*cos(2*f*x + 2*e)^2 + 9
*cos(f*x + e)^2 + 6*(sin(2*f*x + 2*e) + sin(f*x + e))*sin(3*f*x + 3*e) + si
n(3*f*x + 3*e)^2 + 9*sin(2*f*x + 2*e)^2 + 18*sin(2*f*x + 2*e)*sin(f*x + e)
+ 9*sin(f*x + e)^2 + 6*cos(f*x + e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^
2 + 2*cos(f*x + e) + 1) - 2*(f*x + 3*(f*x + e)*cos(f*x + e) + e - sin(2*f*x
 + 2*e) - sin(f*x + e))*sin(3*f*x + 3*e) - 6*(f*x + 3*(f*x + e)*cos(f*x + e
) + e - 2*sin(f*x + e))*sin(2*f*x + 2*e) + 6*sin(2*f*x + 2*e)^2 + 6*sin(f*x
 + e)^2 + 2*cos(f*x + e))*c^2*d/(a^2*f*cos(3*f*x + 3*e)^2 + 9*a^2*f*cos(2*f
*x + 2*e)^2 + 9*a^2*f*cos(f*x + e)^2 + a^2*f*sin(3*f*x + 3*e)^2 + 9*a^2*f*s
in(2*f*x + 2*e)^2 + 18*a^2*f*sin(2*f*x + 2*e)*sin(f*x + e) + 9*a^2*f*sin(f*
x + e)^2 + 6*a^2*f*cos(f*x + e) + a^2*f + 2*(3*a^2*f*cos(2*f*x + 2*e) + 3*a
^2*f*cos(f*x + e) + a^2*f)*cos(3*f*x + 3*e) + 6*(3*a^2*f*cos(f*x + e) + a^2
*f)*cos(2*f*x + 2*e) + 6*(a^2*f*sin(2*f*x + 2*e) + a^2*f*sin(f*x + e))*sin(
3*f*x + 3*e)) + c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(co
s(f*x + e) + 1)^3)/a^2 + 3*c*d^2*e^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + s
```

```
in(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^2*f^2) - 3*c^2*d*e*(3*sin(f*x + e)/(
cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^2*f) - 6*(2*d^3
*e^3 + 12*d^3*e - 12*c*d^2*f - (6*(f*x + e)^2*d^3 + 6*d^3*e^2 + 12*d^3 - 12
*(d^3*e - c*d^2*f)*(f*x + e) + 6*((f*x + e)^2*d^3 + d^3*e^2 + 2*d^3 - 2*(d^
3*e - c*d^2*f)*(f*x + e))*cos(3*f*x + 3*e) + 18*((f*x + e)^2*d^3 + d^3*e^2
+ 2*d^3 - 2*(d^3*e - c*d^2*f)*(f*x + e))*cos(2*f*x + 2*e) + 18*((f*x + e)^2
*d^3 + d^3*e^2 + 2*d^3 - 2*(d^3*e - c*d^2*f)*(f*x + e))*cos(f*x + e) + (6*I
*(f*x + e)^2*d^3 + 6*I*d^3*e^2 + 12*I*d^3 + (-12*I*d^3*e + 12*I*c*d^2*f)*(f
*x + e))*sin(3*f*x + 3*e) + (18*I*(f*x + e)^2*d^3 + 18*I*d^3*e^2 + 36*I*d^3
 + (-36*I*d^3*e + 36*I*c*d^2*f)*(f*x + e))*sin(2*f*x + 2*e) + (18*I*(f*x +
e)^2*d^3 + 18*I*d^3*e^2 + 36*I*d^3 + (-36*I*d^3*e + 36*I*c*d^2*f)*(f*x + e)
)*sin(f*x + e))*arctan2(sin(f*x + e), cos(f*x + e) + 1) + 2*((f*x + e)^3*d^
3 - 3*(d^3*e - c*d^2*f)*(f*x + e)^2 + 3*(d^3*e^2 + 2*d^3)*(f*x + e))*cos(3*
f*x + 3*e) + (6*(f*x + e)^3*d^3 - 6*I*d^3*e^2 + 12*d^3*e - 12*c*d^2*f - (18
*d^3*e - 18*c*d^2*f + 6*I*d^3)*(f*x + e)^2 + (18*d^3*e^2 + 12*I*d^3*e - 12*
I*c*d^2*f + 24*d^3)*(f*x + e))*cos(2*f*x + 2*e) + (6*d^3*e^3 - 6*I*(f*x + e
)^2*d^3 - 6*I*d^3*e^2 + 24*d^3*e - 24*c*d^2*f - (-12*I*d^3*e + 12*I*c*d^2*f
 - 12*d^3)*(f*x + e))*cos(f*x + e) + (12*(f*x + e)*d^3 - 12*d^3*e + 12*c*d^
2*f + 12*((f*x + e)*d^3 - d^3*e + c*d^2*f)*cos(3*f*x + 3*e) + 36*((f*x + e)
*d^3 - d^3*e + c*d^2*f)*cos(2*f*x + 2*e) + 36*((f*x + e)*d^3 - d^3*e + c*d^
2*f)*cos(f*x + e) - (-12*I*(f*x + e)*d^3 + 12*I*d^3*e - 12*I*c*d^2*f)*sin(3
*f*x + 3*e) - (-36*I*(f*x + e)*d^3 + 36*I*d^3*e - 36*I*c*d^2*f)*sin(2*f*x +
 2*e) - (-36*I*(f*x + e)*d^3 + 36*I*d^3*e - 36*I*c*d^2*f)*sin(f*x + e))*dil
og(-e^(I*f*x + I*e)) - (-3*I*(f*x + e)^2*d^3 - 3*I*d^3*e^2 - 6*I*d^3 + (6*I
*d^3*e - 6*I*c*d^2*f)*(f*x + e) + (-3*I*(f*x + e)^2*d^3 - 3*I*d^3*e^2 - 6*I
*d^3 + (6*I*d^3*e - 6*I*c*d^2*f)*(f*x + e))*cos(3*f*x + 3*e) + (-9*I*(f*x +
 e)^2*d^3 - 9*I*d^3*e^2 - 18*I*d^3 + (18*I*d^3*e - 18*I*c*d^2*f)*(f*x + e))
*cos(2*f*x + 2*e) + (-9*I*(f*x + e)^2*d^3 - 9*I*d^3*e^2 - 18*I*d^3 + (18*I*
d^3*e - 18*I*c*d^2*f)*(f*x + e))*cos(f*x + e) + 3*((f*x + e)^2*d^3 + d^3*e^
2 + 2*d^3 - 2*(d^3*e - c*d^2*f)*(f*x + e))*sin(3*f*x + 3*e) + 9*((f*x + e)^
2*d^3 + d^3*e^2 + 2*d^3 - 2*(d^3*e - c*d^2*f)*(f*x + e))*sin(2*f*x + 2*e) +
 9*((f*x + e)^2*d^3 + d^3*e^2 + 2*d^3 - 2*(d^3*e - c*d^2*f)*(f*x + e))*sin(
f*x + e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) - (-12*
I*d^3*cos(3*f*x + 3*e) - 36*I*d^3*cos(2*f*x + 2*e) - 36*I*d^3*cos(f*x + e)
+ 12*d^3*sin(3*f*x + 3*e) + 36*d^3*sin(2*f*x + 2*e) + 36*d^3*sin(f*x + e) -
 12*I*d^3)*polylog(3, -e^(I*f*x + I*e)) - (-2*I*(f*x + e)^3*d^3 + (6*I*d^3*
e - 6*I*c*d^2*f)*(f*x + e)^2 + (-6*I*d^3*e^2 - 12*I*d^3)*(f*x + e))*sin(3*f
*x + 3*e) - (-6*I*(f*x + e)^3*d^3 - 6*d^3*e^2 - 12*I*d^3*e + 12*I*c*d^2*f +
 (18*I*d^3*e - 18*I*c*d^2*f - 6*d^3)*(f*x + e)^2 + (-18*I*d^3*e^2 + 12*d^3*
e - 12*c*d^2*f - 24*I*d^3)*(f*x + e))*sin(2*f*x + 2*e) - (-6*I*d^3*e^3 - 6*
(f*x + e)^2*d^3 - 6*d^3*e^2 - 24*I*d^3*e + 24*I*c*d^2*f + (12*d^3*e - 12*c*
d^2*f - 12*I*d^3)*(f*x + e))*sin(f*x + e))/(-3*I*a^2*f^3*cos(3*f*x + 3*e) -
 9*I*a^2*f^3*cos(2*f*x + 2*e) - 9*I*a^2*f^3*cos(f*x + e) + 3*a^2*f^3*sin(3*
f*x + 3*e) + 9*a^2*f^3*sin(2*f*x + 2*e) + 9*a^2*f^3*sin(f*x + e) - 3*I*a^2*
f^3))/f
```

Fricas [C] time = 1.91105, size = 1796, normalized size = 6.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*d^3*f^2*x^2 + 6*c*d^2*f^2*x + 3*c^2*d*f^2 + 3*(d^3*f^2*x^2 + 2*c*d^
2*f^2*x + c^2*d*f^2)*cos(f*x + e) - (6*I*d^3*f*x + 6*I*c*d^2*f + (6*I*d^3*f
*x + 6*I*c*d^2*f)*cos(f*x + e)^2 + (12*I*d^3*f*x + 12*I*c*d^2*f)*cos(f*x +
```

```
e))*dilog(-cos(f*x + e) + I*sin(f*x + e)) - (-6*I*d^3*f*x - 6*I*c*d^2*f + (
-6*I*d^3*f*x - 6*I*c*d^2*f)*cos(f*x + e)^2 + (-12*I*d^3*f*x - 12*I*c*d^2*f)
*cos(f*x + e))*dilog(-cos(f*x + e) - I*sin(f*x + e)) - 3*(d^3*f^2*x^2 + 2*c
*d^2*f^2*x + c^2*d*f^2 + 2*d^3 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 +
 2*d^3)*cos(f*x + e)^2 + 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + 2*d^3
)*cos(f*x + e))*log(cos(f*x + e) + I*sin(f*x + e) + 1) - 3*(d^3*f^2*x^2 + 2
*c*d^2*f^2*x + c^2*d*f^2 + 2*d^3 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2
 + 2*d^3)*cos(f*x + e)^2 + 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + 2*d
^3)*cos(f*x + e))*log(cos(f*x + e) - I*sin(f*x + e) + 1) - 6*(d^3*cos(f*x +
 e)^2 + 2*d^3*cos(f*x + e) + d^3)*polylog(3, -cos(f*x + e) + I*sin(f*x + e)
) - 6*(d^3*cos(f*x + e)^2 + 2*d^3*cos(f*x + e) + d^3)*polylog(3, -cos(f*x +
 e) - I*sin(f*x + e)) - (2*d^3*f^3*x^3 + 6*c*d^2*f^3*x^2 + 2*c^3*f^3 + 6*c*
d^2*f + 6*(c^2*d*f^3 + d^3*f)*x + (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + c^3*f^3
+ 6*c*d^2*f + 3*(c^2*d*f^3 + 2*d^3*f)*x)*cos(f*x + e))*sin(f*x + e))/(a^2*f
^4*cos(f*x + e)^2 + 2*a^2*f^4*cos(f*x + e) + a^2*f^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c^3}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx + \int \frac{d^3x^3}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx + \int \frac{3cd^2x^2}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx + \int \frac{3c^2dx}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+a*cos(f*x+e))**2,x)
```

```
[Out] (Integral(c**3/(cos(e + f*x)**2 + 2*cos(e + f*x) + 1), x) + Integral(d**3*x
**3/(cos(e + f*x)**2 + 2*cos(e + f*x) + 1), x) + Integral(3*c*d**2*x**2/(co
s(e + f*x)**2 + 2*cos(e + f*x) + 1), x) + Integral(3*c**2*d*x/(cos(e + f*x)
**2 + 2*cos(e + f*x) + 1), x))/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^3}{(a\cos(fx+e)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(a*cos(f*x + e) + a)^2, x)
```

3.134 $\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx$

Optimal. Leaf size=212

$$\frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{3a^2f^2}-\frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2}+\frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}+\frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}$$

```
[Out] ((-I/3)*(c + d*x)^2)/(a^2*f) + (4*d*(c + d*x)*Log[1 + E^(I*(e + f*x))])/(3*
a^2*f^2) - (((4*I)/3)*d^2*PolyLog[2, -E^(I*(e + f*x))])/(a^2*f^3) - (d*(c +
 d*x)*Sec[e/2 + (f*x)/2]^2)/(3*a^2*f^2) + (2*d^2*Tan[e/2 + (f*x)/2])/(3*a^2
*f^3) + ((c + d*x)^2*Tan[e/2 + (f*x)/2])/(3*a^2*f) + ((c + d*x)^2*Sec[e/2 +
 (f*x)/2]^2*Tan[e/2 + (f*x)/2])/(6*a^2*f)
```

Rubi [A] time = 0.255528, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {3318, 4186, 3767, 8, 4184, 3719, 2190, 2279, 2391}

$$\frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{3a^2f^2}-\frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2}+\frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}+\frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + a*Cos[e + f*x])^2,x]
```


```
[Out] ((-I/3)*(c + d*x)^2)/(a^2*f) + (4*d*(c + d*x)*Log[1 + E^(I*(e + f*x))])/(3*
a^2*f^2) - (((4*I)/3)*d^2*PolyLog[2, -E^(I*(e + f*x))])/(a^2*f^3) - (d*(c +
 d*x)*Sec[e/2 + (f*x)/2]^2)/(3*a^2*f^2) + (2*d^2*Tan[e/2 + (f*x)/2])/(3*a^2
*f^3) + ((c + d*x)^2*Tan[e/2 + (f*x)/2])/(3*a^2*f) + ((c + d*x)^2*Sec[e/2 +
 (f*x)/2]^2*Tan[e/2 + (f*x)/2])/(6*a^2*f)
```


Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx = \frac{\int (c+dx)^2 \csc^4\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)dx}{4a^2}$$

$$= -\frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} + \frac{\int (c+dx)^2\sec^2\left(\frac{e}{2}+\right.}{6a^2}$$

$$= -\frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\right.}{6a^2f}$$

$$= -\frac{i(c+dx)^2}{3a^2f} - \frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{2d^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^3} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + -$$

$$= -\frac{i(c+dx)^2}{3a^2f} + \frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{3a^2f^2} - \frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{2d^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^3}$$

$$= -\frac{i(c+dx)^2}{3a^2f} + \frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{3a^2f^2} - \frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{2d^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^3}$$

$$= -\frac{i(c+dx)^2}{3a^2f} + \frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{3a^2f^2} - \frac{4id^2\text{Li}_2\left(-e^{i(e+fx)}\right)}{3a^2f^3} - \frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2}$$

Mathematica [A] time = 1.06372, size = 212, normalized size = 1.

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)\left(\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2+2\right)\right)\cos(e+fx)+2\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2+1\right)\right)\right.\right.}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + a*Cos[e + f*x])^2,x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(-2*d*f*(c + d*x)*Cos[(e + f*x)/2] - (2*I)*f*(c + d*x)*
Cos[(e + f*x)/2]^3*(f*(c + d*x) + (4*I)*d*Log[1 + E^(I*(e + f*x))]) - (8*I)
*d^2*Cos[(e + f*x)/2]^3*PolyLog[2, -E^(I*(e + f*x))] + (2*(c^2*f^2 + 2*c*d*
f^2*x + d^2*(1 + f^2*x^2)) + (c^2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Co
s[e + f*x])*Sin[(e + f*x)/2]))/(3*a^2*f^3*(1 + Cos[e + f*x])^2)
```

Maple [B] time = 0.349, size = 358, normalized size = 1.7

$$\frac{\frac{2i}{3}\left(2\,id^2fx\mathrm{e}^{2\,i(fx+e)}+3\,d^2f^2x^2\mathrm{e}^{i(fx+e)}+2\,icdf\mathrm{e}^{2\,i(fx+e)}+2\,ifd^2x\mathrm{e}^{i(fx+e)}+6\,cdf^2x\mathrm{e}^{i(fx+e)}+f^2x^2d^2+2\,ifcd\mathrm{e}^{i(fx+e)}+\right.}{a^2f^3\left(\mathrm{e}^{i(fx+e)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+a*cos(f*x+e))^2,x)
```

```
[Out] 2/3*I*(2*I*d^2*f*x*exp(2*I*(f*x+e))+3*d^2*f^2*x^2*exp(I*(f*x+e))+2*I*c*d*f*
exp(2*I*(f*x+e))+2*I*f*d^2*x*exp(I*(f*x+e))+6*c*d*f^2*x*exp(I*(f*x+e))+f^2*
x^2*d^2+2*I*f*c*d*exp(I*(f*x+e))+3*c^2*f^2*exp(I*(f*x+e))+2*c*d*f^2*x+c^2*f
^2+2*d^2*exp(2*I*(f*x+e))+4*d^2*exp(I*(f*x+e))+2*d^2)/f^3/a^2/(exp(I*(f*x+e
```

```
))+1)^3+4/3/f^2*d/a^2*c*ln(exp(I*(f*x+e))+1)-4/3/f^2*d/a^2*c*ln(exp(I*(f*x+
e)))-2/3*I/f*d^2/a^2*x^2-4/3*I/f^2*d^2/a^2*e*x-2/3*I/f^3*d^2/a^2*e^2+4/3/f^
2*d^2/a^2*ln(exp(I*(f*x+e))+1)*x-4/3*I*d^2*polylog(2,-exp(I*(f*x+e)))/a^2/f
^3+4/3/f^3*d^2/a^2*e*ln(exp(I*(f*x+e)))
```

Maxima [B] time = 2.65131, size = 1041, normalized size = 4.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (2*c^2*f^2 + 4*d^2 + (4*d^2*f*x + 4*c*d*f + 4*(d^2*f*x + c*d*f)*cos(3*f*x +
 3*e) + 12*(d^2*f*x + c*d*f)*cos(2*f*x + 2*e) + 12*(d^2*f*x + c*d*f)*cos(f*
x + e) + (4*I*d^2*f*x + 4*I*c*d*f)*sin(3*f*x + 3*e) + (12*I*d^2*f*x + 12*I*
c*d*f)*sin(2*f*x + 2*e) + (12*I*d^2*f*x + 12*I*c*d*f)*sin(f*x + e))*arctan2
(sin(f*x + e), cos(f*x + e) + 1) - 2*(d^2*f^2*x^2 + 2*c*d*f^2*x)*cos(3*f*x
+ 3*e) - (6*d^2*f^2*x^2 - 4*I*c*d*f - 4*d^2 + (12*c*d*f^2 - 4*I*d^2*f)*x)*c
os(2*f*x + 2*e) + (6*c^2*f^2 + 4*I*d^2*f*x + 4*I*c*d*f + 8*d^2)*cos(f*x + e
) - (4*d^2*cos(3*f*x + 3*e) + 12*d^2*cos(2*f*x + 2*e) + 12*d^2*cos(f*x + e)
 + 4*I*d^2*sin(3*f*x + 3*e) + 12*I*d^2*sin(2*f*x + 2*e) + 12*I*d^2*sin(f*x
+ e) + 4*d^2)*dilog(-e^(I*f*x + I*e)) + (-2*I*d^2*f*x - 2*I*c*d*f + (-2*I*d
^2*f*x - 2*I*c*d*f)*cos(3*f*x + 3*e) + (-6*I*d^2*f*x - 6*I*c*d*f)*cos(2*f*x
 + 2*e) + (-6*I*d^2*f*x - 6*I*c*d*f)*cos(f*x + e) + 2*(d^2*f*x + c*d*f)*sin
(3*f*x + 3*e) + 6*(d^2*f*x + c*d*f)*sin(2*f*x + 2*e) + 6*(d^2*f*x + c*d*f)*
sin(f*x + e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + (
-2*I*d^2*f^2*x^2 - 4*I*c*d*f^2*x)*sin(3*f*x + 3*e) + (-6*I*d^2*f^2*x^2 - 4*
c*d*f + 4*I*d^2 - 4*(3*I*c*d*f^2 + d^2*f)*x)*sin(2*f*x + 2*e) + (6*I*c^2*f^
2 - 4*d^2*f*x - 4*c*d*f + 8*I*d^2)*sin(f*x + e))/(-3*I*a^2*f^3*cos(3*f*x +
3*e) - 9*I*a^2*f^3*cos(2*f*x + 2*e) - 9*I*a^2*f^3*cos(f*x + e) + 3*a^2*f^3*
sin(3*f*x + 3*e) + 9*a^2*f^3*sin(2*f*x + 2*e) + 9*a^2*f^3*sin(f*x + e) - 3*
I*a^2*f^3)
```

Fricas [B] time = 1.75407, size = 956, normalized size = 4.51

$$-\frac{2 d^2 f x + 2 c d f + 2 \left(d^2 f x + c d f\right) \cos\left(f x + e\right) - \left(2 i d^2 \cos\left(f x + e\right)^2 + 4 i d^2 \cos\left(f x + e\right) + 2 i d^2\right) \mathrm{Li}_2\left(-\cos\left(f x + e\right) + i \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*d^2*f*x + 2*c*d*f + 2*(d^2*f*x + c*d*f)*cos(f*x + e) - (2*I*d^2*cos
(f*x + e)^2 + 4*I*d^2*cos(f*x + e) + 2*I*d^2)*dilog(-cos(f*x + e) + I*sin(f
*x + e)) - (-2*I*d^2*cos(f*x + e)^2 - 4*I*d^2*cos(f*x + e) - 2*I*d^2)*dilog
(-cos(f*x + e) - I*sin(f*x + e)) - 2*(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*c
os(f*x + e)^2 + 2*(d^2*f*x + c*d*f)*cos(f*x + e))*log(cos(f*x + e) + I*sin(
f*x + e) + 1) - 2*(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(f*x + e)^2 + 2*(
d^2*f*x + c*d*f)*cos(f*x + e))*log(cos(f*x + e) - I*sin(f*x + e) + 1) - (2*
d^2*f^2*x^2 + 4*c*d*f^2*x + 2*c^2*f^2 + 2*d^2 + (d^2*f^2*x^2 + 2*c*d*f^2*x
+ c^2*f^2 + 2*d^2)*cos(f*x + e))*sin(f*x + e))/(a^2*f^3*cos(f*x + e)^2 + 2*
a^2*f^3*cos(f*x + e) + a^2*f^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c^2}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx+\int \frac{d^2x^2}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx+\int \frac{2cdx}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+a*cos(f*x+e))**2,x)
```

```
[Out] (Integral(c**2/(cos(e + f*x)**2 + 2*cos(e + f*x) + 1), x) + Integral(d**2*x
**2/(cos(e + f*x)**2 + 2*cos(e + f*x) + 1), x) + Integral(2*c*d*x/(cos(e +
f*x)**2 + 2*cos(e + f*x) + 1), x))/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^2}{\left(a\cos\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(a*cos(f*x + e) + a)^2, x)
```

3.135 $\int \frac{c+dx}{(a+a\cos(e+fx))^2}\,dx$

Optimal. Leaf size=123

$$\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}+\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}-\frac{d\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2}+\frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3a^2f^2}$$

```
[Out] (2*d*Log[Cos[e/2 + (f*x)/2]])/(3*a^2*f^2) - (d*Sec[e/2 + (f*x)/2]^2)/(6*a^2
*f^2) + ((c + d*x)*Tan[e/2 + (f*x)/2])/(3*a^2*f) + ((c + d*x)*Sec[e/2 + (f*
x)/2]^2*Tan[e/2 + (f*x)/2])/(6*a^2*f)
```

Rubi [A] time = 0.0946936, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3318, 4185, 4184, 3475}

$$\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}+\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}-\frac{d\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2}+\frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3a^2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + a*Cos[e + f*x])^2,x]
```

```
[Out] (2*d*Log[Cos[e/2 + (f*x)/2]])/(3*a^2*f^2) - (d*Sec[e/2 + (f*x)/2]^2)/(6*a^2
*f^2) + ((c + d*x)*Tan[e/2 + (f*x)/2])/(3*a^2*f) + ((c + d*x)*Sec[e/2 + (f*
x)/2]^2*Tan[e/2 + (f*x)/2])/(6*a^2*f)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{c+dx}{(a+a\cos(e+fx))^2}\,dx = \frac{\int (c+dx)\csc^4\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)dx}{4a^2}$$
$$= -\frac{d\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2} + \frac{(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} + \frac{\int (c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{6a^2}$$
$$= -\frac{d\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2} + \frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} - d\int t$$
$$= \frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3a^2f^2} - \frac{d\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2} + \frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)\sec^2\left(\frac{e}{2}+\frac{f}{2}\right.}{6a^2f}$$

Mathematica [A] time = 0.507986, size = 113, normalized size = 0.92

$$\frac{\cos\left(\frac{1}{2}(e+fx)\right)\left(f(c+dx)\left(3\sin\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{3}{2}(e+fx)\right)\right)+2d\cos\left(\frac{3}{2}(e+fx)\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+2d\cos\left(\right.\right.}{3a^2f^2(\cos(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + a*Cos[e + f*x])^2,x]
```

```
[Out] (Cos[(e + f*x)/2]*(2*d*Cos[(3*(e + f*x))/2]*Log[Cos[(e + f*x)/2]] + 2*d*Cos
[(e + f*x)/2]*(-1 + 3*Log[Cos[(e + f*x)/2]]) + f*(c + d*x)*(3*Sin[(e + f*x)
/2] + Sin[(3*(e + f*x))/2])))/(3*a^2*f^2*(1 + Cos[e + f*x])^2)
```

Maple [A] time = 0.11, size = 123, normalized size = 1.

$$\frac{c}{6a^2f}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3 + \frac{c}{2a^2f}\tan\left(\frac{fx}{2}+\frac{e}{2}\right) - \frac{d}{6a^2f^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2 + \frac{dx}{2a^2f}\tan\left(\frac{fx}{2}+\frac{e}{2}\right) + \frac{dx}{6a^2f}\left(\tan\left(\frac{fx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+a*cos(f*x+e))^2,x)
```

```
[Out] 1/6/a^2*c/f*tan(1/2*f*x+1/2*e)^3+1/2/a^2*c/f*tan(1/2*f*x+1/2*e)-1/6/a^2*d/f
^2*tan(1/2*f*x+1/2*e)^2+1/2/a^2/f*x*d*tan(1/2*f*x+1/2*e)+1/6/a^2/f*x*d*tan(
1/2*f*x+1/2*e)^3-1/3/a^2*d/f^2*ln(1+tan(1/2*f*x+1/2*e)^2)
```

Maxima [B] time = 1.2705, size = 1030, normalized size = 8.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(2*(3*(f*x + e)*sin(f*x + e) + cos(2*f*x + 2*e) + cos(f*x + e))*cos
(3*f*x + 3*e) + 2*(9*(f*x + e)*sin(f*x + e) + 6*cos(f*x + e) + 1)*cos(2*f*x
 + 2*e) + 6*cos(2*f*x + 2*e)^2 + 6*cos(f*x + e)^2 - (2*(3*cos(2*f*x + 2*e)
```

```
+ 3*cos(f*x + e) + 1)*cos(3*f*x + 3*e) + cos(3*f*x + 3*e)^2 + 6*(3*cos(f*x
+ e) + 1)*cos(2*f*x + 2*e) + 9*cos(2*f*x + 2*e)^2 + 9*cos(f*x + e)^2 + 6*(s
in(2*f*x + 2*e) + sin(f*x + e))*sin(3*f*x + 3*e) + sin(3*f*x + 3*e)^2 + 9*s
in(2*f*x + 2*e)^2 + 18*sin(2*f*x + 2*e)*sin(f*x + e) + 9*sin(f*x + e)^2 + 6
*cos(f*x + e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1
) - 2*(f*x + 3*(f*x + e)*cos(f*x + e) + e - sin(2*f*x + 2*e) - sin(f*x + e)
)*sin(3*f*x + 3*e) - 6*(f*x + 3*(f*x + e)*cos(f*x + e) + e - 2*sin(f*x + e)
)*sin(2*f*x + 2*e) + 6*sin(2*f*x + 2*e)^2 + 6*sin(f*x + e)^2 + 2*cos(f*x +
e))*d/(a^2*f*cos(3*f*x + 3*e)^2 + 9*a^2*f*cos(2*f*x + 2*e)^2 + 9*a^2*f*cos(
f*x + e)^2 + a^2*f*sin(3*f*x + 3*e)^2 + 9*a^2*f*sin(2*f*x + 2*e)^2 + 18*a^2
*f*sin(2*f*x + 2*e)*sin(f*x + e) + 9*a^2*f*sin(f*x + e)^2 + 6*a^2*f*cos(f*x
 + e) + a^2*f + 2*(3*a^2*f*cos(2*f*x + 2*e) + 3*a^2*f*cos(f*x + e) + a^2*f)
*cos(3*f*x + 3*e) + 6*(3*a^2*f*cos(f*x + e) + a^2*f)*cos(2*f*x + 2*e) + 6*(
a^2*f*sin(2*f*x + 2*e) + a^2*f*sin(f*x + e))*sin(3*f*x + 3*e)) - c*(3*sin(f
*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 + d*e
*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/
(a^2*f))/f
```

Fricas [A] time = 1.67137, size = 298, normalized size = 2.42

$$-\frac{d\cos\left(fx+e\right)-\left(d\cos\left(fx+e\right)^{2}+2\,d\cos\left(fx+e\right)+d\right)\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)-\left(2\,dfx+2\,cf+\left(dfx+cf\right)\cos\left(fx\right.\right.}{3\left(a^{2}f^{2}\cos\left(fx+e\right)^{2}+2\,a^{2}f^{2}\cos\left(fx+e\right)+a^{2}f^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(d*cos(f*x + e) - (d*cos(f*x + e)^2 + 2*d*cos(f*x + e) + d)*log(1/2*co
s(f*x + e) + 1/2) - (2*d*f*x + 2*c*f + (d*f*x + c*f)*cos(f*x + e))*sin(f*x
+ e) + d)/(a^2*f^2*cos(f*x + e)^2 + 2*a^2*f^2*cos(f*x + e) + a^2*f^2)
```

Sympy [A] time = 1.43222, size = 146, normalized size = 1.19

$$\begin{cases} \frac{c\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^{2}f}+\frac{c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^{2}f}+\frac{dx\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^{2}f}+\frac{dx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^{2}f}-\frac{d\log\left(\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{3a^{2}f^{2}}-\frac{d\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^{2}f^{2}} & \text{for}\ f\neq 0\\ \frac{cx+\frac{dx^{2}}{2}}{\left(a\cos\left(e\right)+a\right)^{2}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e))**2,x)
```

```
[Out] Piecewise((c*tan(e/2 + f*x/2)**3/(6*a**2*f) + c*tan(e/2 + f*x/2)/(2*a**2*f)
 + d*x*tan(e/2 + f*x/2)**3/(6*a**2*f) + d*x*tan(e/2 + f*x/2)/(2*a**2*f) - d
*log(tan(e/2 + f*x/2)**2 + 1)/(3*a**2*f**2) - d*tan(e/2 + f*x/2)**2/(6*a**2
*f**2), Ne(f, 0)), ((c*x + d*x**2/2)/(a*cos(e) + a)**2, True))
```

Giac [B] time = 1.50304, size = 1022, normalized size = 8.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*d*f*x*tan(1/2*f*x)^3*tan(1/2*e)^2 + 3*d*f*x*tan(1/2*f*x)^2*tan(1/2*
e)^3 - 2*d*log(4*(tan(1/2*e)^2 + 1)/(tan(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/
2*f*x)^3*tan(1/2*e) + tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1/2*f*x)^2 - 2*tan(
1/2*f*x)*tan(1/2*e) + 1))*tan(1/2*f*x)^3*tan(1/2*e)^3 + 3*c*f*tan(1/2*f*x)^
3*tan(1/2*e)^2 + 3*c*f*tan(1/2*f*x)^2*tan(1/2*e)^3 + d*tan(1/2*f*x)^3*tan(1
/2*e)^3 + d*f*x*tan(1/2*f*x)^3 - 3*d*f*x*tan(1/2*f*x)^2*tan(1/2*e) - 3*d*f*
x*tan(1/2*f*x)*tan(1/2*e)^2 + 6*d*log(4*(tan(1/2*e)^2 + 1)/(tan(1/2*f*x)^4*
tan(1/2*e)^2 - 2*tan(1/2*f*x)^3*tan(1/2*e) + tan(1/2*f*x)^2*tan(1/2*e)^2 +
tan(1/2*f*x)^2 - 2*tan(1/2*f*x)*tan(1/2*e) + 1))*tan(1/2*f*x)^2*tan(1/2*e)^
2 + d*f*x*tan(1/2*e)^3 + c*f*tan(1/2*f*x)^3 - 3*c*f*tan(1/2*f*x)^2*tan(1/2*
e) + d*tan(1/2*f*x)^3*tan(1/2*e) - 3*c*f*tan(1/2*f*x)*tan(1/2*e)^2 - d*tan(
1/2*f*x)^2*tan(1/2*e)^2 + c*f*tan(1/2*e)^3 + d*tan(1/2*f*x)*tan(1/2*e)^3 +
3*d*f*x*tan(1/2*f*x) + 3*d*f*x*tan(1/2*e) - 6*d*log(4*(tan(1/2*e)^2 + 1)/(t
an(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^3*tan(1/2*e) + tan(1/2*f*x)^2*t
an(1/2*e)^2 + tan(1/2*f*x)^2 - 2*tan(1/2*f*x)*tan(1/2*e) + 1))*tan(1/2*f*x)
*tan(1/2*e) + 3*c*f*tan(1/2*f*x) - d*tan(1/2*f*x)^2 + 3*c*f*tan(1/2*e) + d*
tan(1/2*f*x)*tan(1/2*e) - d*tan(1/2*e)^2 + 2*d*log(4*(tan(1/2*e)^2 + 1)/(ta
n(1/2*f*x)^4*tan(1/2*e)^2 - 2*tan(1/2*f*x)^3*tan(1/2*e) + tan(1/2*f*x)^2*ta
n(1/2*e)^2 + tan(1/2*f*x)^2 - 2*tan(1/2*f*x)*tan(1/2*e) + 1)) - d)/(a^2*f^2
*tan(1/2*f*x)^3*tan(1/2*e)^3 - 3*a^2*f^2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 3*a^
2*f^2*tan(1/2*f*x)*tan(1/2*e) - a^2*f^2)
```

3.136 $\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)(a\cos(e+fx)+a)^2},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)*(a + a*Cos[e + f*x])^2), x]
```

Rubi [A] time = 0.0541021, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + a*Cos[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a + a*Cos[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx=\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx$$

Mathematica [A] time = 12.1814, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + a*Cos[e + f*x])^2),x]
```

```
[Out] Integrate[1/((c + d*x)*(a + a*Cos[e + f*x])^2), x]
```

Maple [A] time = 2.144, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(a+a\cos\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+a*cos(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)/(a+a*cos(f*x+e))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(6*(d^2*f*x + c*d*f)*cos(2*f*x + 2*e)^2 + 6*(d^2*f*x + c*d*f)*cos(f*x +
 e)^2 + 6*(d^2*f*x + c*d*f)*sin(2*f*x + 2*e)^2 + 4*d^2*sin(f*x + e) + 6*(d^
2*f*x + c*d*f)*sin(f*x + e)^2 - 2*(2*d^2*sin(2*f*x + 2*e) - (d^2*f*x + c*d*
f)*cos(2*f*x + 2*e) - (d^2*f*x + c*d*f)*cos(f*x + e) + (3*d^2*f^2*x^2 + 6*c
*d*f^2*x + 3*c^2*f^2 + 4*d^2)*sin(f*x + e))*cos(3*f*x + 3*e) + 2*(d^2*f*x +
 c*d*f + 6*(d^2*f*x + c*d*f)*cos(f*x + e) - 3*(3*d^2*f^2*x^2 + 6*c*d*f^2*x
+ 3*c^2*f^2 + 2*d^2)*sin(f*x + e))*cos(2*f*x + 2*e) + 2*(d^2*f*x + c*d*f)*c
os(f*x + e) + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x
+ a^2*c^3*f^3 + (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x
+ a^2*c^3*f^3)*cos(3*f*x + 3*e)^2 + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^
2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(2*f*x + 2*e)^2 + 9*(a^2*d^3*f^3*x^
3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f*x + e)^2 +
 (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*
sin(3*f*x + 3*e)^2 + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d
*f^3*x + a^2*c^3*f^3)*sin(2*f*x + 2*e)^2 + 18*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^
2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(2*f*x + 2*e)*sin(f*x + e)
+ 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^
3)*sin(f*x + e)^2 + 2*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*
f^3*x + a^2*c^3*f^3 + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*
d*f^3*x + a^2*c^3*f^3)*cos(2*f*x + 2*e) + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*
f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f*x + e))*cos(3*f*x + 3*e) +
 6*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3
 + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f
^3)*cos(f*x + e))*cos(2*f*x + 2*e) + 6*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x
^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f*x + e) + 6*((a^2*d^3*f^3*x^3 +
3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(2*f*x + 2*e) + (
a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*si
n(f*x + e))*sin(3*f*x + 3*e))*integrate(2/3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x +
c^2*d*f^2 + 6*d^3)*sin(f*x + e)/(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*
a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + (a^2*d^4*f^3*x^4 +
4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f
^3)*cos(f*x + e)^2 + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2
*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*x + e)^2 + 2*(a^2*d^4*f^3
*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^
2*c^4*f^3)*cos(f*x + e)), x) + 2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 2*d
^2*cos(2*f*x + 2*e) + 2*d^2 + (3*d^2*f^2*x^2 + 6*c*d*f^2*x + 3*c^2*f^2 + 4*
d^2)*cos(f*x + e) + (d^2*f*x + c*d*f)*sin(2*f*x + 2*e) + (d^2*f*x + c*d*f)*
sin(f*x + e))*sin(3*f*x + 3*e) + 2*(3*d^2*f^2*x^2 + 6*c*d*f^2*x + 3*c^2*f^2
 + 4*d^2 + 3*(3*d^2*f^2*x^2 + 6*c*d*f^2*x + 3*c^2*f^2 + 2*d^2)*cos(f*x + e)
 + 6*(d^2*f*x + c*d*f)*sin(f*x + e))*sin(2*f*x + 2*e))/(a^2*d^3*f^3*x^3 + 3
*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 + (a^2*d^3*f^3*x^3 + 3
*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(3*f*x + 3*e)^2 +
9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)
*cos(2*f*x + 2*e)^2 + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*
d*f^3*x + a^2*c^3*f^3)*cos(f*x + e)^2 + (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*
x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(3*f*x + 3*e)^2 + 9*(a^2*d^3*f^3*
x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(2*f*x + 2*
e)^2 + 18*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*
c^3*f^3)*sin(2*f*x + 2*e)*sin(f*x + e) + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f
^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(f*x + e)^2 + 2*(a^2*d^3*f^3*x
```

```
^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 + 3*(a^2*d^3*f^3
*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(2*f*x + 2
*e) + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^
3*f^3)*cos(f*x + e))*cos(3*f*x + 3*e) + 6*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^
3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*
f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f*x + e))*cos(2*f*x + 2*e) +
 6*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3
)*cos(f*x + e) + 6*((a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^
3*x + a^2*c^3*f^3)*sin(2*f*x + 2*e) + (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^
2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(f*x + e))*sin(3*f*x + 3*e))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^2dx + a^2c + \left(a^2dx + a^2c\right)\cos\left(fx + e\right)^2 + 2\left(a^2dx + a^2c\right)\cos\left(fx + e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d*x + a^2*c + (a^2*d*x + a^2*c)*cos(f*x + e)^2 + 2*(a^2*d*x
 + a^2*c)*cos(f*x + e)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{c\cos^2\left(e+fx\right)+2c\cos\left(e+fx\right)+c+dx\cos^2\left(e+fx\right)+2dx\cos\left(e+fx\right)+dx}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e))**2,x)
```

```
[Out] Integral(1/(c*cos(e + f*x)**2 + 2*c*cos(e + f*x) + c + d*x*cos(e + f*x)**2
+ 2*d*x*cos(e + f*x) + d*x), x)/a**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx + c)\left(a\cos\left(fx + e\right) + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(a*cos(f*x + e) + a)^2), x)
```

3.137 $\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)^2(a\cos(e+fx)+a)^2},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)^2*(a + a*Cos[e + f*x])^2), x]
```

Rubi [A] time = 0.0521773, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + a*Cos[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + a*Cos[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx=\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx$$

Mathematica [A] time = 13.2323, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + a*Cos[e + f*x])^2),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + a*Cos[e + f*x])^2), x]
```

Maple [A] time = 2.551, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2\left(a+a\cos\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+a*cos(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)^2/(a+a*cos(f*x+e))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(12*(d^2*f*x + c*d*f)*cos(2*f*x + 2*e)^2 + 12*(d^2*f*x + c*d*f)*cos(f*x
 + e)^2 + 12*(d^2*f*x + c*d*f)*sin(2*f*x + 2*e)^2 + 12*d^2*sin(f*x + e) + 1
2*(d^2*f*x + c*d*f)*sin(f*x + e)^2 - 2*(6*d^2*sin(2*f*x + 2*e) - 2*(d^2*f*x
 + c*d*f)*cos(2*f*x + 2*e) - 2*(d^2*f*x + c*d*f)*cos(f*x + e) + 3*(d^2*f^2*
x^2 + 2*c*d*f^2*x + c^2*f^2 + 4*d^2)*sin(f*x + e))*cos(3*f*x + 3*e) + 2*(2*
d^2*f*x + 2*c*d*f + 12*(d^2*f*x + c*d*f)*cos(f*x + e) - 9*(d^2*f^2*x^2 + 2*
c*d*f^2*x + c^2*f^2 + 2*d^2)*sin(f*x + e))*cos(2*f*x + 2*e) + 4*(d^2*f*x +
c*d*f)*cos(f*x + e) + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*
d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + (a^2*d^4*f^3*x^4 + 4*a^2*c*
d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(
3*f*x + 3*e)^2 + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f
^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(2*f*x + 2*e)^2 + 9*(a^2*d^4*f
^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x +
a^2*c^4*f^3)*cos(f*x + e)^2 + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^
2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(3*f*x + 3*e)^2 + 9
*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3
*d*f^3*x + a^2*c^4*f^3)*sin(2*f*x + 2*e)^2 + 18*(a^2*d^4*f^3*x^4 + 4*a^2*c*
d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(
2*f*x + 2*e)*sin(f*x + e) + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^
2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*x + e)^2 + 2*(a^
2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f
^3*x + a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d
^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(2*f*x + 2*e) + 3*(a^2*d^4
*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x
+ a^2*c^4*f^3)*cos(f*x + e))*cos(3*f*x + 3*e) + 6*(a^2*d^4*f^3*x^4 + 4*a^2*
c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + 3
*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3
*d*f^3*x + a^2*c^4*f^3)*cos(f*x + e))*cos(2*f*x + 2*e) + 6*(a^2*d^4*f^3*x^4
 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^
4*f^3)*cos(f*x + e) + 6*((a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2
*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(2*f*x + 2*e) + (a^2*d^4
*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x
+ a^2*c^4*f^3)*sin(f*x + e))*sin(3*f*x + 3*e))*integrate(4/3*(d^3*f^2*x^2 +
 2*c*d^2*f^2*x + c^2*d*f^2 + 12*d^3)*sin(f*x + e)/(a^2*d^5*f^3*x^5 + 5*a^2*
c*d^4*f^3*x^4 + 10*a^2*c^2*d^3*f^3*x^3 + 10*a^2*c^3*d^2*f^3*x^2 + 5*a^2*c^4
*d*f^3*x + a^2*c^5*f^3 + (a^2*d^5*f^3*x^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*c^
2*d^3*f^3*x^3 + 10*a^2*c^3*d^2*f^3*x^2 + 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3)*c
os(f*x + e)^2 + (a^2*d^5*f^3*x^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*c^2*d^3*f^3
*x^3 + 10*a^2*c^3*d^2*f^3*x^2 + 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3)*sin(f*x +
e)^2 + 2*(a^2*d^5*f^3*x^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*c^2*d^3*f^3*x^3 +
10*a^2*c^3*d^2*f^3*x^2 + 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3)*cos(f*x + e)), x)
 + 2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 6*d^2*cos(2*f*x + 2*e) + 6*d^2
+ 3*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 4*d^2)*cos(f*x + e) + 2*(d^2*f*x
 + c*d*f)*sin(2*f*x + 2*e) + 2*(d^2*f*x + c*d*f)*sin(f*x + e))*sin(3*f*x +
3*e) + 6*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 4*d^2 + 3*(d^2*f^2*x^2 + 2*
c*d*f^2*x + c^2*f^2 + 2*d^2)*cos(f*x + e) + 4*(d^2*f*x + c*d*f)*sin(f*x + e
))*sin(2*f*x + 2*e))/(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2
*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3
*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(3*f
*x + 3*e)^2 + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*
```

```
x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(2*f*x + 2*e)^2 + 9*(a^2*d^4*f^3*
x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2
*c^4*f^3)*cos(f*x + e)^2 + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c
^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(3*f*x + 3*e)^2 + 9*(a
^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*
f^3*x + a^2*c^4*f^3)*sin(2*f*x + 2*e)^2 + 18*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3
*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(2*f
*x + 2*e)*sin(f*x + e) + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c
^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*x + e)^2 + 2*(a^2*d
^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*
x + a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*
f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(2*f*x + 2*e) + 3*(a^2*d^4*f^
3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a
^2*c^4*f^3)*cos(f*x + e))*cos(3*f*x + 3*e) + 6*(a^2*d^4*f^3*x^4 + 4*a^2*c*d
^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + 3*(a
^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*
f^3*x + a^2*c^4*f^3)*cos(f*x + e))*cos(2*f*x + 2*e) + 6*(a^2*d^4*f^3*x^4 +
4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f
^3)*cos(f*x + e) + 6*((a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^
2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(2*f*x + 2*e) + (a^2*d^4*f^
3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a
^2*c^4*f^3)*sin(f*x + e))*sin(3*f*x + 3*e))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^2d^2x^2 + 2\,a^2cdx + a^2c^2 + \left(a^2d^2x^2 + 2\,a^2cdx + a^2c^2\right)\cos\left(fx+e\right)^2 + 2\left(a^2d^2x^2 + 2\,a^2cdx + a^2c^2\right)\cos\left(fx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (a^2*d^2*x^2 + 2*a^2*c*d*
x + a^2*c^2)*cos(f*x + e)^2 + 2*(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2)*cos(f
*x + e)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{c^2\cos^2\left(e+fx\right)+2c^2\cos\left(e+fx\right)+c^2+2cdx\cos^2\left(e+fx\right)+4cdx\cos\left(e+fx\right)+2cdx+d^2x^2\cos^2\left(e+fx\right)+2d^2x^2\cos\left(e+fx\right)+d^2x^2}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+a*cos(f*x+e))**2,x)
```

```
[Out] Integral(1/(c**2*cos(e + f*x)**2 + 2*c**2*cos(e + f*x) + c**2 + 2*c*d*x*cos
(e + f*x)**2 + 4*c*d*x*cos(e + f*x) + 2*c*d*x + d**2*x**2*cos(e + f*x)**2 +
 2*d**2*x**2*cos(e + f*x) + d**2*x**2), x)/a**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2\left(a\cos\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(a*cos(f*x + e) + a)^2), x)
```

3.138 $\int \frac{(c+dx)^3}{a-a\cos(e+fx)}\,dx$

Optimal. Leaf size=133

$$-\frac{12id^2(c+dx)\text{Li}_2\left(e^{i(e+fx)}\right)}{af^3}+\frac{6d(c+dx)^2\log\left(1-e^{i(e+fx)}\right)}{af^2}-\frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{i(c+dx)^3}{af}+\frac{12d^3\text{Li}_3\left(e^{i(e+fx)}\right)}{af^4}$$

```
[Out] ((-I)*(c + d*x)^3)/(a*f) - ((c + d*x)^3*Cot[e/2 + (f*x)/2])/(a*f) + (6*d*(c
 + d*x)^2*Log[1 - E^(I*(e + f*x))])/(a*f^2) - ((12*I)*d^2*(c + d*x)*PolyLog
[2, E^(I*(e + f*x))])/(a*f^3) + (12*d^3*PolyLog[3, E^(I*(e + f*x))])/(a*f^4
)
```

Rubi [A] time = 0.283706, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3318, 4184, 3717, 2190, 2531, 2282, 6589}

$$-\frac{12id^2(c+dx)\text{Li}_2\left(e^{i(e+fx)}\right)}{af^3}+\frac{6d(c+dx)^2\log\left(1-e^{i(e+fx)}\right)}{af^2}-\frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{i(c+dx)^3}{af}+\frac{12d^3\text{Li}_3\left(e^{i(e+fx)}\right)}{af^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a - a*Cos[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)^3)/(a*f) - ((c + d*x)^3*Cot[e/2 + (f*x)/2])/(a*f) + (6*d*(c
 + d*x)^2*Log[1 - E^(I*(e + f*x))])/(a*f^2) - ((12*I)*d^2*(c + d*x)*PolyLog
[2, E^(I*(e + f*x))])/(a*f^3) + (12*d^3*PolyLog[3, E^(I*(e + f*x))])/(a*f^4
)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a-a\cos(e+fx)}\,dx = \frac{\int (c+dx)^3\csc^2\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{2a}$$

$$= -\frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{(3d)\int (c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{af}$$

$$= -\frac{i(c+dx)^3}{af}-\frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{(6id)\int \frac{e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)^2}{1-e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}}dx}{af}$$

$$= -\frac{i(c+dx)^3}{af}-\frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{6d(c+dx)^2\log\left(1-e^{i(e+fx)}\right)}{af^2}-\frac{\left(12d^2\right)\int (c+dx)\log\left(1\right.}{af^2}$$

$$= -\frac{i(c+dx)^3}{af}-\frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{6d(c+dx)^2\log\left(1-e^{i(e+fx)}\right)}{af^2}-\frac{12id^2(c+dx)\mathrm{Li}_2\left(e^{i(e+f\cdot}\right.}{af^3}$$

$$= -\frac{i(c+dx)^3}{af}-\frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{6d(c+dx)^2\log\left(1-e^{i(e+fx)}\right)}{af^2}-\frac{12id^2(c+dx)\mathrm{Li}_2\left(e^{i(e+f\cdot}\right.}{af^3}$$

$$= -\frac{i(c+dx)^3}{af}-\frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{6d(c+dx)^2\log\left(1-e^{i(e+fx)}\right)}{af^2}-\frac{12id^2(c+dx)\mathrm{Li}_2\left(e^{i(e+f\cdot}\right.}{af^3}$$

Mathematica [A] time = 1.23837, size = 164, normalized size = 1.23

$$\frac{2\sin\left(\frac{1}{2}(e+fx)\right)\left(f^3\csc\left(\frac{e}{2}\right)(c+dx)^3\sin\left(\frac{fx}{2}\right)+2\sin\left(\frac{1}{2}(e+fx)\right)\left(6id^2f(c+dx)\mathrm{Li}_2\left(e^{-i(e+fx)}\right)-\frac{if^3(c+dx)^3}{-1+e^{ie}}+3df^2(c+dx\right.\right.}{f^4(a-a\cos(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a - a*Cos[e + f*x]),x]
```

```
[Out] (2*Sin[(e + f*x)/2]*(f^3*(c + d*x)^3*Csc[e/2]*Sin[(f*x)/2] + 2*(((-I)*f^3*(
c + d*x)^3)/(-1 + E^(I*e)) + 3*d*f^2*(c + d*x)^2*Log[1 - E^((-I)*(e + f*x))
] + (6*I)*d^2*f*(c + d*x)*PolyLog[2, E^((-I)*(e + f*x))] + 6*d^3*PolyLog[3,
 E^((-I)*(e + f*x))])*Sin[(e + f*x)/2]))/(f^4*(a - a*Cos[e + f*x]))
```

Maple [B] time = 0.46, size = 468, normalized size = 3.5

$$\frac{-12\,id^2c\mathrm{polylog}\left(2,\mathrm{e}^{i(fx+e)}\right)}{af^3}-6\,\frac{c^2d\ln\left(\mathrm{e}^{i(fx+e)}\right)}{af^2}+6\,\frac{c^2d\ln\left(\mathrm{e}^{i(fx+e)}-1\right)}{af^2}+6\,\frac{d^3e^2\ln\left(\mathrm{e}^{i(fx+e)}-1\right)}{f^4a}-6\,\frac{d^3e^2\ln\left(\mathrm{e}^{i(fx+e}\right.}{f^4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a-a*cos(f*x+e)),x)
```

```
[Out] -12*I*d^2/f^3/a*c*polylog(2,exp(I*(f*x+e)))-6*d/f^2/a*c^2*ln(exp(I*(f*x+e))
)+6*d/f^2/a*c^2*ln(exp(I*(f*x+e))-1)+6*d^3/f^4/a*e^2*ln(exp(I*(f*x+e))-1)-6
*d^3/f^4/a*e^2*ln(exp(I*(f*x+e)))-6*I*d^2/f/a*c*x^2-2*I*d^3/f/a*x^3+6*I*d^3
/f^3/a*e^2*x-12*I*d^3/f^3/a*polylog(2,exp(I*(f*x+e)))*x+6*d^3/f^2/a*ln(1-ex
p(I*(f*x+e)))*x^2-6*d^3/f^4/a*ln(1-exp(I*(f*x+e)))*e^2+4*I*d^3/f^4/a*e^3+12
*d^3*polylog(3,exp(I*(f*x+e)))/a/f^4-12*d^2/f^3/a*c*e*ln(exp(I*(f*x+e))-1)+
12*d^2/f^3/a*c*e*ln(exp(I*(f*x+e)))+12*d^2/f^2/a*c*ln(1-exp(I*(f*x+e)))*x+1
2*d^2/f^3/a*c*ln(1-exp(I*(f*x+e)))*e-12*I*d^2/f^2/a*c*e*x-2*I*(d^3*x^3+3*c*
d^2*x^2+3*c^2*d*x+c^3)/f/a/(exp(I*(f*x+e))-1)-6*I*d^2/f^3/a*c*e^2
```

Maxima [B] time = 2.06757, size = 1295, normalized size = 9.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a-a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] -(6*((cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1)*log(cos(f*x + e
)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1) - 2*(f*x + e)*sin(f*x + e))*c*d^
2*e/(a*f^2*cos(f*x + e)^2 + a*f^2*sin(f*x + e)^2 - 2*a*f^2*cos(f*x + e) + a
*f^2) - 3*((cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1)*log(cos(f
*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1) - 2*(f*x + e)*sin(f*x + e)
)*c^2*d/(a*f*cos(f*x + e)^2 + a*f*sin(f*x + e)^2 - 2*a*f*cos(f*x + e) + a*f
) + c^3*(cos(f*x + e) + 1)/(a*sin(f*x + e)) + 3*c*d^2*e^2*(cos(f*x + e) + 1
)/(a*f^2*sin(f*x + e)) - 3*c^2*d*e*(cos(f*x + e) + 1)/(a*f*sin(f*x + e)) -
(2*d^3*e^3 + (6*d^3*e^2*cos(f*x + e) + 6*I*d^3*e^2*sin(f*x + e) - 6*d^3*e^2
)*arctan2(sin(f*x + e), cos(f*x + e) - 1) + (6*(f*x + e)^2*d^3 - 12*(d^3*e
- c*d^2*f)*(f*x + e) - 6*((f*x + e)^2*d^3 - 2*(d^3*e - c*d^2*f)*(f*x + e))*
cos(f*x + e) + (-6*I*(f*x + e)^2*d^3 + (12*I*d^3*e - 12*I*c*d^2*f)*(f*x + e
))*sin(f*x + e))*arctan2(sin(f*x + e), -cos(f*x + e) + 1) - 2*((f*x + e)^3*
d^3 + 3*(f*x + e)*d^3*e^2 - 3*(d^3*e - c*d^2*f)*(f*x + e)^2)*cos(f*x + e) +
 (12*(f*x + e)*d^3 - 12*d^3*e + 12*c*d^2*f - 12*((f*x + e)*d^3 - d^3*e + c*
d^2*f)*cos(f*x + e) + (-12*I*(f*x + e)*d^3 + 12*I*d^3*e - 12*I*c*d^2*f)*sin
(f*x + e))*dilog(e^(I*f*x + I*e)) + (3*I*(f*x + e)^2*d^3 + 3*I*d^3*e^2 + (-
6*I*d^3*e + 6*I*c*d^2*f)*(f*x + e) + (-3*I*(f*x + e)^2*d^3 - 3*I*d^3*e^2 +
(6*I*d^3*e - 6*I*c*d^2*f)*(f*x + e))*cos(f*x + e) + 3*((f*x + e)^2*d^3 + d^
```

```
3*e^2 - 2*(d^3*e - c*d^2*f)*(f*x + e))*sin(f*x + e))*log(cos(f*x + e)^2 + s
in(f*x + e)^2 - 2*cos(f*x + e) + 1) + (-12*I*d^3*cos(f*x + e) + 12*d^3*sin(
f*x + e) + 12*I*d^3)*polylog(3, e^(I*f*x + I*e)) + (-2*I*(f*x + e)^3*d^3 -
6*I*(f*x + e)*d^3*e^2 + (6*I*d^3*e - 6*I*c*d^2*f)*(f*x + e)^2)*sin(f*x + e)
)/(-I*a*f^3*cos(f*x + e) + a*f^3*sin(f*x + e) + I*a*f^3))/f
```

Fricas [C] time = 1.77678, size = 1173, normalized size = 8.82

$$-\frac{d^3f^3x^3+3\,cd^2f^3x^2+3\,c^2df^3x+c^3f^3-6\,d^3\mathrm{polylog}\left(3,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\sin\left(fx+e\right)-6\,d^3\mathrm{polylog}\left(3,\mathrm{cc}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a-a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + c^3*f^3 - 6*d^3*polylog(3
, cos(f*x + e) + I*sin(f*x + e))*sin(f*x + e) - 6*d^3*polylog(3, cos(f*x +
e) - I*sin(f*x + e))*sin(f*x + e) - (-6*I*d^3*f*x - 6*I*c*d^2*f)*dilog(cos(
f*x + e) + I*sin(f*x + e))*sin(f*x + e) - (6*I*d^3*f*x + 6*I*c*d^2*f)*dilog
(cos(f*x + e) - I*sin(f*x + e))*sin(f*x + e) - 3*(d^3*e^2 - 2*c*d^2*e*f + c
^2*d*f^2)*log(-1/2*cos(f*x + e) + 1/2*I*sin(f*x + e) + 1/2)*sin(f*x + e) -
3*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*log(-1/2*cos(f*x + e) - 1/2*I*sin(f*x
 + e) + 1/2)*sin(f*x + e) - 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*
d^2*e*f)*log(-cos(f*x + e) + I*sin(f*x + e) + 1)*sin(f*x + e) - 3*(d^3*f^2*
x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*log(-cos(f*x + e) - I*sin(f*x
+ e) + 1)*sin(f*x + e) + (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + c
^3*f^3)*cos(f*x + e))/(a*f^4*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{c^3}{\cos\left(e+fx\right)-1}\,dx+\int \frac{d^3x^3}{\cos\left(e+fx\right)-1}\,dx+\int \frac{3cd^2x^2}{\cos\left(e+fx\right)-1}\,dx+\int \frac{3c^2dx}{\cos\left(e+fx\right)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a-a*cos(f*x+e)),x)
```

```
[Out] -(Integral(c**3/(cos(e + f*x) - 1), x) + Integral(d**3*x**3/(cos(e + f*x) -
 1), x) + Integral(3*c*d**2*x**2/(cos(e + f*x) - 1), x) + Integral(3*c**2*d
*x/(cos(e + f*x) - 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(dx+c)^3}{a\cos\left(fx+e\right)-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a-a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(-(d*x + c)^3/(a*cos(f*x + e) - a), x)
```

3.139 $\int \frac{(c+dx)^2}{a-a\cos(e+fx)}\,dx$

Optimal. Leaf size=102

$$\frac{4d(c+dx)\log\left(1-e^{i(e+fx)}\right)}{af^2}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{i(c+dx)^2}{af}-\frac{4id^2\text{Li}_2\left(e^{i(e+fx)}\right)}{af^3}$$

```
[Out] ((-I)*(c + d*x)^2)/(a*f) - ((c + d*x)^2*Cot[e/2 + (f*x)/2])/(a*f) + (4*d*(c
 + d*x)*Log[1 - E^(I*(e + f*x))])/(a*f^2) - ((4*I)*d^2*PolyLog[2, E^(I*(e +
 f*x))])/(a*f^3)
```

Rubi [A] time = 0.202976, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3318, 4184, 3717, 2190, 2279, 2391}

$$\frac{4d(c+dx)\log\left(1-e^{i(e+fx)}\right)}{af^2}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{i(c+dx)^2}{af}-\frac{4id^2\text{Li}_2\left(e^{i(e+fx)}\right)}{af^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a - a*Cos[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)^2)/(a*f) - ((c + d*x)^2*Cot[e/2 + (f*x)/2])/(a*f) + (4*d*(c
 + d*x)*Log[1 - E^(I*(e + f*x))])/(a*f^2) - ((4*I)*d^2*PolyLog[2, E^(I*(e +
 f*x))])/(a*f^3)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(c+dx)^2}{a-a\cos(e+fx)}\,dx = \frac{\int (c+dx)^2\csc^2\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{2a}$$

$$= -\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{(2d)\int (c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{af}$$

$$= -\frac{i(c+dx)^2}{af}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{(4id)\int \frac{e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)}{1-e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}}dx}{af}$$

$$= -\frac{i(c+dx)^2}{af}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{4d(c+dx)\log\left(1-e^{i(e+fx)}\right)}{af^2}-\frac{(4d^2)\int \log\left(1-e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}\right.$$

$$= -\frac{i(c+dx)^2}{af}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{4d(c+dx)\log\left(1-e^{i(e+fx)}\right)}{af^2}+\frac{(4id^2)\,\mathrm{Subst}\left(\int \frac{\log(1-x)}{x}\right.}{af^3}$$

$$= -\frac{i(c+dx)^2}{af}-\frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{4d(c+dx)\log\left(1-e^{i(e+fx)}\right)}{af^2}-\frac{4id^2\mathrm{Li}_2\left(e^{i(e+fx)}\right)}{af^3}$$

Mathematica [B] time = 5.48236, size = 292, normalized size = 2.86

$$\frac{2\csc\left(\frac{e}{2}\right)\sin\left(\frac{1}{2}(e+fx)\right)\left(-2cdf\sin\left(\frac{1}{2}(e+fx)\right)\left(fx\cos\left(\frac{e}{2}\right)-2\sin\left(\frac{e}{2}\right)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+f^2(c+dx)^2\sin\left(\frac{fx}{2}\right)+\ldots}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2/(a - a*Cos[e + f*x]),x]
```

```
[Out] (2*Csc[e/2]*Sin[(e + f*x)/2]*(f^2*(c + d*x)^2*Sin[(f*x)/2] - 2*c*d*f*(f*x*C
os[e/2] - 2*Log[Sin[(e + f*x)/2]]*Sin[e/2])*Sin[(e + f*x)/2] + d^2*(-(E^(I*
ArcTan[Tan[e/2]])*f^2*x^2*Cos[e/2]*Sqrt[Sec[e/2]^2]) - 4*((-I/2)*f*x*(Pi -
2*ArcTan[Tan[e/2]]) - Pi*Log[1 + E^((-I)*f*x)] - (f*x + 2*ArcTan[Tan[e/2]])
*Log[1 - E^(I*(f*x + 2*ArcTan[Tan[e/2]]))] + Pi*Log[Cos[(f*x)/2]] + 2*ArcTa
n[Tan[e/2]]*Log[Sin[(f*x)/2 + ArcTan[Tan[e/2]]]] + I*PolyLog[2, E^(I*(f*x +
 2*ArcTan[Tan[e/2]]))])*Sin[e/2])*Sin[(e + f*x)/2]))/(f^3*(a - a*Cos[e + f*
x]))
```

Maple [B] time = 0.398, size = 247, normalized size = 2.4

$$\frac{-2\,i\left(d^2x^2+2\,cdx+c^2\right)}{af\left(\mathrm{e}^{i(fx+e)}-1\right)}-4\,\frac{cd\ln\left(\mathrm{e}^{i(fx+e)}\right)}{af^2}+4\,\frac{cd\ln\left(\mathrm{e}^{i(fx+e)}-1\right)}{af^2}-\frac{2\,id^2x^2}{af}-\frac{4\,id^2ex}{af^2}-\frac{2\,id^2e^2}{f^3a}+4\,\frac{d^2\ln\left(1-\mathrm{e}^{i(fx+e)}\right.}{af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a-a*cos(f*x+e)),x)
```

```
[Out] -2*I*(d^2*x^2+2*c*d*x+c^2)/f/a/(exp(I*(f*x+e))-1)-4*d/f^2/a*c*ln(exp(I*(f*x
+e)))+4*d/f^2/a*c*ln(exp(I*(f*x+e))-1)-2*I*d^2/f/a*x^2-4*I*d^2/f^2/a*e*x-2*
I*d^2/f^3/a*e^2+4*d^2/f^2/a*ln(1-exp(I*(f*x+e)))*x+4*d^2/f^3/a*ln(1-exp(I*(
f*x+e)))*e-4*I*d^2*polylog(2,exp(I*(f*x+e)))/a/f^3+4*d^2/f^3/a*e*ln(exp(I*(
f*x+e)))-4*d^2/f^3/a*e*ln(exp(I*(f*x+e))-1)
```

Maxima [B] time = 1.95884, size = 425, normalized size = 4.17

$$-\frac{2\,c^2f^2-\left(4\,cdf\cos\left(fx+e\right)+4i\,cdf\sin\left(fx+e\right)-4\,cdf\right)\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)-1\right)+4\left(d^2fx\cos\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] -(2*c^2*f^2 - (4*c*d*f*cos(f*x + e) + 4*I*c*d*f*sin(f*x + e) - 4*c*d*f)*arc
tan2(sin(f*x + e), cos(f*x + e) - 1) + 4*(d^2*f*x*cos(f*x + e) + I*d^2*f*x*
sin(f*x + e) - d^2*f*x)*arctan2(sin(f*x + e), -cos(f*x + e) + 1) + 2*(d^2*f
^2*x^2 + 2*c*d*f^2*x)*cos(f*x + e) + 4*(d^2*cos(f*x + e) + I*d^2*sin(f*x +
e) - d^2)*dilog(e^(I*f*x + I*e)) - (2*I*d^2*f*x + 2*I*c*d*f + (-2*I*d^2*f*x
 - 2*I*c*d*f)*cos(f*x + e) + 2*(d^2*f*x + c*d*f)*sin(f*x + e))*log(cos(f*x
+ e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1) - (-2*I*d^2*f^2*x^2 - 4*I*c*d
*f^2*x)*sin(f*x + e))/(-I*a*f^3*cos(f*x + e) + a*f^3*sin(f*x + e) + I*a*f^3
)
```

Fricas [B] time = 1.71963, size = 743, normalized size = 7.28

$$-\frac{d^2f^2x^2+2\,cdf^2x+c^2f^2+2i\,d^2\mathrm{Li}_2\left(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\sin\left(fx+e\right)-2i\,d^2\mathrm{Li}_2\left(\cos\left(fx+e\right)-i\,\sin\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 2*I*d^2*dilog(cos(f*x + e) + I*sin(
f*x + e))*sin(f*x + e) - 2*I*d^2*dilog(cos(f*x + e) - I*sin(f*x + e))*sin(f
*x + e) + 2*(d^2*e - c*d*f)*log(-1/2*cos(f*x + e) + 1/2*I*sin(f*x + e) + 1/
2)*sin(f*x + e) + 2*(d^2*e - c*d*f)*log(-1/2*cos(f*x + e) - 1/2*I*sin(f*x +
 e) + 1/2)*sin(f*x + e) - 2*(d^2*f*x + d^2*e)*log(-cos(f*x + e) + I*sin(f*x
 + e) + 1)*sin(f*x + e) - 2*(d^2*f*x + d^2*e)*log(-cos(f*x + e) - I*sin(f*x
 + e) + 1)*sin(f*x + e) + (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*cos(f*x + e
```

```
))/(a*f^3*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{c^2}{\cos\left(e+fx\right)-1}\,dx + \int \frac{d^2x^2}{\cos\left(e+fx\right)-1}\,dx + \int \frac{2cdx}{\cos\left(e+fx\right)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a-a*cos(f*x+e)),x)
```

```
[Out] -(Integral(c**2/(cos(e + f*x) - 1), x) + Integral(d**2*x**2/(cos(e + f*x) -
 1), x) + Integral(2*c*d*x/(cos(e + f*x) - 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(dx+c)^2}{a\cos\left(fx+e\right)-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(-(d*x + c)^2/(a*cos(f*x + e) - a), x)
```

3.140 $\int \frac{c+dx}{a-a\cos(e+fx)}\,dx$

Optimal. Leaf size=50

$$\frac{2d\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2}-\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}$$

```
[Out] -(((c + d*x)*Cot[e/2 + (f*x)/2])/(a*f)) + (2*d*Log[Sin[e/2 + (f*x)/2]])/(a*
f^2)
```

Rubi [A] time = 0.0652033, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3318, 4184, 3475}

$$\frac{2d\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2}-\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a - a*Cos[e + f*x]),x]
```

```
[Out] -(((c + d*x)*Cot[e/2 + (f*x)/2])/(a*f)) + (2*d*Log[Sin[e/2 + (f*x)/2]])/(a*
f^2)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{a-a\cos(e+fx)}\,dx &= \frac{\int (c+dx)\csc^2\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{2a}\\ &= -\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{d\int \cot\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{af}\\ &= -\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{2d\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2}\end{aligned}$$

Mathematica [A] time = 0.262909, size = 57, normalized size = 1.14

$$\frac{f(c+dx)\sin(e+fx)-4d\sin^2\left(\frac{1}{2}(e+fx)\right)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{af^2(\cos(e+fx)-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a - a*Cos[e + f*x]),x]
```

```
[Out] (-4*d*Log[Sin[(e + f*x)/2]]*Sin[(e + f*x)/2]^2 + f*(c + d*x)*Sin[e + f*x])/
(a*f^2*(-1 + Cos[e + f*x]))
```

Maple [A] time = 0.063, size = 85, normalized size = 1.7

$$-\frac{c}{af}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-1}-\frac{dx}{af}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-1}-\frac{d}{af^2}\ln\left(1+\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2\right)+2\frac{d\ln\left(\tan\left(1/2\,fx+e/2\right)\right)}{af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a-a*cos(f*x+e)),x)
```

```
[Out] -1/a*c/f/tan(1/2*f*x+1/2*e)-1/a*d*x/f/tan(1/2*f*x+1/2*e)-1/a*d/f^2*ln(1+tan
(1/2*f*x+1/2*e)^2)+2/a*d/f^2*ln(tan(1/2*f*x+1/2*e))
```

Maxima [B] time = 1.16745, size = 216, normalized size = 4.32

$$\frac{\frac{\left(\left(\cos(fx+e)^2+\sin(fx+e)^2-2\cos(fx+e)+1\right)\log\left(\cos(fx+e)^2+\sin(fx+e)^2-2\cos(fx+e)+1\right)-2(fx+e)\sin(fx+e)\right)d}{af\cos(fx+e)^2+af\sin(fx+e)^2-2af\cos(fx+e)+af}-\frac{c\left(\cos(fx+e)+1\right)}{a\sin(fx+e)}+\frac{de\left(\cos(fx+e)+1\right)}{af\sin(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a-a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] (((cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1)*log(cos(f*x + e)^2
 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1) - 2*(f*x + e)*sin(f*x + e))*d/(a*f*
cos(f*x + e)^2 + a*f*sin(f*x + e)^2 - 2*a*f*cos(f*x + e) + a*f) - c*(cos(f*
x + e) + 1)/(a*sin(f*x + e)) + d*e*(cos(f*x + e) + 1)/(a*f*sin(f*x + e)))/f
```

Fricas [A] time = 1.63022, size = 151, normalized size = 3.02

$$-\frac{dfx-d\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)+cf+\left(dfx+cf\right)\cos\left(fx+e\right)}{af^2\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a-a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d*f*x - d*log(-1/2*cos(f*x + e) + 1/2)*sin(f*x + e) + c*f + (d*f*x + c*f)
*cos(f*x + e))/(a*f^2*sin(f*x + e))
```

Sympy [A] time = 0.896187, size = 90, normalized size = 1.8

$$\begin{cases} -\frac{c}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)} - \frac{dx}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)} - \frac{d\log\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{af^2} + \frac{2d\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2} & \text{for } f \neq 0 \\ \frac{cx+\frac{dx^2}{2}}{-a\cos(e)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a-a*cos(f*x+e)),x)
```

```
[Out] Piecewise((-c/(a*f*tan(e/2 + f*x/2)) - d*x/(a*f*tan(e/2 + f*x/2)) - d*log(t
an(e/2 + f*x/2)**2 + 1)/(a*f**2) + 2*d*log(tan(e/2 + f*x/2))/(a*f**2), Ne(f
, 0)), ((c*x + d*x**2/2)/(-a*cos(e) + a), True))
```

Giac [B] time = 1.16138, size = 309, normalized size = 6.18

$$dfx\tan\left(\frac{1}{2}fx\right)\tan\left(\frac{1}{2}e\right) + cf\tan\left(\frac{1}{2}fx\right)\tan\left(\frac{1}{2}e\right) - dfx + d\log\left(\frac{4\left(\tan\left(\frac{1}{2}e\right)^2+1\right)}{\tan\left(\frac{1}{2}fx\right)^4+2\tan\left(\frac{1}{2}fx\right)^3\tan\left(\frac{1}{2}e\right)+\tan\left(\frac{1}{2}fx\right)^2\tan\left(\frac{1}{2}e\right)^2+\tan(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a-a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] (d*f*x*tan(1/2*f*x)*tan(1/2*e) + c*f*tan(1/2*f*x)*tan(1/2*e) - d*f*x + d*lo
g(4*(tan(1/2*e)^2 + 1)/(tan(1/2*f*x)^4 + 2*tan(1/2*f*x)^3*tan(1/2*e) + tan(
1/2*f*x)^2*tan(1/2*e)^2 + tan(1/2*f*x)^2 + 2*tan(1/2*f*x)*tan(1/2*e) + tan(
1/2*e)^2))*tan(1/2*f*x) + d*log(4*(tan(1/2*e)^2 + 1)/(tan(1/2*f*x)^4 + 2*ta
n(1/2*f*x)^3*tan(1/2*e) + tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1/2*f*x)^2 + 2*
tan(1/2*f*x)*tan(1/2*e) + tan(1/2*e)^2))*tan(1/2*e) - c*f)/(a*f^2*tan(1/2*f
*x) + a*f^2*tan(1/2*e))
```

3.141 $\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{1}{(c+dx)(a-a\cos(e+fx))},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)*(a - a*Cos[e + f*x])), x]
```

Rubi [A] time = 0.062457, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(a - a*Cos[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a - a*Cos[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx = \int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx$$

Mathematica [A] time = 2.54058, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a - a*Cos[e + f*x])),x]
```

```
[Out] Integrate[1/((c + d*x)*(a - a*Cos[e + f*x])), x]
```

Maple [A] time = 0.295, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(a-a\cos\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a-a*cos(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)/(a-a*cos(f*x+e)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a-a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{adx + ac - (adx + ac)\cos\left(fx + e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a-a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d*x + a*c - (a*d*x + a*c)*cos(f*x + e)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{1}{c\cos\left(e+fx\right)-c+dx\cos\left(e+fx\right)-dx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a-a*cos(f*x+e)),x)
```

```
[Out] -Integral(1/(c*cos(e + f*x) - c + d*x*cos(e + f*x) - d*x), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(dx + c)\left(a\cos\left(fx + e\right) - a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a-a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(-1/((d*x + c)*(a*cos(f*x + e) - a)), x)
```

3.142 $\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{1}{(c+dx)^2(a-a\cos(e+fx))},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)^2*(a - a*Cos[e + f*x])), x]
```

Rubi [A] time = 0.057798, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a - a*Cos[e + f*x])),x]

[Out] Defer[Int][1/((c + d*x)^2*(a - a*Cos[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx=\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx$$

Mathematica [A] time = 2.40513, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a - a*Cos[e + f*x])),x]

[Out] Integrate[1/((c + d*x)^2*(a - a*Cos[e + f*x])), x]
```

Maple [A] time = 0.328, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2\left(a-a\cos\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a-a*cos(f*x+e)),x)

[Out] int(1/(d*x+c)^2/(a-a*cos(f*x+e)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{ad^2x^2 + 2\,acdx + ac^2 - \left(ad^2x^2 + 2\,acdx + ac^2\right)\cos\left(fx + e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 - (a*d^2*x^2 + 2*a*c*d*x + a*c^2)
*cos(f*x + e)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{1}{c^2\cos\left(e+fx\right)-c^2+2cdx\cos\left(e+fx\right)-2cdx+d^2x^2\cos\left(e+fx\right)-d^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a-a*cos(f*x+e)),x)
```

```
[Out] -Integral(1/(c**2*cos(e + f*x) - c**2 + 2*c*d*x*cos(e + f*x) - 2*c*d*x + d*
*2*x**2*cos(e + f*x) - d**2*x**2), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(dx + c)^2\left(a\cos\left(fx + e\right) - a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(-1/((d*x + c)^2*(a*cos(f*x + e) - a)), x)
```

3.143 $\int x^3\sqrt{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=110

$$\frac{12x^2\sqrt{a\cos(c+dx)+a}}{d^2}-\frac{96\sqrt{a\cos(c+dx)+a}}{d^4}-\frac{48x\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}}{d^3}+\frac{2x^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+a}}{d}$$

```
[Out] (-96*Sqrt[a + a*Cos[c + d*x]])/d^4 + (12*x^2*Sqrt[a + a*Cos[c + d*x]])/d^2
- (48*x*Sqrt[a + a*Cos[c + d*x]]*Tan[c/2 + (d*x)/2])/d^3 + (2*x^3*Sqrt[a +
a*Cos[c + d*x]]*Tan[c/2 + (d*x)/2])/d
```

Rubi [A] time = 0.133953, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3319, 3296, 2638}

$$\frac{12x^2\sqrt{a\cos(c+dx)+a}}{d^2}-\frac{96\sqrt{a\cos(c+dx)+a}}{d^4}-\frac{48x\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}}{d^3}+\frac{2x^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (-96*Sqrt[a + a*Cos[c + d*x]])/d^4 + (12*x^2*Sqrt[a + a*Cos[c + d*x]])/d^2
- (48*x*Sqrt[a + a*Cos[c + d*x]]*Tan[c/2 + (d*x)/2])/d^3 + (2*x^3*Sqrt[a +
a*Cos[c + d*x]]*Tan[c/2 + (d*x)/2])/d
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x^3\sqrt{a+a\cos(c+dx)}\,dx = \left(\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x^3\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)dx$$
$$= \frac{2x^3\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} - \frac{\left(6\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\ldots}{d}$$
$$= \frac{12x^2\sqrt{a+a\cos(c+dx)}}{d^2} + \frac{2x^3\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} - \frac{\left(24\sqrt{a+a\cos(c+dx}\ldots}{\ldots}$$
$$= \frac{12x^2\sqrt{a+a\cos(c+dx)}}{d^2} - \frac{48x\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3} + \frac{2x^3\sqrt{a+a\cos(c+dx}\ldots}{d}$$
$$= -\frac{96\sqrt{a+a\cos(c+dx)}}{d^4} + \frac{12x^2\sqrt{a+a\cos(c+dx)}}{d^2} - \frac{48x\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\ldots}{d^3}$$

Mathematica [A] time = 0.2103, size = 53, normalized size = 0.48

$$\frac{2\left(dx\left(d^2x^2-24\right)\tan\left(\frac{1}{2}(c+dx)\right)+6\left(d^2x^2-8\right)\right)\sqrt{a(\cos(c+dx)+1)}}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(6*(-8 + d^2*x^2) + d*x*(-24 + d^2*x^2)*Tan[(
c + d*x)/2]))/d^4
```

Maple [C] time = 0.29, size = 132, normalized size = 1.2

$$\frac{-i\sqrt{2}\left(d^3x^3e^{i(dx+c)}+6\,id^2x^2e^{i(dx+c)}-d^3x^3+6\,id^2x^2-24\,dxe^{i(dx+c)}-48\,ie^{i(dx+c)}+24\,dx-48\,i\right)}{\left(e^{i(dx+c)}+1\right)d^4}\sqrt{a\left(e^{i(dx+c)}+1\right)^2e^{-i(d\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -I*2^(1/2)*(a*(exp(I*(d*x+c))+1)^2*exp(-I*(d*x+c)))^(1/2)/(exp(I*(d*x+c))+1
)*(d^3*x^3*exp(I*(d*x+c))+6*I*d^2*x^2*exp(I*(d*x+c))-d^3*x^3+6*I*d^2*x^2-24
*d*x*exp(I*(d*x+c))-48*I*exp(I*(d*x+c))+24*d*x-48*I)/d^4
```

Maxima [B] time = 2.86868, size = 278, normalized size = 2.53

$$-\frac{2\left(\sqrt{2}\sqrt{a}c^3\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3\left(\sqrt{2}(dx+c)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sqrt{a}c^2+3\left(\sqrt{2}(dx+c)^2\sin\left(\frac{1}{2}d\ldots\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*(sqrt(2)*sqrt(a)*c^3*sin(1/2*d*x + 1/2*c) - 3*(sqrt(2)*(d*x + c)*sin(1/2
*d*x + 1/2*c) + 2*sqrt(2)*cos(1/2*d*x + 1/2*c))*sqrt(a)*c^2 + 3*(sqrt(2)*(d
```

```
*x + c)^2*sin(1/2*d*x + 1/2*c) + 4*sqrt(2)*(d*x + c)*cos(1/2*d*x + 1/2*c) -
 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a)*c - (sqrt(2)*(d*x + c)^3*sin(1/2*d
*x + 1/2*c) + 6*sqrt(2)*(d*x + c)^2*cos(1/2*d*x + 1/2*c) - 24*sqrt(2)*(d*x
+ c)*sin(1/2*d*x + 1/2*c) - 48*sqrt(2)*cos(1/2*d*x + 1/2*c))*sqrt(a))/d^4
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cos\left(dx+c\right)+a}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)*x^3, x)
```

3.144 $\int x^2\sqrt{a + a\cos(c + dx)}\,dx$

Optimal. Leaf size=88

$$\frac{8x\sqrt{a\cos(c+dx)+a}}{d^2}-\frac{16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}}{d^3}+\frac{2x^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}}{d}$$

```
[Out] (8*x*Sqrt[a + a*Cos[c + d*x]])/d^2 - (16*Sqrt[a + a*Cos[c + d*x]]*Tan[c/2 +
 (d*x)/2])/d^3 + (2*x^2*Sqrt[a + a*Cos[c + d*x]]*Tan[c/2 + (d*x)/2])/d
```

Rubi [A] time = 0.112431, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3319, 3296, 2637}

$$\frac{8x\sqrt{a\cos(c+dx)+a}}{d^2}-\frac{16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}}{d^3}+\frac{2x^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (8*x*Sqrt[a + a*Cos[c + d*x]])/d^2 - (16*Sqrt[a + a*Cos[c + d*x]]*Tan[c/2 +
 (d*x)/2])/d^3 + (2*x^2*Sqrt[a + a*Cos[c + d*x]]*Tan[c/2 + (d*x)/2])/d
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^2\sqrt{a+a\cos(c+dx)}\,dx &= \left(\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x^2\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)dx\\
&= \frac{2x^2\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{\left(4\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)}{d}\\
&= \frac{8x\sqrt{a+a\cos(c+dx)}}{d^2}+\frac{2x^2\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{\left(8\sqrt{a+a\cos(c+dx)}\,\mathrm{cs}\right.}{}\\
&= \frac{8x\sqrt{a+a\cos(c+dx)}}{d^2}-\frac{16\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3}+\frac{2x^2\sqrt{a+a\cos(c+dx)}\,\mathrm{ta}}{d}
\end{aligned}$$

Mathematica [A] time = 0.157771, size = 44, normalized size = 0.5

$$\frac{2\left(\left(d^2x^2-8\right)\tan\left(\frac{1}{2}(c+dx)\right)+4dx\right)\sqrt{a(\cos(c+dx)+1)}}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(4*d*x + (-8 + d^2*x^2)*Tan[(c + d*x)/2]))/d^
3
```

Maple [C] time = 0.231, size = 105, normalized size = 1.2

$$\frac{-i\sqrt{2}\left(d^2x^2\mathrm{e}^{i(dx+c)}+4\,idx\mathrm{e}^{i(dx+c)}-d^2x^2+4\,idx-8\,\mathrm{e}^{i(dx+c)}+8\right)}{\left(\mathrm{e}^{i(dx+c)}+1\right)d^3}\sqrt{a\left(\mathrm{e}^{i(dx+c)}+1\right)^2\mathrm{e}^{-i(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -I*2^(1/2)*(a*(exp(I*(d*x+c))+1)^2*exp(-I*(d*x+c)))^(1/2)/(exp(I*(d*x+c))+1
)*(d^2*x^2*exp(I*(d*x+c))+4*I*d*x*exp(I*(d*x+c))-d^2*x^2+4*I*d*x-8*exp(I*(d
*x+c))+8)/d^3
```

Maxima [A] time = 2.81163, size = 165, normalized size = 1.88

$$\frac{2\left(\sqrt{2}\sqrt{a}c^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\left(\sqrt{2}(dx+c)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,\sqrt{2}\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}c+\left(\sqrt{2}(dx+c)^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(2)*sqrt(a)*c^2*sin(1/2*d*x + 1/2*c) - 2*(sqrt(2)*(d*x + c)*sin(1/2*
d*x + 1/2*c) + 2*sqrt(2)*cos(1/2*d*x + 1/2*c))*sqrt(a)*c + (sqrt(2)*(d*x +
c)^2*sin(1/2*d*x + 1/2*c) + 4*sqrt(2)*(d*x + c)*cos(1/2*d*x + 1/2*c) - 8*sq
rt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a))/d^3
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cos\left(dx+c\right)+a}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)*x^2, x)
```

3.145 $\int x\sqrt{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=53

$$\frac{4\sqrt{a\cos(c+dx)+a}}{d^2}+\frac{2x\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}}{d}$$

```
[Out] (4*Sqrt[a + a*Cos[c + d*x]])/d^2 + (2*x*Sqrt[a + a*Cos[c + d*x]]*Tan[c/2 +
(d*x)/2])/d
```

Rubi [A] time = 0.0608678, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3319, 3296, 2638}

$$\frac{4\sqrt{a\cos(c+dx)+a}}{d^2}+\frac{2x\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (4*Sqrt[a + a*Cos[c + d*x]])/d^2 + (2*x*Sqrt[a + a*Cos[c + d*x]]*Tan[c/2 +
(d*x)/2])/d
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+a\cos(c+dx)}\,dx &= \left(\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)dx\\ &= \frac{2x\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{\left(2\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int \sin}{d}\\ &= \frac{4\sqrt{a+a\cos(c+dx)}}{d^2}+\frac{2x\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.128065, size = 34, normalized size = 0.64

$$\frac{2\left(dx\tan\left(\frac{1}{2}(c+dx)\right)+2\right)\sqrt{a(\cos(c+dx)+1)}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(2 + d*x*Tan[(c + d*x)/2]))/d^2
```

Maple [C] time = 0.218, size = 80, normalized size = 1.5

$$\frac{-i\sqrt{2}\left(dx\mathrm{e}^{i(dx+c)}+2\,i\mathrm{e}^{i(dx+c)}-dx+2\,i\right)}{\left(\mathrm{e}^{i(dx+c)}+1\right)d^2}\sqrt{a\left(\mathrm{e}^{i(dx+c)}+1\right)^2\mathrm{e}^{-i(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -I*2^(1/2)*(a*(exp(I*(d*x+c))+1)^2*exp(-I*(d*x+c)))^(1/2)/(exp(I*(d*x+c))+1
)*(d*x*exp(I*(d*x+c))+2*I*exp(I*(d*x+c))-d*x+2*I)/d^2
```

Maxima [A] time = 2.52002, size = 82, normalized size = 1.55

$$-\frac{2\left(\sqrt{2}\sqrt{a}c\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\left(\sqrt{2}(dx+c)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,\sqrt{2}\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*(sqrt(2)*sqrt(a)*c*sin(1/2*d*x + 1/2*c) - (sqrt(2)*(d*x + c)*sin(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*cos(1/2*d*x + 1/2*c))*sqrt(a))/d^2
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cos(dx+c)+a}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)*x, x)
```

3.146 $\int \sqrt{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=26

$$\frac{2a\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*a*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.0131807, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2646}

$$\frac{2a\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*a*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,dx = \frac{2a\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.0326455, size = 29, normalized size = 1.12

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*Tan[(c + d*x)/2])/d
```

Maple [A] time = 0.496, size = 43, normalized size = 1.7

$$2\,{\frac{a\cos\left(1/2\,dx+c/2\right)\sin\left(1/2\,dx+c/2\right)\sqrt{2}}{\sqrt{\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}a}d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 2*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*2^(1/2)/(cos(1/2*d*x+1/2*c)^2*a)^
(1/2)/d
```

Maxima [A] time = 2.67626, size = 27, normalized size = 1.04

$$\frac{2\sqrt{2}\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*sqrt(a)*sin(1/2*d*x + 1/2*c)/d
```

Fricas [A] time = 1.50136, size = 84, normalized size = 3.23

$$\frac{2\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{d\cos\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\cos\left(c+dx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*cos(c + d*x) + a), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a), x)
```

3.147 $\int \frac{\sqrt{a+a\cos(c+dx)}}{x}\,dx$

Optimal. Leaf size=84

$$\cos\left(\frac{c}{2}\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}-\sin\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}$$

```
[Out] Cos[c/2]*Sqrt[a + a*Cos[c + d*x]]*CosIntegral[(d*x)/2]*Sec[c/2 + (d*x)/2] -
 Sqrt[a + a*Cos[c + d*x]]*Sec[c/2 + (d*x)/2]*Sin[c/2]*SinIntegral[(d*x)/2]
```

Rubi [A] time = 0.121311, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3319, 3303, 3299, 3302}

$$\cos\left(\frac{c}{2}\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}-\sin\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]/x,x]
```

```
[Out] Cos[c/2]*Sqrt[a + a*Cos[c + d*x]]*CosIntegral[(d*x)/2]*Sec[c/2 + (d*x)/2] -
 Sqrt[a + a*Cos[c + d*x]]*Sec[c/2 + (d*x)/2]*Sin[c/2]*SinIntegral[(d*x)/2]
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x}\,dx = \left(\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int \frac{\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)}{x}\,dx$$

$$= \left(\cos\left(\frac{c}{2}\right)\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int \frac{\cos\left(\frac{dx}{2}\right)}{x}\,dx - \left(\sqrt{a+a\cos(c+d\right.$$

$$= \cos\left(\frac{c}{2}\right)\sqrt{a+a\cos(c+dx)}\operatorname{Ci}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right) - \sqrt{a+a\cos(c+dx)}\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}\right.$$

Mathematica [A] time = 0.0907398, size = 55, normalized size = 0.65

$$\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\cos\left(\frac{c}{2}\right)\text{CosIntegral}\left(\frac{dx}{2}\right)-\sin\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/x,x]
```

```
[Out] Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Cos[c/2]*CosIntegral[(d*x)/2]
- Sin[c/2]*SinIntegral[(d*x)/2])
```

Maple [F] time = 0.268, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{a+\cos(dx+c)\,a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2)/x,x)
```

```
[Out] int((a+cos(d*x+c)*a)^(1/2)/x,x)
```

Maxima [C] time = 2.58361, size = 82, normalized size = 0.98

$$-\frac{1}{2}\left(\left(\sqrt{2}E_1\left(\frac{1}{2}i\,dx\right)+\sqrt{2}E_1\left(-\frac{1}{2}i\,dx\right)\right)\cos\left(\frac{1}{2}c\right)-\left(i\sqrt{2}E_1\left(\frac{1}{2}i\,dx\right)-i\sqrt{2}E_1\left(-\frac{1}{2}i\,dx\right)\right)\sin\left(\frac{1}{2}c\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -1/2*((sqrt(2)*exp_integral_e(1, 1/2*I*d*x) + sqrt(2)*exp_integral_e(1, -1/
2*I*d*x))*cos(1/2*c) - (I*sqrt(2)*exp_integral_e(1, 1/2*I*d*x) - I*sqrt(2)*
exp_integral_e(1, -1/2*I*d*x))*sin(1/2*c))*sqrt(a)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\cos\left(dx+c\right)+a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)/x, x)
```

3.148 $\int \frac{\sqrt{a+a\cos(c+dx)}}{x^2}\,dx$

Optimal. Leaf size=110

$$-\frac{1}{2}d\sin\left(\frac{c}{2}\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}-\frac{1}{2}d\cos\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}$$

```
[Out] -(Sqrt[a + a*Cos[c + d*x]]/x) - (d*Sqrt[a + a*Cos[c + d*x]]*CosIntegral[(d*
x)/2]*Sec[c/2 + (d*x)/2]*Sin[c/2])/2 - (d*Cos[c/2]*Sqrt[a + a*Cos[c + d*x]]
*Sec[c/2 + (d*x)/2]*SinIntegral[(d*x)/2])/2
```

Rubi [A] time = 0.133418, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3319, 3297, 3303, 3299, 3302}

$$-\frac{1}{2}d\sin\left(\frac{c}{2}\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}-\frac{1}{2}d\cos\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]/x^2,x]
```

```
[Out] -(Sqrt[a + a*Cos[c + d*x]]/x) - (d*Sqrt[a + a*Cos[c + d*x]]*CosIntegral[(d*
x)/2]*Sec[c/2 + (d*x)/2]*Sin[c/2])/2 - (d*Cos[c/2]*Sqrt[a + a*Cos[c + d*x]]
*Sec[c/2 + (d*x)/2]*SinIntegral[(d*x)/2])/2
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^2}\,dx = \left(\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int \frac{\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)}{x^2}\,dx$$

$$= -\frac{\sqrt{a+a\cos(c+dx)}}{x} - \frac{1}{2}\left(d\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int \frac{\sin\left(\frac{c}{2}+\frac{d}{2}\right.}{x}$$

$$= -\frac{\sqrt{a+a\cos(c+dx)}}{x} - \frac{1}{2}\left(d\cos\left(\frac{c}{2}\right)\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int \frac{\mathrm{si}}{-}$$

$$= -\frac{\sqrt{a+a\cos(c+dx)}}{x} - \frac{1}{2}d\sqrt{a+a\cos(c+dx)}\mathrm{Ci}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}\right) - \frac{1}{2}d\cos\left(\frac{c}{2}\right)$$

Mathematica [A] time = 0.162369, size = 75, normalized size = 0.68

$$-\frac{\sqrt{a(\cos(c+dx)+1)}\left(dx\sin\left(\frac{c}{2}\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+dx\cos\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+2\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/x^2,x]
```

```
[Out] -(Sqrt[a*(1 + Cos[c + d*x])]*(2 + d*x*CosIntegral[(d*x)/2]*Sec[(c + d*x)/2]
*Sin[c/2] + d*x*Cos[c/2]*Sec[(c + d*x)/2]*SinIntegral[(d*x)/2]))/(2*x)
```

Maple [F] time = 0.215, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt{a+\cos(dx+c)a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2)/x^2,x)
```

```
[Out] int((a+cos(d*x+c)*a)^(1/2)/x^2,x)
```

Maxima [C] time = 2.65022, size = 267, normalized size = 2.43

$$-\frac{\left(4\left(E_2\left(\frac{1}{2}i\,dx\right)+E_2\left(-\frac{1}{2}i\,dx\right)\right)\cos\left(\frac{1}{2}c\right)^3+4\left(E_2\left(\frac{1}{2}i\,dx\right)+E_2\left(-\frac{1}{2}i\,dx\right)\right)\cos\left(\frac{1}{2}c\right)\sin\left(\frac{1}{2}c\right)^2-\left(4i\,E_2\left(\frac{1}{2}i\,dx\right)-4i\,E_2\left(\right.\right.}{8\left(\left(\sqrt{2}\cos\left(\frac{1}{2}c\right)^2+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/8*(4*(exp_integral_e(2, 1/2*I*d*x) + exp_integral_e(2, -1/2*I*d*x))*cos(
1/2*c)^3 + 4*(exp_integral_e(2, 1/2*I*d*x) + exp_integral_e(2, -1/2*I*d*x))
*cos(1/2*c)*sin(1/2*c)^2 - (4*I*exp_integral_e(2, 1/2*I*d*x) - 4*I*exp_inte
gral_e(2, -1/2*I*d*x))*sin(1/2*c)^3 + 4*(exp_integral_e(2, 1/2*I*d*x) + exp
_integral_e(2, -1/2*I*d*x))*cos(1/2*c) - ((4*I*exp_integral_e(2, 1/2*I*d*x)
 - 4*I*exp_integral_e(2, -1/2*I*d*x))*cos(1/2*c)^2 + 4*I*exp_integral_e(2,
1/2*I*d*x) - 4*I*exp_integral_e(2, -1/2*I*d*x))*sin(1/2*c))*sqrt(a)*d/((sqr
t(2)*cos(1/2*c)^2 + sqrt(2)*sin(1/2*c)^2)*(d*x + c) - (sqrt(2)*cos(1/2*c)^2
 + sqrt(2)*sin(1/2*c)^2)*c)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\cos\left(dx+c\right)+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)/x^2, x)
```

3.149 $\int \frac{\sqrt{a+a\cos(c+dx)}}{x^3}\,dx$

Optimal. Leaf size=151

$$-\frac{1}{8}d^2\cos\left(\frac{c}{2}\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}+\frac{1}{8}d^2\sin\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)}$$

```
[Out] -Sqrt[a + a*Cos[c + d*x]]/(2*x^2) - (d^2*Cos[c/2]*Sqrt[a + a*Cos[c + d*x]]*
CosIntegral[(d*x)/2]*Sec[c/2 + (d*x)/2])/8 + (d^2*Sqrt[a + a*Cos[c + d*x]]*
Sec[c/2 + (d*x)/2]*Sin[c/2]*SinIntegral[(d*x)/2])/8 + (d*Sqrt[a + a*Cos[c +
 d*x]]*Tan[c/2 + (d*x)/2])/(4*x)
```

Rubi [A] time = 0.161713, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3319, 3297, 3303, 3299, 3302}

$$-\frac{1}{8}d^2\cos\left(\frac{c}{2}\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}+\frac{1}{8}d^2\sin\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]/x^3,x]
```

```
[Out] -Sqrt[a + a*Cos[c + d*x]]/(2*x^2) - (d^2*Cos[c/2]*Sqrt[a + a*Cos[c + d*x]]*
CosIntegral[(d*x)/2]*Sec[c/2 + (d*x)/2])/8 + (d^2*Sqrt[a + a*Cos[c + d*x]]*
Sec[c/2 + (d*x)/2]*Sin[c/2]*SinIntegral[(d*x)/2])/8 + (d*Sqrt[a + a*Cos[c +
 d*x]]*Tan[c/2 + (d*x)/2])/(4*x)
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^3}\,dx = \left(\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int \frac{\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)}{x^3}\,dx$$

$$= -\frac{\sqrt{a+a\cos(c+dx)}}{2x^2} - \frac{1}{4}\left(d\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int \frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{x^2}$$

$$= -\frac{\sqrt{a+a\cos(c+dx)}}{2x^2} + \frac{d\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4x} - \frac{1}{8}\left(d^2\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\right.\right.$$

$$= -\frac{\sqrt{a+a\cos(c+dx)}}{2x^2} + \frac{d\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4x} - \frac{1}{8}\left(d^2\cos\left(\frac{c}{2}\right)\sqrt{a+a\cos(c+dx}\right.$$

$$= -\frac{\sqrt{a+a\cos(c+dx)}}{2x^2} - \frac{1}{8}d^2\cos\left(\frac{c}{2}\right)\sqrt{a+a\cos(c+dx)}\text{Ci}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right) + \frac{1}{8}d^2\sqrt{a+a\,c}$$

Mathematica [A] time = 0.252596, size = 98, normalized size = 0.65

$$\frac{\sqrt{a(\cos(c+dx)+1)}\left(-d^2x^2\cos\left(\frac{c}{2}\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+d^2x^2\sin\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+2dx\tan\left(\frac{1}{2}\right.\right.}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/x^3,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(-4 - d^2*x^2*Cos[c/2]*CosIntegral[(d*x)/2]*Sec
[(c + d*x)/2] + d^2*x^2*Sec[(c + d*x)/2]*Sin[c/2]*SinIntegral[(d*x)/2] + 2*
d*x*Tan[(c + d*x)/2]))/(8*x^2)
```

Maple [F] time = 0.223, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\sqrt{a+\cos(dx+c)\,a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2)/x^3,x)
```

```
[Out] int((a+cos(d*x+c)*a)^(1/2)/x^3,x)
```

Maxima [C] time = 2.51852, size = 313, normalized size = 2.07

$$-\frac{\left(4\left(E_3\left(\frac{1}{2}i\,dx\right)+E_3\left(-\frac{1}{2}i\,dx\right)\right)\cos\left(\frac{1}{2}c\right)^3+4\left(E_3\left(\frac{1}{2}i\,dx\right)+E_3\left(-\frac{1}{2}i\,dx\right)\right)\cos\left(\frac{1}{2}c\right)\sin\left(\frac{1}{2}c\right)^2-\left(4i\,E_3\left(\frac{1}{2}i\,dx\right)-4i\,E_3\left(-\frac{1}{2}i\right.\right.}{8\left(\left(\sqrt{2}\cos\left(\frac{1}{2}c\right)^2+\sqrt{2}\sin\left(\frac{1}{2}c\right)^2\right)(dx+c)^2-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*(4*(exp_integral_e(3, 1/2*I*d*x) + exp_integral_e(3, -1/2*I*d*x))*cos(
1/2*c)^3 + 4*(exp_integral_e(3, 1/2*I*d*x) + exp_integral_e(3, -1/2*I*d*x))
*cos(1/2*c)*sin(1/2*c)^2 - (4*I*exp_integral_e(3, 1/2*I*d*x) - 4*I*exp_inte
gral_e(3, -1/2*I*d*x))*sin(1/2*c)^3 + 4*(exp_integral_e(3, 1/2*I*d*x) + exp
_integral_e(3, -1/2*I*d*x))*cos(1/2*c) - ((4*I*exp_integral_e(3, 1/2*I*d*x)
 - 4*I*exp_integral_e(3, -1/2*I*d*x))*cos(1/2*c)^2 + 4*I*exp_integral_e(3,
1/2*I*d*x) - 4*I*exp_integral_e(3, -1/2*I*d*x))*sin(1/2*c))*sqrt(a)*d^2/((s
qrt(2)*cos(1/2*c)^2 + sqrt(2)*sin(1/2*c)^2)*(d*x + c)^2 - 2*(sqrt(2)*cos(1/
2*c)^2 + sqrt(2)*sin(1/2*c)^2)*(d*x + c)*c + (sqrt(2)*cos(1/2*c)^2 + sqrt(2
)*sin(1/2*c)^2)*c^2)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\cos\left(dx+c\right)+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)/x^3, x)
```

3.150 $\int x^3\sqrt{a+a\cos(x)}\,dx$

Optimal. Leaf size=68

$$12x^2\sqrt{a\cos(x)+a}+2x^3\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}-96\sqrt{a\cos(x)+a}-48x\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

```
[Out] -96*Sqrt[a + a*Cos[x]] + 12*x^2*Sqrt[a + a*Cos[x]] - 48*x*Sqrt[a + a*Cos[x]
]*Tan[x/2] + 2*x^3*Sqrt[a + a*Cos[x]]*Tan[x/2]
```

Rubi [A] time = 0.110838, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3319, 3296, 2638}

$$12x^2\sqrt{a\cos(x)+a}+2x^3\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}-96\sqrt{a\cos(x)+a}-48x\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + a*Cos[x]],x]
```

```
[Out] -96*Sqrt[a + a*Cos[x]] + 12*x^2*Sqrt[a + a*Cos[x]] - 48*x*Sqrt[a + a*Cos[x]
]*Tan[x/2] + 2*x^3*Sqrt[a + a*Cos[x]]*Tan[x/2]
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{a+a\cos(x)}\,dx &= \left(\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x^3\cos\left(\frac{x}{2}\right)\,dx\\
&= 2x^3\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)-\left(6\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x^2\sin\left(\frac{x}{2}\right)\,dx\\
&= 12x^2\sqrt{a+a\cos(x)}+2x^3\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)-\left(24\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x\cos\left(\frac{x}{2}\right)\,dx\\
&= 12x^2\sqrt{a+a\cos(x)}-48x\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)+2x^3\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)+\left(48\sqrt{a+a\cos(x)}\right.\\
&= -96\sqrt{a+a\cos(x)}+12x^2\sqrt{a+a\cos(x)}-48x\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)+2x^3\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0550919, size = 33, normalized size = 0.49

$$2\left(6\left(x^2-8\right)+x\left(x^2-24\right)\tan\left(\frac{x}{2}\right)\right)\sqrt{a(\cos(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + a*Cos[x]],x]
```

```
[Out] 2*Sqrt[a*(1 + Cos[x])]*(6*(-8 + x^2) + x*(-24 + x^2)*Tan[x/2])
```

Maple [C] time = 0.214, size = 87, normalized size = 1.3

$$\frac{-i\sqrt{2}\left(6\,ix^2e^{ix}+x^3e^{ix}+6\,ix^2-x^3-48\,ie^{ix}-24\,xe^{ix}-48\,i+24\,x\right)}{e^{ix}+1}\sqrt{a\left(e^{ix}+1\right)^2e^{-ix}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+a*cos(x))^(1/2),x)
```

```
[Out] -I*2^(1/2)*(a*(exp(I*x)+1)^2*exp(-I*x))^(1/2)/(exp(I*x)+1)*(6*I*x^2*exp(I*x
)+x^3*exp(I*x)+6*I*x^2-x^3-48*I*exp(I*x)-24*x*exp(I*x)-48*I+24*x)
```

Maxima [A] time = 2.33819, size = 65, normalized size = 0.96

$$2\left(\sqrt{2}x^3\sin\left(\frac{1}{2}x\right)+6\sqrt{2}x^2\cos\left(\frac{1}{2}x\right)-24\sqrt{2}x\sin\left(\frac{1}{2}x\right)-48\sqrt{2}\cos\left(\frac{1}{2}x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(2)*x^3*sin(1/2*x) + 6*sqrt(2)*x^2*cos(1/2*x) - 24*sqrt(2)*x*sin(1/2
*x) - 48*sqrt(2)*cos(1/2*x))*sqrt(a)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{a\left(\cos\left(x\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+a*cos(x))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*(cos(x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cos(x) + a}x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(x) + a)*x^3, x)
```

3.151 $\int x^2\sqrt{a+a\cos(x)}\,dx$

Optimal. Leaf size=53

$$2x^2\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+8x\sqrt{a\cos(x)+a}-16\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

```
[Out] 8*x*Sqrt[a + a*Cos[x]] - 16*Sqrt[a + a*Cos[x]]*Tan[x/2] + 2*x^2*Sqrt[a + a*
Cos[x]]*Tan[x/2]
```

Rubi [A] time = 0.0959995, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3319, 3296, 2637}

$$2x^2\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+8x\sqrt{a\cos(x)+a}-16\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + a*Cos[x]],x]
```

```
[Out] 8*x*Sqrt[a + a*Cos[x]] - 16*Sqrt[a + a*Cos[x]]*Tan[x/2] + 2*x^2*Sqrt[a + a*
Cos[x]]*Tan[x/2]
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^2\sqrt{a+a\cos(x)}\,dx &= \left(\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x^2\cos\left(\frac{x}{2}\right)\,dx\\
&= 2x^2\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)-\left(4\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x\sin\left(\frac{x}{2}\right)\,dx\\
&= 8x\sqrt{a+a\cos(x)}+2x^2\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)-\left(8\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int\cos\left(\frac{x}{2}\right)\,dx\\
&= 8x\sqrt{a+a\cos(x)}-16\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)+2x^2\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0440321, size = 29, normalized size = 0.55

$$8\left(\frac{1}{4}\left(x^2-8\right)\tan\left(\frac{x}{2}\right)+x\right)\sqrt{a(\cos(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + a*Cos[x]],x]
```

```
[Out] 8*Sqrt[a*(1 + Cos[x])]*(x + ((-8 + x^2)*Tan[x/2])/4)
```

Maple [C] time = 0.155, size = 70, normalized size = 1.3

$$\frac{-i\sqrt{2}\left(4\,ix\mathrm{e}^{ix}+x^{2}\mathrm{e}^{ix}+4\,ix-x^{2}-8\,\mathrm{e}^{ix}+8\right)}{\mathrm{e}^{ix}+1}\sqrt{a\left(\mathrm{e}^{ix}+1\right)^{2}\mathrm{e}^{-ix}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+a*cos(x))^(1/2),x)
```

```
[Out] -I*2^(1/2)*(a*(exp(I*x)+1)^2*exp(-I*x))^(1/2)/(exp(I*x)+1)*(4*I*x*exp(I*x)+
x^2*exp(I*x)+4*I*x-x^2-8*exp(I*x)+8)
```

Maxima [A] time = 2.17184, size = 49, normalized size = 0.92

$$2\left(\sqrt{2}x^{2}\sin\left(\frac{1}{2}\,x\right)+4\,\sqrt{2}x\cos\left(\frac{1}{2}\,x\right)-8\,\sqrt{2}\sin\left(\frac{1}{2}\,x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(2)*x^2*sin(1/2*x) + 4*sqrt(2)*x*cos(1/2*x) - 8*sqrt(2)*sin(1/2*x))*
sqrt(a)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{2}\sqrt{a\left(\cos\left(x\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+a*cos(x))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*(cos(x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cos(x) + a}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(x) + a)*x^2, x)
```

3.152 $\int x\sqrt{a + a\cos(x)}\,dx$

Optimal. Leaf size=32

$$4\sqrt{a\cos(x)+a}+2x\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

```
[Out] 4*Sqrt[a + a*Cos[x]] + 2*x*Sqrt[a + a*Cos[x]]*Tan[x/2]
```

Rubi [A] time = 0.0500326, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3319, 3296, 2638}

$$4\sqrt{a\cos(x)+a}+2x\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + a*Cos[x]],x]
```

```
[Out] 4*Sqrt[a + a*Cos[x]] + 2*x*Sqrt[a + a*Cos[x]]*Tan[x/2]
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+a\cos(x)}\,dx &= \left(\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x\cos\left(\frac{x}{2}\right)\,dx\\ &= 2x\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)-\left(2\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \sin\left(\frac{x}{2}\right)\,dx\\ &= 4\sqrt{a+a\cos(x)}+2x\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.021216, size = 22, normalized size = 0.69

$$2\left(x\tan\left(\frac{x}{2}\right)+2\right)\sqrt{a(\cos(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + a*Cos[x]],x]

[Out] 2*Sqrt[a*(1 + Cos[x])]*(2 + x*Tan[x/2])
```

Maple [C] time = 0.154, size = 55, normalized size = 1.7

$$\frac{-i\sqrt{2}\left(2\,i\mathrm{e}^{ix}+x\mathrm{e}^{ix}+2\,i-x\right)}{\mathrm{e}^{ix}+1}\sqrt{a\left(\mathrm{e}^{ix}+1\right)^{2}\mathrm{e}^{-ix}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+a*cos(x))^(1/2),x)

[Out] -I*2^(1/2)*(a*(exp(I*x)+1)^2*exp(-I*x))^(1/2)/(exp(I*x)+1)*(2*I*exp(I*x)+x*
exp(I*x)+2*I-x)
```

Maxima [A] time = 2.30039, size = 32, normalized size = 1.

$$2\left(\sqrt{2}x\sin\left(\frac{1}{2}\,x\right)+2\,\sqrt{2}\cos\left(\frac{1}{2}\,x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(x))^(1/2),x, algorithm="maxima")

[Out] 2*(sqrt(2)*x*sin(1/2*x) + 2*sqrt(2)*cos(1/2*x))*sqrt(a)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(x))^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{a\left(\cos\left(x\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(x))**(1/2),x)

[Out] Integral(x*sqrt(a*(cos(x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cos(x) + a}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(x) + a)*x, x)
```

3.153 $\int \sqrt{a+a\cos(x)}\,dx$

Optimal. Leaf size=15

$$\frac{2a\sin(x)}{\sqrt{a\cos(x)+a}}$$

```
[Out] (2*a*Sin[x])/Sqrt[a + a*Cos[x]]
```

Rubi [A] time = 0.0111368, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2646}

$$\frac{2a\sin(x)}{\sqrt{a\cos(x)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[x]],x]

[Out] (2*a*Sin[x])/Sqrt[a + a*Cos[x]]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(x)}\,dx=\frac{2a\sin(x)}{\sqrt{a+a\cos(x)}}$$

Mathematica [A] time = 0.0082106, size = 18, normalized size = 1.2

$$2\tan\left(\frac{x}{2}\right)\sqrt{a(\cos(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[x]],x]

[Out] 2*Sqrt[a*(1 + Cos[x])]*Tan[x/2]
```

Maple [A] time = 0.373, size = 25, normalized size = 1.7

$$2\,\frac{a\cos(x/2)\sin(x/2)\sqrt{2}}{\sqrt{(\cos(x/2))^2 a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^(1/2),x)
```

```
[Out] 2*a*cos(1/2*x)*sin(1/2*x)*2^(1/2)/(cos(1/2*x)^2*a)^(1/2)
```

Maxima [A] time = 2.38141, size = 16, normalized size = 1.07

$$2\sqrt{2}\sqrt{a}\sin\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*sqrt(a)*sin(1/2*x)
```

Fricas [A] time = 1.56093, size = 57, normalized size = 3.8

$$\frac{2\sqrt{a\cos\left(x\right)+a}\sin\left(x\right)}{\cos\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(a*cos(x) + a)*sin(x)/(cos(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\cos\left(x\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(1/2),x)
```

```
[Out] Integral(sqrt(a*cos(x) + a), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\cos\left(x\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(x) + a), x)
```

3.154 $\int \frac{\sqrt{a+a\cos(x)}}{x}\,dx$

Optimal. Leaf size=23

$$\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

```
[Out] Sqrt[a + a*Cos[x]]*CosIntegral[x/2]*Sec[x/2]
```

Rubi [A] time = 0.087079, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3319, 3302}

$$\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[x]]/x,x]
```

```
[Out] Sqrt[a + a*Cos[x]]*CosIntegral[x/2]*Sec[x/2]
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\cos(x)}}{x}\,dx &= \left(\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos\left(\frac{x}{2}\right)}{x}\,dx\\ &= \sqrt{a+a\cos(x)}\,\text{Ci}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.0065143, size = 23, normalized size = 1.

$$\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a(\cos(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[x]]/x,x]
```

```
[Out] Sqrt[a*(1 + Cos[x])]*CosIntegral[x/2]*Sec[x/2]
```

Maple [F] time = 0.197, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{a+a\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^(1/2)/x,x)
```

```
[Out] int((a+a*cos(x))^(1/2)/x,x)
```

Maxima [C] time = 2.53467, size = 23, normalized size = 1.

$$\frac{1}{2}\sqrt{2}\sqrt{a}\left(\mathrm{Ei}\left(\frac{1}{2}ix\right)+\mathrm{Ei}\left(-\frac{1}{2}ix\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*sqrt(a)*(Ei(1/2*I*x) + Ei(-1/2*I*x))
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a(\cos(x)+1)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a*(cos(x) + 1))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\cos(x)+a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(x) + a)/x, x)
```

3.155 $\int \frac{\sqrt{a+a\cos(x)}}{x^2}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{2}\text{Si}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}-\frac{\sqrt{a\cos(x)+a}}{x}$$

```
[Out] -(Sqrt[a + a*Cos[x]]/x) - (Sqrt[a + a*Cos[x]]*Sec[x/2]*SinIntegral[x/2])/2
```

Rubi [A] time = 0.0906291, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3319, 3297, 3299}

$$-\frac{1}{2}\text{Si}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}-\frac{\sqrt{a\cos(x)+a}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[x]]/x^2,x]
```

```
[Out] -(Sqrt[a + a*Cos[x]]/x) - (Sqrt[a + a*Cos[x]]*Sec[x/2]*SinIntegral[x/2])/2
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\cos(x)}}{x^2}\,dx &= \left(\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos\left(\frac{x}{2}\right)}{x^2}\,dx\\ &= -\frac{\sqrt{a+a\cos(x)}}{x}-\frac{1}{2}\left(\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\sin\left(\frac{x}{2}\right)}{x}\,dx\\ &= -\frac{\sqrt{a+a\cos(x)}}{x}-\frac{1}{2}\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\text{Si}\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.057508, size = 33, normalized size = 0.79

$$-\frac{\sqrt{a(\cos(x)+1)}\left(x\text{Si}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)+2\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[x]]/x^2,x]
```

```
[Out] -(Sqrt[a*(1 + Cos[x])]*(2 + x*Sec[x/2]*SinIntegral[x/2]))/(2*x)
```

Maple [F] time = 0.149, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt{a + a\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^(1/2)/x^2,x)
```

```
[Out] int((a+a*cos(x))^(1/2)/x^2,x)
```

Maxima [C] time = 2.18569, size = 31, normalized size = 0.74

$$-\frac{1}{4}\sqrt{2}\sqrt{a}\left(i\Gamma\left(-1,\frac{1}{2}ix\right) - i\Gamma\left(-1,-\frac{1}{2}ix\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*sqrt(a)*(I*gamma(-1, 1/2*I*x) - I*gamma(-1, -1/2*I*x))
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a(\cos(x) + 1)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a*(cos(x) + 1))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\cos(x) + a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(x) + a)/x^2, x)
```

3.156 $\int \frac{\sqrt{a+a\cos(x)}}{x^3}\,dx$

Optimal. Leaf size=67

$$-\frac{1}{8}\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}-\frac{\sqrt{a\cos(x)+a}}{2x^2}+\frac{\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}}{4x}$$

```
[Out] -Sqrt[a + a*Cos[x]]/(2*x^2) - (Sqrt[a + a*Cos[x]]*CosIntegral[x/2]*Sec[x/2]
)/8 + (Sqrt[a + a*Cos[x]]*Tan[x/2])/(4*x)
```

Rubi [A] time = 0.106145, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3319, 3297, 3302}

$$-\frac{1}{8}\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}-\frac{\sqrt{a\cos(x)+a}}{2x^2}+\frac{\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[x]]/x^3,x]
```

```
[Out] -Sqrt[a + a*Cos[x]]/(2*x^2) - (Sqrt[a + a*Cos[x]]*CosIntegral[x/2]*Sec[x/2]
)/8 + (Sqrt[a + a*Cos[x]]*Tan[x/2])/(4*x)
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(x)}}{x^3}\,dx = \left(\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos\left(\frac{x}{2}\right)}{x^3}\,dx$$
$$= -\frac{\sqrt{a+a\cos(x)}}{2x^2} - \frac{1}{4}\left(\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\sin\left(\frac{x}{2}\right)}{x^2}\,dx$$
$$= -\frac{\sqrt{a+a\cos(x)}}{2x^2} + \frac{\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)}{4x} - \frac{1}{8}\left(\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos\left(\frac{x}{2}\right)}{x}\,dx$$
$$= -\frac{\sqrt{a+a\cos(x)}}{2x^2} - \frac{1}{8}\sqrt{a+a\cos(x)}\,\mathrm{Ci}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right) + \frac{\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)}{4x}$$

Mathematica [A] time = 0.0749154, size = 44, normalized size = 0.66

$$-\frac{\sqrt{a(\cos(x)+1)}\left(x^2\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)-2x\tan\left(\frac{x}{2}\right)+4\right)}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[x]]/x^3,x]
```

```
[Out] -(Sqrt[a*(1 + Cos[x])]*(4 + x^2*CosIntegral[x/2]*Sec[x/2] - 2*x*Tan[x/2]))/
(8*x^2)
```

Maple [F] time = 0.152, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\sqrt{a+a\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^(1/2)/x^3,x)
```

```
[Out] int((a+a*cos(x))^(1/2)/x^3,x)
```

Maxima [C] time = 2.13404, size = 26, normalized size = 0.39

$$\frac{1}{8}\sqrt{2}\sqrt{a}\left(\Gamma\left(-2,\frac{1}{2}ix\right)+\Gamma\left(-2,-\frac{1}{2}ix\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*sqrt(a)*(gamma(-2, 1/2*I*x) + gamma(-2, -1/2*I*x))
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a(\cos(x)+1)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a*(cos(x) + 1))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\cos(x)+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(x) + a)/x^3, x)
```

3.157 $\int x^3\sqrt{a-a\cos(x)}\,dx$

Optimal. Leaf size=72

$$12x^2\sqrt{a-a\cos(x)}-2x^3\cot\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}-96\sqrt{a-a\cos(x)}+48x\cot\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}$$

```
[Out] -96*Sqrt[a - a*Cos[x]] + 12*x^2*Sqrt[a - a*Cos[x]] + 48*x*Sqrt[a - a*Cos[x]
]*Cot[x/2] - 2*x^3*Sqrt[a - a*Cos[x]]*Cot[x/2]
```

Rubi [A] time = 0.114764, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3319, 3296, 2637}

$$12x^2\sqrt{a-a\cos(x)}-2x^3\cot\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}-96\sqrt{a-a\cos(x)}+48x\cot\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a - a*Cos[x]],x]
```

```
[Out] -96*Sqrt[a - a*Cos[x]] + 12*x^2*Sqrt[a - a*Cos[x]] + 48*x*Sqrt[a - a*Cos[x]
]*Cot[x/2] - 2*x^3*Sqrt[a - a*Cos[x]]*Cot[x/2]
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{a-a\cos(x)}\,dx &= \left(\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int x^3\sin\left(\frac{x}{2}\right)dx\\
&= -2x^3\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)+\left(6\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int x^2\cos\left(\frac{x}{2}\right)dx\\
&= 12x^2\sqrt{a-a\cos(x)}-2x^3\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)-\left(24\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int x\sin\left(\frac{x}{2}\right)dx\\
&= 12x^2\sqrt{a-a\cos(x)}+48x\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)-2x^3\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)-\left(48\sqrt{a-a\cos(x)}\,c\right.\\
&= -96\sqrt{a-a\cos(x)}+12x^2\sqrt{a-a\cos(x)}+48x\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)-2x^3\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] time = 0.051273, size = 34, normalized size = 0.47

$$-2\left(x\left(x^2-24\right)\cot\left(\frac{x}{2}\right)-6\left(x^2-8\right)\right)\sqrt{a-a\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a - a*Cos[x]],x]
```

```
[Out] -2*Sqrt[a - a*Cos[x]]*(-6*(-8 + x^2) + x*(-24 + x^2)*Cot[x/2])
```

Maple [C] time = 0.163, size = 86, normalized size = 1.2

$$\frac{-i\sqrt{2}\left(6\,ix^2\mathrm{e}^{ix}+x^3\mathrm{e}^{ix}-6\,ix^2+x^3-48\,i\mathrm{e}^{ix}-24\,x\mathrm{e}^{ix}+48\,i-24\,x\right)}{\mathrm{e}^{ix}-1}\sqrt{-a\left(\mathrm{e}^{ix}-1\right)^2\mathrm{e}^{-ix}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a-a*cos(x))^(1/2),x)
```

```
[Out] -I*2^(1/2)*(-a*(exp(I*x)-1)^2*exp(-I*x))^(1/2)/(exp(I*x)-1)*(6*I*x^2*exp(I*
x)+x^3*exp(I*x)-6*I*x^2+x^3-48*I*exp(I*x)-24*x*exp(I*x)+48*I-24*x)
```

Maxima [B] time = 1.97059, size = 174, normalized size = 2.42

$$-\left(\left(6\sqrt{2}x^2-6\left(\sqrt{2}x^2-8\sqrt{2}\right)\cos(x)-\left(\sqrt{2}x^3-24\sqrt{2}x\right)\sin(x)-48\sqrt{2}\right)\cos\left(\frac{1}{2}\pi+\frac{1}{2}\arctan(\sin(x),\cos(x))\right)+\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -((6*sqrt(2)*x^2 - 6*(sqrt(2)*x^2 - 8*sqrt(2))*cos(x) - (sqrt(2)*x^3 - 24*s
qrt(2)*x)*sin(x) - 48*sqrt(2))*cos(1/2*pi + 1/2*arctan2(sin(x), cos(x))) +
(sqrt(2)*x^3 + (sqrt(2)*x^3 - 24*sqrt(2)*x)*cos(x) - 6*(sqrt(2)*x^2 - 8*sqr
t(2))*sin(x) - 24*sqrt(2)*x)*sin(1/2*pi + 1/2*arctan2(sin(x), cos(x))))*sqr
t(a)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{-a\left(\cos(x)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a-a*cos(x))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-a*(cos(x) - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a\cos(x)+a}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*cos(x) + a)*x^3, x)
```

3.158 $\int x^2\sqrt{a-a\cos(x)}\,dx$

Optimal. Leaf size=56

$$-2x^2\cot\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}+8x\sqrt{a-a\cos(x)}+16\cot\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}$$

```
[Out] 8*x*Sqrt[a - a*Cos[x]] + 16*Sqrt[a - a*Cos[x]]*Cot[x/2] - 2*x^2*Sqrt[a - a*
Cos[x]]*Cot[x/2]
```

Rubi [A] time = 0.0983111, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3319, 3296, 2638}

$$-2x^2\cot\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}+8x\sqrt{a-a\cos(x)}+16\cot\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a - a*Cos[x]],x]
```

```
[Out] 8*x*Sqrt[a - a*Cos[x]] + 16*Sqrt[a - a*Cos[x]]*Cot[x/2] - 2*x^2*Sqrt[a - a*
Cos[x]]*Cot[x/2]
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a-a\cos(x)}\,dx &= \left(\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int x^2\sin\left(\frac{x}{2}\right)\,dx\\ &= -2x^2\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)+\left(4\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int x\cos\left(\frac{x}{2}\right)\,dx\\ &= 8x\sqrt{a-a\cos(x)}-2x^2\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)-\left(8\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int\sin\left(\frac{x}{2}\right)\,dx\\ &= 8x\sqrt{a-a\cos(x)}+16\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)-2x^2\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.0423441, size = 30, normalized size = 0.54

$$8\left(x-\frac{1}{4}\left(x^2-8\right)\cot\left(\frac{x}{2}\right)\right)\sqrt{a-a\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a - a*Cos[x]],x]
```

```
[Out] 8*Sqrt[a - a*Cos[x]]*(x - ((-8 + x^2)*Cot[x/2])/4)
```

Maple [C] time = 0.096, size = 69, normalized size = 1.2

$$\frac{-i\sqrt{2}\left(4\,ixe^{ix}+x^2e^{ix}-4\,ix+x^2-8\,e^{ix}-8\right)}{e^{ix}-1}\sqrt{-a\left(e^{ix}-1\right)^2e^{-ix}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a-a*cos(x))^(1/2),x)
```

```
[Out] -I*2^(1/2)*(-a*(exp(I*x)-1)^2*exp(-I*x))^(1/2)/(exp(I*x)-1)*(4*I*x*exp(I*x)
+x^2*exp(I*x)-4*I*x+x^2-8*exp(I*x)-8)
```

Maxima [B] time = 1.98723, size = 135, normalized size = 2.41

$$\left(\left(4\,\sqrt{2}x\cos\left(x\right)+\left(\sqrt{2}x^2-8\,\sqrt{2}\right)\sin\left(x\right)-4\,\sqrt{2}x\right)\cos\left(\frac{1}{2}\,\pi+\frac{1}{2}\,\arctan\left(\sin\left(x\right),\cos\left(x\right)\right)\right)-\left(\sqrt{2}x^2-4\,\sqrt{2}x\sin\left(x\right)+\left(\sqrt{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] ((4*sqrt(2)*x*cos(x) + (sqrt(2)*x^2 - 8*sqrt(2))*sin(x) - 4*sqrt(2)*x)*cos(
1/2*pi + 1/2*arctan2(sin(x), cos(x))) - (sqrt(2)*x^2 - 4*sqrt(2)*x*sin(x) +
 (sqrt(2)*x^2 - 8*sqrt(2))*cos(x) - 8*sqrt(2))*sin(1/2*pi + 1/2*arctan2(sin
(x), cos(x))))*sqrt(a)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{-a\left(\cos\left(x\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a-a*cos(x))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-a*(cos(x) - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a\cos(x) + a}x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*cos(x) + a)*x^2, x)
```

3.159 $\int x\sqrt{a - a\cos(x)}\,dx$

Optimal. Leaf size=34

$$4\sqrt{a - a\cos(x)} - 2x\cot\left(\frac{x}{2}\right)\sqrt{a - a\cos(x)}$$

```
[Out] 4*Sqrt[a - a*Cos[x]] - 2*x*Sqrt[a - a*Cos[x]]*Cot[x/2]
```

Rubi [A] time = 0.0507503, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3319, 3296, 2637}

$$4\sqrt{a - a\cos(x)} - 2x\cot\left(\frac{x}{2}\right)\sqrt{a - a\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a - a*Cos[x]],x]

[Out] 4*Sqrt[a - a*Cos[x]] - 2*x*Sqrt[a - a*Cos[x]]*Cot[x/2]
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a - a\cos(x)}\,dx &= \left(\sqrt{a - a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int x\sin\left(\frac{x}{2}\right)\,dx\\ &= -2x\sqrt{a - a\cos(x)}\cot\left(\frac{x}{2}\right) + \left(2\sqrt{a - a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int \cos\left(\frac{x}{2}\right)\,dx\\ &= 4\sqrt{a - a\cos(x)} - 2x\sqrt{a - a\cos(x)}\cot\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.0242597, size = 23, normalized size = 0.68

$$-2\left(x\cot\left(\frac{x}{2}\right) - 2\right)\sqrt{a - a\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a - a*Cos[x]],x]
```

```
[Out] -2*Sqrt[a - a*Cos[x]]*(-2 + x*Cot[x/2])
```

Maple [C] time = 0.095, size = 54, normalized size = 1.6

$$\frac{-i\sqrt{2}\left(2\,i\mathrm{e}^{ix} + x\mathrm{e}^{ix} - 2\,i + x\right)}{\mathrm{e}^{ix} - 1}\sqrt{-a\left(\mathrm{e}^{ix} - 1\right)^{2}\mathrm{e}^{-ix}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a-a*cos(x))^(1/2),x)
```

```
[Out] -I*2^(1/2)*(-a*(exp(I*x)-1)^2*exp(-I*x))^(1/2)/(exp(I*x)-1)*(2*I*exp(I*x)+x
*exp(I*x)-2*I+x)
```

Maxima [B] time = 1.82291, size = 97, normalized size = 2.85

$$\left(\left(\sqrt{2}x\sin\left(x\right) + 2\,\sqrt{2}\cos\left(x\right) - 2\,\sqrt{2}\right)\cos\left(\frac{1}{2}\,\pi + \frac{1}{2}\,\arctan\left(\sin\left(x\right),\cos\left(x\right)\right)\right) - \left(\sqrt{2}x\cos\left(x\right) + \sqrt{2}x - 2\,\sqrt{2}\sin\left(x\right)\right)\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] ((sqrt(2)*x*sin(x) + 2*sqrt(2)*cos(x) - 2*sqrt(2))*cos(1/2*pi + 1/2*arctan2
(sin(x), cos(x))) - (sqrt(2)*x*cos(x) + sqrt(2)*x - 2*sqrt(2)*sin(x))*sin(1
/2*pi + 1/2*arctan2(sin(x), cos(x))))*sqrt(a)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{-a\left(\cos\left(x\right) - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*cos(x))**(1/2),x)
```

```
[Out] Integral(x*sqrt(-a*(cos(x) - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a\cos(x) + a}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*cos(x) + a)*x, x)
```

3.160 $\int \sqrt{a - a\cos(x)}\,dx$

Optimal. Leaf size=16

$$-\frac{2a\sin(x)}{\sqrt{a - a\cos(x)}}$$

```
[Out] (-2*a*Sin[x])/Sqrt[a - a*Cos[x]]
```

Rubi [A] time = 0.011432, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2646}

$$-\frac{2a\sin(x)}{\sqrt{a - a\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Cos[x]],x]
```

```
[Out] (-2*a*Sin[x])/Sqrt[a - a*Cos[x]]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a - a\cos(x)}\,dx = -\frac{2a\sin(x)}{\sqrt{a - a\cos(x)}}$$

Mathematica [A] time = 0.0068481, size = 19, normalized size = 1.19

$$-2\cot\left(\frac{x}{2}\right)\sqrt{a - a\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Cos[x]],x]
```

```
[Out] -2*Sqrt[a - a*Cos[x]]*Cot[x/2]
```

Maple [A] time = 0.961, size = 25, normalized size = 1.6

$$-2\,\frac{\sin(x/2)\,a\cos(x/2)\,\sqrt{2}}{\sqrt{a\,(\sin(x/2))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cos(x))^(1/2),x)
```

```
[Out] -2*sin(1/2*x)*a*cos(1/2*x)*2^(1/2)/(a*sin(1/2*x)^2)^(1/2)
```

Maxima [A] time = 1.94552, size = 31, normalized size = 1.94

$$-\frac{2\sqrt{2}\sqrt{a}}{\sqrt{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(2)*sqrt(a)/sqrt(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A] time = 1.55202, size = 59, normalized size = 3.69

$$-\frac{2\sqrt{-a\cos(x)+a}(\cos(x)+1)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-a*cos(x) + a)*(cos(x) + 1)/sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a\cos(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*cos(x) + a), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a\cos(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*cos(x) + a), x)
```

3.161 $\int \frac{\sqrt{a-a\cos(x)}}{x}\,dx$

Optimal. Leaf size=24

$$\mathrm{Si}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}$$

```
[Out] Sqrt[a - a*Cos[x]]*Csc[x/2]*SinIntegral[x/2]
```

Rubi [A] time = 0.087634, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3319, 3299}

$$\mathrm{Si}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Cos[x]]/x,x]
```

```
[Out] Sqrt[a - a*Cos[x]]*Csc[x/2]*SinIntegral[x/2]
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a-a\cos(x)}}{x}\,dx &= \left(\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int \frac{\sin\left(\frac{x}{2}\right)}{x}\,dx\\ &= \sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\mathrm{Si}\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.020777, size = 24, normalized size = 1.

$$\mathrm{Si}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Cos[x]]/x,x]
```

```
[Out] Sqrt[a - a*Cos[x]]*Csc[x/2]*SinIntegral[x/2]
```

Maple [F] time = 0.137, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{a-a\cos\left(x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cos(x))^(1/2)/x,x)
```

```
[Out] int((a-a*cos(x))^(1/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\cos\left(x\right)+a}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*cos(x) + a)/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\left(\cos\left(x\right)-1\right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-a*(cos(x) - 1))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\cos\left(x\right)+a}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*cos(x) + a)/x, x)
```

3.162 $\int \frac{\sqrt{a-a\cos(x)}}{x^2}\,dx$

Optimal. Leaf size=44

$$\frac{1}{2}\text{CosIntegral}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}-\frac{\sqrt{a-a\cos(x)}}{x}$$

```
[Out] -(Sqrt[a - a*Cos[x]]/x) + (Sqrt[a - a*Cos[x]]*CosIntegral[x/2]*Csc[x/2])/2
```

Rubi [A] time = 0.0917802, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3319, 3297, 3302}

$$\frac{1}{2}\text{CosIntegral}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}-\frac{\sqrt{a-a\cos(x)}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Cos[x]]/x^2,x]
```

```
[Out] -(Sqrt[a - a*Cos[x]]/x) + (Sqrt[a - a*Cos[x]]*CosIntegral[x/2]*Csc[x/2])/2
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a-a\cos(x)}}{x^2}\,dx &= \left(\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int \frac{\sin\left(\frac{x}{2}\right)}{x^2}\,dx\\
&= -\frac{\sqrt{a-a\cos(x)}}{x}+\frac{1}{2}\left(\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int \frac{\cos\left(\frac{x}{2}\right)}{x}\,dx\\
&= -\frac{\sqrt{a-a\cos(x)}}{x}+\frac{1}{2}\sqrt{a-a\cos(x)}\text{Ci}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0264103, size = 34, normalized size = 0.77

$$\frac{\sqrt{a-a\cos(x)}\left(x\text{CosIntegral}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)-2\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Cos[x]]/x^2,x]
```

```
[Out] (Sqrt[a - a*Cos[x]]*(-2 + x*CosIntegral[x/2]*Csc[x/2]))/(2*x)
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt{a-a\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cos(x))^(1/2)/x^2,x)
```

```
[Out] int((a-a*cos(x))^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\cos(x)+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*cos(x) + a)/x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a(\cos(x)-1)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-a*(cos(x) - 1))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\cos(x)+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*cos(x) + a)/x^2, x)
```

3.163 $\int \frac{\sqrt{a-a\cos(x)}}{x^3}\,dx$

Optimal. Leaf size=70

$$-\frac{1}{8}\text{Si}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}-\frac{\sqrt{a-a\cos(x)}}{2x^2}-\frac{\cot\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}}{4x}$$

```
[Out] -Sqrt[a - a*Cos[x]]/(2*x^2) - (Sqrt[a - a*Cos[x]]*Cot[x/2])/(4*x) - (Sqrt[a
 - a*Cos[x]]*Csc[x/2]*SinIntegral[x/2])/8
```

Rubi [A] time = 0.105748, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3319, 3297, 3299}

$$-\frac{1}{8}\text{Si}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}-\frac{\sqrt{a-a\cos(x)}}{2x^2}-\frac{\cot\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Cos[x]]/x^3,x]
```


```
[Out] -Sqrt[a - a*Cos[x]]/(2*x^2) - (Sqrt[a - a*Cos[x]]*Cot[x/2])/(4*x) - (Sqrt[a
 - a*Cos[x]]*Csc[x/2]*SinIntegral[x/2])/8
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a-a\cos(x)}}{x^3}\,dx &= \left(\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int \frac{\sin\left(\frac{x}{2}\right)}{x^3}\,dx\\
&= -\frac{\sqrt{a-a\cos(x)}}{2x^2}+\frac{1}{4}\left(\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int \frac{\cos\left(\frac{x}{2}\right)}{x^2}\,dx\\
&= -\frac{\sqrt{a-a\cos(x)}}{2x^2}-\frac{\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)}{4x}-\frac{1}{8}\left(\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int \frac{\sin\left(\frac{x}{2}\right)}{x}\,dx\\
&= -\frac{\sqrt{a-a\cos(x)}}{2x^2}-\frac{\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)}{4x}-\frac{1}{8}\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\text{Si}\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0569305, size = 45, normalized size = 0.64

$$-\frac{\sqrt{a-a\cos(x)}\left(x^2\text{Si}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)+2x\cot\left(\frac{x}{2}\right)+4\right)}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Cos[x]]/x^3,x]
```

```
[Out] -(Sqrt[a - a*Cos[x]]*(4 + 2*x*Cot[x/2] + x^2*Csc[x/2]*SinIntegral[x/2]))/(8
*x^2)
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\sqrt{a-a\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cos(x))^(1/2)/x^3,x)
```

```
[Out] int((a-a*cos(x))^(1/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\cos(x)+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*cos(x) + a)/x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a(\cos(x)-1)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-a*(cos(x) - 1))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\cos(x) + a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(x))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*cos(x) + a)/x^3, x)
```

3.164 $\int x^3(a+a\cos(x))^{3/2}\,dx$

Optimal. Leaf size=185

$$\frac{8}{3}ax^2\cos^2\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+16ax^2\sqrt{a\cos(x)+a}+\frac{4}{3}ax^3\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+\frac{8}{3}ax^3\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)}$$

```
[Out] (-1280*a*Sqrt[a + a*Cos[x]])/9 + 16*a*x^2*Sqrt[a + a*Cos[x]] - (64*a*Cos[x/
2]^2*Sqrt[a + a*Cos[x]])/27 + (8*a*x^2*Cos[x/2]^2*Sqrt[a + a*Cos[x]])/3 - (
32*a*x*Cos[x/2]*Sqrt[a + a*Cos[x]]*Sin[x/2])/9 + (4*a*x^3*Cos[x/2]*Sqrt[a +
 a*Cos[x]]*Sin[x/2])/3 - (640*a*x*Sqrt[a + a*Cos[x]]*Tan[x/2])/9 + (8*a*x^3
*Sqrt[a + a*Cos[x]]*Tan[x/2])/3
```

Rubi [A] time = 0.183217, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3319, 3311, 3296, 2638, 3310}

$$\frac{8}{3}ax^2\cos^2\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+16ax^2\sqrt{a\cos(x)+a}+\frac{4}{3}ax^3\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+\frac{8}{3}ax^3\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + a*Cos[x])^(3/2),x]
```


```
[Out] (-1280*a*Sqrt[a + a*Cos[x]])/9 + 16*a*x^2*Sqrt[a + a*Cos[x]] - (64*a*Cos[x/
2]^2*Sqrt[a + a*Cos[x]])/27 + (8*a*x^2*Cos[x/2]^2*Sqrt[a + a*Cos[x]])/3 - (
32*a*x*Cos[x/2]*Sqrt[a + a*Cos[x]]*Sin[x/2])/9 + (4*a*x^3*Cos[x/2]*Sqrt[a +
 a*Cos[x]]*Sin[x/2])/3 - (640*a*x*Sqrt[a + a*Cos[x]]*Tan[x/2])/9 + (8*a*x^3
*Sqrt[a + a*Cos[x]]*Tan[x/2])/3
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int x^3(a+a\cos(x))^{3/2}\,dx = \left(2a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x^3\cos^3\left(\frac{x}{2}\right)dx$$
$$=\frac{8}{3}ax^2\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}+\frac{4}{3}ax^3\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right)+\frac{1}{3}\left(4a\sqrt{a+a\cos(x)}\sec\right.$$
$$=-\frac{64}{27}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}+\frac{8}{3}ax^2\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}-\frac{32}{9}ax\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\,\mathrm{si}$$
$$=16ax^2\sqrt{a+a\cos(x)}-\frac{64}{27}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}+\frac{8}{3}ax^2\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}-\frac{32}{9}ax\cos$$
$$=-\frac{128}{9}a\sqrt{a+a\cos(x)}+16ax^2\sqrt{a+a\cos(x)}-\frac{64}{27}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}+\frac{8}{3}ax^2\cos^2\left(\frac{x}{2}\right)\sqrt{a}$$
$$=-\frac{1280}{9}a\sqrt{a+a\cos(x)}+16ax^2\sqrt{a+a\cos(x)}-\frac{64}{27}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}+\frac{8}{3}ax^2\cos^2\left(\frac{x}{2}\right)\sqrt{}$$

Mathematica [A] time = 0.29763, size = 67, normalized size = 0.36

$$\frac{2}{27}a\sqrt{a(\cos(x)+1)}\left(234x^2+3\left(15x^2-328\right)x\tan\left(\frac{x}{2}\right)+\cos(x)\left(2\left(9x^2-8\right)+3x\left(3x^2-8\right)\tan\left(\frac{x}{2}\right)\right)-1936\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + a*Cos[x])^(3/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cos[x])]*(-1936 + 234*x^2 + 3*x*(-328 + 15*x^2)*Tan[x/2] +
 Cos[x]*(2*(-8 + 9*x^2) + 3*x*(-8 + 3*x^2)*Tan[x/2])))/27
```

Maple [F] time = 0.129, size = 0, normalized size = 0.

$$\int x^3\left(a+a\cos(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+a*cos(x))^(3/2),x)
```

```
[Out] int(x^3*(a+a*cos(x))^(3/2),x)
```

Maxima [A] time = 2.25295, size = 132, normalized size = 0.71

$$\frac{1}{27}\left(81\sqrt{2}ax^3\sin\left(\frac{1}{2}x\right)+486\sqrt{2}ax^2\cos\left(\frac{1}{2}x\right)-1944\sqrt{2}ax\sin\left(\frac{1}{2}x\right)-3888\sqrt{2}a\cos\left(\frac{1}{2}x\right)+2\left(9\sqrt{2}ax^2-8\sqrt{2}a\right)\cos\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/27*(81*sqrt(2)*a*x^3*sin(1/2*x) + 486*sqrt(2)*a*x^2*cos(1/2*x) - 1944*sqr
t(2)*a*x*sin(1/2*x) - 3888*sqrt(2)*a*cos(1/2*x) + 2*(9*sqrt(2)*a*x^2 - 8*sq
rt(2)*a)*cos(3/2*x) + 3*(3*sqrt(2)*a*x^3 - 8*sqrt(2)*a*x)*sin(3/2*x))*sqrt(
a)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+a*cos(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(x) + a)^{\frac{3}{2}} x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cos(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(x) + a)^(3/2)*x^3, x)
```

3.165 $\int x^2(a + a\cos(x))^{3/2}\, dx$

Optimal. Leaf size=145

$$\frac{4}{3}ax^2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+\frac{8}{3}ax^2\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+\frac{16}{9}ax\cos^2\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+\frac{32}{3}ax\sqrt{a\cos(x)+a}$$

```
[Out] (32*a*x*Sqrt[a + a*Cos[x]])/3 + (16*a*x*Cos[x/2]^2*Sqrt[a + a*Cos[x]])/9 +
(4*a*x^2*Cos[x/2]*Sqrt[a + a*Cos[x]]*Sin[x/2])/3 - (224*a*Sqrt[a + a*Cos[x]
]*Tan[x/2])/9 + (8*a*x^2*Sqrt[a + a*Cos[x]]*Tan[x/2])/3 + (32*a*Sqrt[a + a*
Cos[x]]*Sin[x/2]^2*Tan[x/2])/27
```

Rubi [A] time = 0.144266, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3319, 3311, 3296, 2637, 2633}

$$\frac{4}{3}ax^2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+\frac{8}{3}ax^2\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+\frac{16}{9}ax\cos^2\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+\frac{32}{3}ax\sqrt{a\cos(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + a*Cos[x])^(3/2),x]
```


```
[Out] (32*a*x*Sqrt[a + a*Cos[x]])/3 + (16*a*x*Cos[x/2]^2*Sqrt[a + a*Cos[x]])/9 +
(4*a*x^2*Cos[x/2]*Sqrt[a + a*Cos[x]]*Sin[x/2])/3 - (224*a*Sqrt[a + a*Cos[x]
]*Tan[x/2])/9 + (8*a*x^2*Sqrt[a + a*Cos[x]]*Tan[x/2])/3 + (32*a*Sqrt[a + a*
Cos[x]]*Sin[x/2]^2*Tan[x/2])/27
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int x^2(a+a\cos(x))^{3/2}\,dx &= \left(2a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x^2\cos^3\left(\frac{x}{2}\right)dx\\ &= \frac{16}{9}ax\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}+\frac{4}{3}ax^2\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right)+\frac{1}{3}\left(4a\sqrt{a+a\cos(x)}\right.\\ &= \frac{16}{9}ax\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}+\frac{4}{3}ax^2\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right)+\frac{8}{3}ax^2\sqrt{a+a\cos(x)}\\ &= \frac{32}{3}ax\sqrt{a+a\cos(x)}+\frac{16}{9}ax\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}+\frac{4}{3}ax^2\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right)\\ &= \frac{32}{3}ax\sqrt{a+a\cos(x)}+\frac{16}{9}ax\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}+\frac{4}{3}ax^2\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.241013, size = 54, normalized size = 0.37

$$\frac{2}{27}a\sqrt{a(\cos(x)+1)}\left(\left(45x^2-328\right)\tan\left(\frac{x}{2}\right)+\cos(x)\left(\left(9x^2-8\right)\tan\left(\frac{x}{2}\right)+12x\right)+156x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + a*Cos[x])^(3/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cos[x])]*(156*x + (-328 + 45*x^2)*Tan[x/2] + Cos[x]*(12*x
+ (-8 + 9*x^2)*Tan[x/2])))/27
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int x^2\left(a+a\cos(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+a*cos(x))^(3/2),x)
```

```
[Out] int(x^2*(a+a*cos(x))^(3/2),x)
```

Maxima [A] time = 1.98863, size = 97, normalized size = 0.67

$$\frac{1}{27}\left(81\sqrt{2}ax^2\sin\left(\frac{1}{2}x\right)+12\sqrt{2}ax\cos\left(\frac{3}{2}x\right)+324\sqrt{2}ax\cos\left(\frac{1}{2}x\right)-648\sqrt{2}a\sin\left(\frac{1}{2}x\right)+\left(9\sqrt{2}ax^2-8\sqrt{2}a\right)\sin\left(\frac{3}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/27*(81*sqrt(2)*a*x^2*sin(1/2*x) + 12*sqrt(2)*a*x*cos(3/2*x) + 324*sqrt(2)
*a*x*cos(1/2*x) - 648*sqrt(2)*a*sin(1/2*x) + (9*sqrt(2)*a*x^2 - 8*sqrt(2)*a
)*sin(3/2*x))*sqrt(a)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+a*cos(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(x) + a)^{\frac{3}{2}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cos(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(x) + a)^(3/2)*x^2, x)
```

3.166 $\int x(a+a\cos(x))^{3/2}\,dx$

Optimal. Leaf size=89

$$\frac{8}{9}a\cos^2\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+\frac{16}{3}a\sqrt{a\cos(x)+a}+\frac{4}{3}ax\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+\frac{8}{3}ax\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

```
[Out] (16*a*Sqrt[a + a*Cos[x]])/3 + (8*a*Cos[x/2]^2*Sqrt[a + a*Cos[x]])/9 + (4*a*
x*Cos[x/2]*Sqrt[a + a*Cos[x]]*Sin[x/2])/3 + (8*a*x*Sqrt[a + a*Cos[x]]*Tan[x
/2])/3
```

Rubi [A] time = 0.0720092, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3319, 3310, 3296, 2638}

$$\frac{8}{9}a\cos^2\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+\frac{16}{3}a\sqrt{a\cos(x)+a}+\frac{4}{3}ax\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+\frac{8}{3}ax\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + a*Cos[x])^(3/2),x]
```

```
[Out] (16*a*Sqrt[a + a*Cos[x]])/3 + (8*a*Cos[x/2]^2*Sqrt[a + a*Cos[x]])/9 + (4*a*
x*Cos[x/2]*Sqrt[a + a*Cos[x]]*Sin[x/2])/3 + (8*a*x*Sqrt[a + a*Cos[x]]*Tan[x
/2])/3
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x(a+a\cos(x))^{3/2}\,dx = \left(2a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x\cos^3\left(\frac{x}{2}\right)\,dx$$
$$= \frac{8}{9}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} + \frac{4}{3}ax\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) + \frac{1}{3}\left(4a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right.$$
$$= \frac{8}{9}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} + \frac{4}{3}ax\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) + \frac{8}{3}ax\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)$$
$$= \frac{16}{3}a\sqrt{a+a\cos(x)} + \frac{8}{9}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} + \frac{4}{3}ax\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) + \frac{8}{3}ax\sqrt{a}$$

Mathematica [A] time = 0.0661176, size = 45, normalized size = 0.51

$$\frac{1}{9}a\sqrt{a(\cos(x)+1)}\left(4\cos(x)+27x\tan\left(\frac{x}{2}\right)+3x\sin\left(\frac{3x}{2}\right)\sec\left(\frac{x}{2}\right)+52\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + a*Cos[x])^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[x])]*(52 + 4*Cos[x] + 3*x*Sec[x/2]*Sin[(3*x)/2] + 27*x*T
an[x/2]))/9
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int x\left(a+a\cos(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+a*cos(x))^(3/2),x)
```

```
[Out] int(x*(a+a*cos(x))^(3/2),x)
```

Maxima [A] time = 2.34028, size = 65, normalized size = 0.73

$$\frac{1}{9}\left(3\sqrt{2}ax\sin\left(\frac{3}{2}x\right)+27\sqrt{2}ax\sin\left(\frac{1}{2}x\right)+2\sqrt{2}a\cos\left(\frac{3}{2}x\right)+54\sqrt{2}a\cos\left(\frac{1}{2}x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*(3*sqrt(2)*a*x*sin(3/2*x) + 27*sqrt(2)*a*x*sin(1/2*x) + 2*sqrt(2)*a*cos
(3/2*x) + 54*sqrt(2)*a*cos(1/2*x))*sqrt(a)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(x) + a)^{\frac{3}{2}} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cos(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(x) + a)^(3/2)*x, x)
```

3.167 $\int \frac{(a+a\cos(x))^{3/2}}{x}\,dx$

Optimal. Leaf size=55

$$\frac{3}{2}a\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+\frac{1}{2}a\text{CosIntegral}\left(\frac{3x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

```
[Out] (3*a*Sqrt[a + a*Cos[x]]*CosIntegral[x/2]*Sec[x/2])/2 + (a*Sqrt[a + a*Cos[x]
]*CosIntegral[(3*x)/2]*Sec[x/2])/2
```

Rubi [A] time = 0.127667, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3319, 3312, 3302}

$$\frac{3}{2}a\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}+\frac{1}{2}a\text{CosIntegral}\left(\frac{3x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[x])^(3/2)/x,x]
```

```
[Out] (3*a*Sqrt[a + a*Cos[x]]*CosIntegral[x/2]*Sec[x/2])/2 + (a*Sqrt[a + a*Cos[x]
]*CosIntegral[(3*x)/2]*Sec[x/2])/2
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(x))^{3/2}}{x}\,dx = \left(2a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos^3\left(\frac{x}{2}\right)}{x}\,dx$$
$$= \left(2a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \left(\frac{3\cos\left(\frac{x}{2}\right)}{4x}+\frac{\cos\left(\frac{3x}{2}\right)}{4x}\right)dx$$
$$= \frac{1}{2}\left(a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos\left(\frac{3x}{2}\right)}{x}\,dx+\frac{1}{2}\left(3a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos\left(\frac{x}{2}\right)}{x}\,dx$$
$$= \frac{3}{2}a\sqrt{a+a\cos(x)}\text{Ci}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)+\frac{1}{2}a\sqrt{a+a\cos(x)}\text{Ci}\left(\frac{3x}{2}\right)\sec\left(\frac{x}{2}\right)$$

Mathematica [A] time = 0.0199382, size = 36, normalized size = 0.65

$$\frac{1}{2}a\left(3\text{CosIntegral}\left(\frac{x}{2}\right)+\text{CosIntegral}\left(\frac{3x}{2}\right)\right)\sec\left(\frac{x}{2}\right)\sqrt{a(\cos(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])^(3/2)/x,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[x])]*(3*CosIntegral[x/2] + CosIntegral[(3*x)/2])*Sec[x/2
])/2
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+a\cos(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^(3/2)/x,x)
```

```
[Out] int((a+a*cos(x))^(3/2)/x,x)
```

Maxima [C] time = 2.42917, size = 39, normalized size = 0.71

$$\frac{1}{4}\sqrt{2}a^{\frac{3}{2}}\left(\text{Ei}\left(\frac{3}{2}ix\right)+3\,\text{Ei}\left(\frac{1}{2}ix\right)+3\,\text{Ei}\left(-\frac{1}{2}ix\right)+\text{Ei}\left(-\frac{3}{2}ix\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*a^(3/2)*(Ei(3/2*I*x) + 3*Ei(1/2*I*x) + 3*Ei(-1/2*I*x) + Ei(-3/2
*I*x))
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(x)+a)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((a*cos(x) + a)^(3/2)/x, x)
```

3.168 $\int \frac{(a+a\cos(x))^{3/2}}{x^2}\,dx$

Optimal. Leaf size=79

$$-\frac{3}{4}a\text{Si}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}-\frac{3}{4}a\text{Si}\left(\frac{3x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}-\frac{2a\cos^2\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}}{x}$$

```
[Out] (-2*a*Cos[x/2]^2*Sqrt[a + a*Cos[x]])/x - (3*a*Sqrt[a + a*Cos[x]]*Sec[x/2]*S
inIntegral[x/2])/4 - (3*a*Sqrt[a + a*Cos[x]]*Sec[x/2]*SinIntegral[(3*x)/2])
/4
```

Rubi [A] time = 0.12649, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3319, 3313, 3299}

$$-\frac{3}{4}a\text{Si}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}-\frac{3}{4}a\text{Si}\left(\frac{3x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}-\frac{2a\cos^2\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[x])^(3/2)/x^2,x]
```

```
[Out] (-2*a*Cos[x/2]^2*Sqrt[a + a*Cos[x]])/x - (3*a*Sqrt[a + a*Cos[x]]*Sec[x/2]*S
inIntegral[x/2])/4 - (3*a*Sqrt[a + a*Cos[x]]*Sec[x/2]*SinIntegral[(3*x)/2])
/4
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{(a + a\cos(x))^{3/2}}{x^2}\,dx = \left(2a\sqrt{a + a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos^3\left(\frac{x}{2}\right)}{x^2}\,dx$$
$$= -\frac{2a\cos^2\left(\frac{x}{2}\right)\sqrt{a + a\cos(x)}}{x} + \left(3a\sqrt{a + a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \left(-\frac{\sin\left(\frac{x}{2}\right)}{4x} - \frac{\sin\left(\frac{3x}{2}\right)}{4x}\right)dx$$
$$= -\frac{2a\cos^2\left(\frac{x}{2}\right)\sqrt{a + a\cos(x)}}{x} - \frac{1}{4}\left(3a\sqrt{a + a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\sin\left(\frac{x}{2}\right)}{x}\,dx - \frac{1}{4}\left(3a\sqrt{a + a\cos(x)}\right.$$
$$= -\frac{2a\cos^2\left(\frac{x}{2}\right)\sqrt{a + a\cos(x)}}{x} - \frac{3}{4}a\sqrt{a + a\cos(x)}\sec\left(\frac{x}{2}\right)\text{Si}\left(\frac{x}{2}\right) - \frac{3}{4}a\sqrt{a + a\cos(x)}\sec\left(\frac{x}{2}\right)\text{Si}\left(\frac{3}{2}\right.$$

Mathematica [A] time = 0.0846497, size = 53, normalized size = 0.67

$$-\frac{a\sec\left(\frac{x}{2}\right)\sqrt{a(\cos(x)+1)}\left(3x\text{Si}\left(\frac{x}{2}\right) + 3x\text{Si}\left(\frac{3x}{2}\right) + 8\cos^3\left(\frac{x}{2}\right)\right)}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])^(3/2)/x^2,x]
```

```
[Out] -(a*Sqrt[a*(1 + Cos[x])]*Sec[x/2]*(8*Cos[x/2]^3 + 3*x*SinIntegral[x/2] + 3*
x*SinIntegral[(3*x)/2]))/(4*x)
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a + a\cos(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^(3/2)/x^2,x)
```

```
[Out] int((a+a*cos(x))^(3/2)/x^2,x)
```

Maxima [C] time = 2.10422, size = 50, normalized size = 0.63

$$-\frac{1}{8}\sqrt{2}a^{\frac{3}{2}}\left(3i\Gamma\left(-1, \frac{3}{2}ix\right) + 3i\Gamma\left(-1, \frac{1}{2}ix\right) - 3i\Gamma\left(-1, -\frac{1}{2}ix\right) - 3i\Gamma\left(-1, -\frac{3}{2}ix\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*a^(3/2)*(3*I*gamma(-1, 3/2*I*x) + 3*I*gamma(-1, 1/2*I*x) - 3*I
*gamma(-1, -1/2*I*x) - 3*I*gamma(-1, -3/2*I*x))
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(x) + a)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((a*cos(x) + a)^(3/2)/x^2, x)
```

3.169 $\int \frac{(a+a\cos(x))^{3/2}}{x^3}\,dx$

Optimal. Leaf size=109

$$-\frac{3}{16}a\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}-\frac{9}{16}a\text{CosIntegral}\left(\frac{3x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}-\frac{a\cos^2\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}}{x^2}$$

```
[Out] -((a*Cos[x/2]^2*Sqrt[a + a*Cos[x]])/x^2) - (3*a*Sqrt[a + a*Cos[x]]*CosInteg
ral[x/2]*Sec[x/2])/16 - (9*a*Sqrt[a + a*Cos[x]]*CosIntegral[(3*x)/2]*Sec[x/
2])/16 + (3*a*Cos[x/2]*Sqrt[a + a*Cos[x]]*Sin[x/2])/(2*x)
```

Rubi [A] time = 0.165469, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3319, 3314, 3302, 3312}

$$-\frac{3}{16}a\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}-\frac{9}{16}a\text{CosIntegral}\left(\frac{3x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}-\frac{a\cos^2\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[x])^(3/2)/x^3,x]
```


```
[Out] -((a*Cos[x/2]^2*Sqrt[a + a*Cos[x]])/x^2) - (3*a*Sqrt[a + a*Cos[x]]*CosInteg
ral[x/2]*Sec[x/2])/16 - (9*a*Sqrt[a + a*Cos[x]]*CosIntegral[(3*x)/2]*Sec[x/
2])/16 + (3*a*Cos[x/2]*Sqrt[a + a*Cos[x]]*Sin[x/2])/(2*x)
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{(a+a\cos(x))^{3/2}}{x^3}\,dx = \left(2a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos^3\left(\frac{x}{2}\right)}{x^3}\,dx$$
$$= -\frac{a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x^2} + \frac{3a\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right)}{2x} + \frac{1}{2}\left(3a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right.$$
$$= -\frac{a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x^2} + \frac{3}{2}a\sqrt{a+a\cos(x)}\text{Ci}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right) + \frac{3a\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin}{2x}$$
$$= -\frac{a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x^2} + \frac{3}{2}a\sqrt{a+a\cos(x)}\text{Ci}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right) + \frac{3a\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin}{2x}$$
$$= -\frac{a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x^2} - \frac{3}{16}a\sqrt{a+a\cos(x)}\text{Ci}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right) - \frac{9}{16}a\sqrt{a+a\cos(x)}\text{Ci}\left(\frac{3x}{2}\right)\text{s}$$

Mathematica [A] time = 0.0548408, size = 66, normalized size = 0.61

$$-\frac{(a(\cos(x)+1))^{3/2}\left(3x^2\text{CosIntegral}\left(\frac{x}{2}\right)\sec^3\left(\frac{x}{2}\right)+9x^2\text{CosIntegral}\left(\frac{3x}{2}\right)\sec^3\left(\frac{x}{2}\right)-24x\tan\left(\frac{x}{2}\right)+16\right)}{32x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])^(3/2)/x^3,x]
```

```
[Out] -((a*(1 + Cos[x]))^(3/2)*(16 + 3*x^2*CosIntegral[x/2]*Sec[x/2]^3 + 9*x^2*Co
sIntegral[(3*x)/2]*Sec[x/2]^3 - 24*x*Tan[x/2]))/(32*x^2)
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}(a+a\cos(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^(3/2)/x^3,x)
```

```
[Out] int((a+a*cos(x))^(3/2)/x^3,x)
```

Maxima [C] time = 2.24817, size = 45, normalized size = 0.41

$$\frac{3}{16}\sqrt{2}a^{\frac{3}{2}}\left(3\,\Gamma\left(-2,\frac{3}{2}i\,x\right)+\Gamma\left(-2,\frac{1}{2}i\,x\right)+\Gamma\left(-2,-\frac{1}{2}i\,x\right)+3\,\Gamma\left(-2,-\frac{3}{2}i\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/16*sqrt(2)*a^(3/2)*(3*gamma(-2, 3/2*I*x) + gamma(-2, 1/2*I*x) + gamma(-2,
 -1/2*I*x) + 3*gamma(-2, -3/2*I*x))
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(x) + a)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((a*cos(x) + a)^(3/2)/x^3, x)
```

3.170 $\int \frac{x^3}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=374

$$\frac{12ix^2\text{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cos(c+dx)+a}}-\frac{12ix^2\text{Li}_2\left(ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cos(c+dx)+a}}-\frac{48x\text{Li}_3\left(-ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3\sqrt{a\cos(c+dx)+a}}+\frac{48x\text{Li}_3}{d^3}$$

```
[Out] ((-4*I)*x^3*ArcTan[E^((I/2)*(c + d*x))]*Cos[c/2 + (d*x)/2])/(d*Sqrt[a + a*C
os[c + d*x]]) + ((12*I)*x^2*Cos[c/2 + (d*x)/2]*PolyLog[2, (-I)*E^((I/2)*(c
+ d*x))])/(d^2*Sqrt[a + a*Cos[c + d*x]]) - ((12*I)*x^2*Cos[c/2 + (d*x)/2]*P
olyLog[2, I*E^((I/2)*(c + d*x))])/(d^2*Sqrt[a + a*Cos[c + d*x]]) - (48*x*Co
s[c/2 + (d*x)/2]*PolyLog[3, (-I)*E^((I/2)*(c + d*x))])/(d^3*Sqrt[a + a*Cos[
c + d*x]]) + (48*x*Cos[c/2 + (d*x)/2]*PolyLog[3, I*E^((I/2)*(c + d*x))])/(d
^3*Sqrt[a + a*Cos[c + d*x]]) - ((96*I)*Cos[c/2 + (d*x)/2]*PolyLog[4, (-I)*E
^((I/2)*(c + d*x))])/(d^4*Sqrt[a + a*Cos[c + d*x]]) + ((96*I)*Cos[c/2 + (d*
x)/2]*PolyLog[4, I*E^((I/2)*(c + d*x))])/(d^4*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.211637, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3319, 4181, 2531, 6609, 2282, 6589}

$$\frac{12ix^2\text{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cos(c+dx)+a}}-\frac{12ix^2\text{Li}_2\left(ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cos(c+dx)+a}}-\frac{48x\text{Li}_3\left(-ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3\sqrt{a\cos(c+dx)+a}}+\frac{48x\text{Li}_3}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((-4*I)*x^3*ArcTan[E^((I/2)*(c + d*x))]*Cos[c/2 + (d*x)/2])/(d*Sqrt[a + a*C
os[c + d*x]]) + ((12*I)*x^2*Cos[c/2 + (d*x)/2]*PolyLog[2, (-I)*E^((I/2)*(c
+ d*x))])/(d^2*Sqrt[a + a*Cos[c + d*x]]) - ((12*I)*x^2*Cos[c/2 + (d*x)/2]*P
olyLog[2, I*E^((I/2)*(c + d*x))])/(d^2*Sqrt[a + a*Cos[c + d*x]]) - (48*x*Co
s[c/2 + (d*x)/2]*PolyLog[3, (-I)*E^((I/2)*(c + d*x))])/(d^3*Sqrt[a + a*Cos[
c + d*x]]) + (48*x*Cos[c/2 + (d*x)/2]*PolyLog[3, I*E^((I/2)*(c + d*x))])/(d
^3*Sqrt[a + a*Cos[c + d*x]]) - ((96*I)*Cos[c/2 + (d*x)/2]*PolyLog[4, (-I)*E
^((I/2)*(c + d*x))])/(d^4*Sqrt[a + a*Cos[c + d*x]]) + ((96*I)*Cos[c/2 + (d*
x)/2]*PolyLog[4, I*E^((I/2)*(c + d*x))])/(d^4*Sqrt[a + a*Cos[c + d*x]])
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\int x^3\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)dx}{\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{4ix^3\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\left(6\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x^2\log\left(1-ie^{i\left(\frac{c}{2}+\frac{dx}{2}\right)}\right) d}{d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{4ix^3\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{12ix^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\mathrm{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} - \frac{12ix^2\cos\left(\frac{c}{2}+\right.}{d^2\sqrt{a+a}}$$

$$= -\frac{4ix^3\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{12ix^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\mathrm{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} - \frac{12ix^2\cos\left(\frac{c}{2}+\right.}{d^2\sqrt{a+a}}$$

$$= -\frac{4ix^3\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{12ix^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\mathrm{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} - \frac{12ix^2\cos\left(\frac{c}{2}+\right.}{d^2\sqrt{a+a}}$$

$$= -\frac{4ix^3\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{12ix^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\mathrm{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} - \frac{12ix^2\cos\left(\frac{c}{2}+\right.}{d^2\sqrt{a+a}}$$

Mathematica [A] time = 0.126843, size = 199, normalized size = 0.53

$$-\frac{4i\cos\left(\frac{1}{2}(c+dx)\right)\left(-3d^2x^2\mathrm{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)+3d^2x^2\mathrm{Li}_2\left(ie^{\frac{1}{2}i(c+dx)}\right)+d^3x^3\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)-12idx\mathrm{Li}_3\left(-ie^{\frac{1}{2}i(c+dx)}\right)+12\right.}{d^4\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((-4*I)*Cos[(c + d*x)/2]*(d^3*x^3*ArcTan[E^((I/2)*(c + d*x))] - 3*d^2*x^2*P
olyLog[2, (-I)*E^((I/2)*(c + d*x))] + 3*d^2*x^2*PolyLog[2, I*E^((I/2)*(c +
d*x))] - (12*I)*d*x*PolyLog[3, (-I)*E^((I/2)*(c + d*x))] + (12*I)*d*x*PolyL
og[3, I*E^((I/2)*(c + d*x))] + 24*PolyLog[4, (-I)*E^((I/2)*(c + d*x))] - 24
*PolyLog[4, I*E^((I/2)*(c + d*x))]))/(d^4*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [F] time = 0.197, size = 0, normalized size = 0.

$$\int x^3 \frac{1}{\sqrt{a + \cos(dx + c)\, a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] int(x^3/(a+cos(d*x+c)*a)^(1/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{\sqrt{a\cos(dx + c) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3/sqrt(a*cos(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a(\cos(c + dx) + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(a*cos(d*x + c) + a), x)
```

3.171 $\int \frac{x^2}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=262

$$\frac{8ix\text{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cos(c+dx)+a}}-\frac{8ix\text{Li}_2\left(ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cos(c+dx)+a}}-\frac{16\text{Li}_3\left(-ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3\sqrt{a\cos(c+dx)+a}}+\frac{16\text{Li}_3\left(ie^{\frac{1}{2}i(c+\cdots}\right.}{d^3\sqrt{a\cos\cdots}}$$

```
[Out] ((-4*I)*x^2*ArcTan[E^((I/2)*(c + d*x))]*Cos[c/2 + (d*x)/2])/(d*Sqrt[a + a*C
os[c + d*x]]) + ((8*I)*x*Cos[c/2 + (d*x)/2]*PolyLog[2, (-I)*E^((I/2)*(c + d
*x))])/(d^2*Sqrt[a + a*Cos[c + d*x]]) - ((8*I)*x*Cos[c/2 + (d*x)/2]*PolyLog
[2, I*E^((I/2)*(c + d*x))])/(d^2*Sqrt[a + a*Cos[c + d*x]]) - (16*Cos[c/2 +
(d*x)/2]*PolyLog[3, (-I)*E^((I/2)*(c + d*x))])/(d^3*Sqrt[a + a*Cos[c + d*x]
]) + (16*Cos[c/2 + (d*x)/2]*PolyLog[3, I*E^((I/2)*(c + d*x))])/(d^3*Sqrt[a
+ a*Cos[c + d*x]])
```

Rubi [A] time = 0.167395, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3319, 4181, 2531, 2282, 6589}

$$\frac{8ix\text{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cos(c+dx)+a}}-\frac{8ix\text{Li}_2\left(ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cos(c+dx)+a}}-\frac{16\text{Li}_3\left(-ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3\sqrt{a\cos(c+dx)+a}}+\frac{16\text{Li}_3\left(ie^{\frac{1}{2}i(c+\cdots}\right.}{d^3\sqrt{a\cos\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((-4*I)*x^2*ArcTan[E^((I/2)*(c + d*x))]*Cos[c/2 + (d*x)/2])/(d*Sqrt[a + a*C
os[c + d*x]]) + ((8*I)*x*Cos[c/2 + (d*x)/2]*PolyLog[2, (-I)*E^((I/2)*(c + d
*x))])/(d^2*Sqrt[a + a*Cos[c + d*x]]) - ((8*I)*x*Cos[c/2 + (d*x)/2]*PolyLog
[2, I*E^((I/2)*(c + d*x))])/(d^2*Sqrt[a + a*Cos[c + d*x]]) - (16*Cos[c/2 +
(d*x)/2]*PolyLog[3, (-I)*E^((I/2)*(c + d*x))])/(d^3*Sqrt[a + a*Cos[c + d*x]
]) + (16*Cos[c/2 + (d*x)/2]*PolyLog[3, I*E^((I/2)*(c + d*x))])/(d^3*Sqrt[a
+ a*Cos[c + d*x]])
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\int x^2\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)dx}{\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{4ix^2\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\left(4\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x\log\left(1-ie^{i\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)dx}{d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{4ix^2\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{8ix\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} - \frac{8ix\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{L}}{d^2\sqrt{a+a\cos}}$$

$$= -\frac{4ix^2\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{8ix\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} - \frac{8ix\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{L}}{d^2\sqrt{a+a\cos}}$$

$$= -\frac{4ix^2\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{8ix\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} - \frac{8ix\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{L}}{d^2\sqrt{a+a\cos}}$$

Mathematica [A] time = 0.0766249, size = 146, normalized size = 0.56

$$\frac{4\cos\left(\frac{1}{2}(c+dx)\right)\left(-id^2x^2\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)+2idx\text{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)-2idx\text{Li}_2\left(ie^{\frac{1}{2}i(c+dx)}\right)-4\text{Li}_3\left(-ie^{\frac{1}{2}i(c+dx)}\right)+4\text{Li}_3\left(ie^{\frac{1}{2}i(c}\right.\right.}{d^3\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (4*Cos[(c + d*x)/2]*((-I)*d^2*x^2*ArcTan[E^((I/2)*(c + d*x))] + (2*I)*d*x*P
olyLog[2, (-I)*E^((I/2)*(c + d*x))] - (2*I)*d*x*PolyLog[2, I*E^((I/2)*(c +
d*x))] - 4*PolyLog[3, (-I)*E^((I/2)*(c + d*x))] + 4*PolyLog[3, I*E^((I/2)*(
c + d*x))]))/(d^3*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [F] time = 0.184, size = 0, normalized size = 0.

$$\int x^2\frac{1}{\sqrt{a+\cos(dx+c)\,a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] int(x^2/(a+cos(d*x+c)*a)^(1/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\sqrt{a\cos(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2/sqrt(a*cos(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(a*cos(d*x + c) + a), x)
```

3.172 $\int \frac{x}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=156

$$\frac{4i\text{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cos(c+dx)+a}}-\frac{4i\text{Li}_2\left(ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cos(c+dx)+a}}-\frac{4ix\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] ((-4*I)*x*ArcTan[E^((I/2)*(c + d*x))]*Cos[c/2 + (d*x)/2])/(d*Sqrt[a + a*Cos
[c + d*x]]) + ((4*I)*Cos[c/2 + (d*x)/2]*PolyLog[2, (-I)*E^((I/2)*(c + d*x))
])/(d^2*Sqrt[a + a*Cos[c + d*x]]) - ((4*I)*Cos[c/2 + (d*x)/2]*PolyLog[2, I*
E^((I/2)*(c + d*x))])/(d^2*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.0856533, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3319, 4181, 2279, 2391}

$$\frac{4i\text{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cos(c+dx)+a}}-\frac{4i\text{Li}_2\left(ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cos(c+dx)+a}}-\frac{4ix\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((-4*I)*x*ArcTan[E^((I/2)*(c + d*x))]*Cos[c/2 + (d*x)/2])/(d*Sqrt[a + a*Cos
[c + d*x]]) + ((4*I)*Cos[c/2 + (d*x)/2]*PolyLog[2, (-I)*E^((I/2)*(c + d*x))
])/(d^2*Sqrt[a + a*Cos[c + d*x]]) - ((4*I)*Cos[c/2 + (d*x)/2]*PolyLog[2, I*
E^((I/2)*(c + d*x))])/(d^2*Sqrt[a + a*Cos[c + d*x]])
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\int x\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)dx}{\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{4ix\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\left(2\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int\log\left(1-ie^{i\left(\frac{c}{2}+\frac{dx}{2}\right)}\right) d}{d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{4ix\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\left(4i\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\text{Subst}\left(\int\frac{\log(1-ix)}{x}dx,\right.}{d^2\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{4ix\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{4i\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} - \frac{4i\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{L}}{d^2\sqrt{a+a\cos}}$$

Mathematica [A] time = 0.0507512, size = 89, normalized size = 0.57

$$-\frac{4i\cos\left(\frac{1}{2}(c+dx)\right)\left(-\text{Li}_2\left(-ie^{\frac{1}{2}i(c+dx)}\right)+\text{Li}_2\left(ie^{\frac{1}{2}i(c+dx)}\right)+dx\tan^{-1}\left(e^{\frac{1}{2}i(c+dx)}\right)\right)}{d^2\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((-4*I)*Cos[(c + d*x)/2]*(d*x*ArcTan[E^((I/2)*(c + d*x))] - PolyLog[2, (-I)
*E^((I/2)*(c + d*x))] + PolyLog[2, I*E^((I/2)*(c + d*x))]))/(d^2*Sqrt[a*(1
+ Cos[c + d*x])])
```

Maple [F] time = 0.178, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt{a+\cos(dx+c)\,a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] int(x/(a+cos(d*x+c)*a)^(1/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{a\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(a*cos(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(a*cos(d*x + c) + a), x)
```

3.173 $\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]
)/(Sqrt[a]*d)
```

Rubi [A] time = 0.0216911, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2649, 206}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]
)/(Sqrt[a]*d)
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}\\ &= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}\end{aligned}$$

Mathematica [A] time = 0.0154564, size = 40, normalized size = 0.87

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x])
])
```

Maple [C] time = 0.243, size = 54, normalized size = 1.2

$$\frac{\sqrt{2}}{d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\mathrm{InverseJacobiAM}\left(\frac{dx}{2}+\frac{c}{2},1\right)\frac{1}{\sqrt{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}a}}\left(\mathrm{csgn}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/csgn(cos(1/2*d*x+1/2*c))*cos(1/2
*d*x+1/2*c)*InverseJacobiAM(1/2*d*x+1/2*c,1)
```

Maxima [B] time = 2.13944, size = 122, normalized size = 2.65

$$\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\right.}{2\,\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/(sqrt(a)*d)
```

Fricas [A] time = 1.66261, size = 347, normalized size = 7.54

$$\left[\frac{\sqrt{2}\log\left(-\frac{\cos(dx+c)^{2}-\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)}{2\sqrt{a}d},-\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{-\frac{1}{a}}}{\sin(dx+c)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(
d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
))/(sqrt(a)*d), -sqrt(2)*sqrt(-1/a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)
*sqrt(-1/a)/sin(d*x + c))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cos(c+dx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*cos(c + d*x) + a), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*cos(d*x + c) + a), x)
```

3.174 $\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{a\cos(c+dx)+a}},x\right)$$

```
[Out] Unintegrable[1/(x*Sqrt[a + a*Cos[c + d*x]]), x]
```

Rubi [A] time = 0.0685169, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[a + a*Cos[c + d*x]]),x]

[Out] Defer[Int][1/(x*Sqrt[a + a*Cos[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx$$

Mathematica [A] time = 0.97155, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[a + a*Cos[c + d*x]]),x]

[Out] Integrate[1/(x*Sqrt[a + a*Cos[c + d*x]]), x]
```

Maple [A] time = 0.178, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt{a+\cos(dx+c)a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+cos(d*x+c)*a)^(1/2),x)

[Out] int(1/x/(a+cos(d*x+c)*a)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cos(dx+c)+a}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\cos(dx+c)+a}}{ax\cos(dx+c)+ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cos(d*x + c) + a)/(a*x*cos(d*x + c) + a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a*(cos(c + d*x) + 1))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cos(dx+c)+a}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*x), x)
```

3.175 $\int \frac{x^3}{\sqrt{a-a\cos(x)}}\,dx$

Optimal. Leaf size=235

$$\frac{12ix^2\text{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{12ix^2\text{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{48x\text{Li}_3\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}+\frac{48x\text{Li}_3\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{96i\text{Li}_4\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}$$

```
[Out] (-4*x^3*ArcTanh[E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] + ((12*I)*x^2*Pol
yLog[2, -E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] - ((12*I)*x^2*PolyLog[2,
 E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] - (48*x*PolyLog[3, -E^((I/2)*x)]
*Sin[x/2])/Sqrt[a - a*Cos[x]] + (48*x*PolyLog[3, E^((I/2)*x)]*Sin[x/2])/Sqr
t[a - a*Cos[x]] - ((96*I)*PolyLog[4, -E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos
[x]] + ((96*I)*PolyLog[4, E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

Rubi [A] time = 0.173309, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3319, 4183, 2531, 6609, 2282, 6589}

$$\frac{12ix^2\text{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{12ix^2\text{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{48x\text{Li}_3\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}+\frac{48x\text{Li}_3\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{96i\text{Li}_4\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a - a*Cos[x]],x]
```

```
[Out] (-4*x^3*ArcTanh[E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] + ((12*I)*x^2*Pol
yLog[2, -E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] - ((12*I)*x^2*PolyLog[2,
 E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] - (48*x*PolyLog[3, -E^((I/2)*x)]
*Sin[x/2])/Sqrt[a - a*Cos[x]] + (48*x*PolyLog[3, E^((I/2)*x)]*Sin[x/2])/Sqr
t[a - a*Cos[x]] - ((96*I)*PolyLog[4, -E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos
[x]] + ((96*I)*PolyLog[4, E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a-a\cos(x)}}\,dx = \frac{\sin\left(\frac{x}{2}\right)\int x^3\csc\left(\frac{x}{2}\right)dx}{\sqrt{a-a\cos(x)}}$$

$$= -\frac{4x^3\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{\left(6\sin\left(\frac{x}{2}\right)\right)\int x^2\log\left(1-e^{\frac{ix}{2}}\right)dx}{\sqrt{a-a\cos(x)}} + \frac{\left(6\sin\left(\frac{x}{2}\right)\right)\int x^2\log\left(1+e^{\frac{ix}{2}}\right)d}{\sqrt{a-a\cos(x)}}$$

$$= -\frac{4x^3\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{12ix^2\mathrm{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{12ix^2\mathrm{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{\left(24i\sin\left(\frac{x}{2}\right)\right)\int}{\sqrt{a-}}$$

$$= -\frac{4x^3\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{12ix^2\mathrm{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{12ix^2\mathrm{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{48x\mathrm{Li}_3\left(-e^{\frac{ix}{2}}\right)}{\sqrt{a-a\cos}}$$

$$= -\frac{4x^3\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{12ix^2\mathrm{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{12ix^2\mathrm{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{48x\mathrm{Li}_3\left(-e^{\frac{ix}{2}}\right)}{\sqrt{a-a\cos}}$$

$$= -\frac{4x^3\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{12ix^2\mathrm{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{12ix^2\mathrm{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{48x\mathrm{Li}_3\left(-e^{\frac{ix}{2}}\right)}{\sqrt{a-a\cos}}$$

Mathematica [A] time = 0.0978472, size = 170, normalized size = 0.72

$$-\frac{i\sin\left(\frac{x}{2}\right)\left(-48x^2\mathrm{Li}_2\left(e^{-\frac{ix}{2}}\right)-48x^2\mathrm{Li}_2\left(-e^{\frac{ix}{2}}\right)+192ix\mathrm{Li}_3\left(e^{-\frac{ix}{2}}\right)-192ix\mathrm{Li}_3\left(-e^{\frac{ix}{2}}\right)+384\mathrm{Li}_4\left(e^{-\frac{ix}{2}}\right)+384\mathrm{Li}_4\left(-e^{\frac{ix}{2}}\right)-\right.}{4\sqrt{a-a\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a - a*Cos[x]],x]
```

```
[Out] ((-I/4)*(8*Pi^4 - x^4 + (8*I)*x^3*Log[1 - E^((-I/2)*x)] - (8*I)*x^3*Log[1 +
 E^((I/2)*x)] - 48*x^2*PolyLog[2, E^((-I/2)*x)] - 48*x^2*PolyLog[2, -E^((I/
```

```
2)*x)] + (192*I)*x*PolyLog[3, E^((-I/2)*x)] - (192*I)*x*PolyLog[3, -E^((I/2
)*x)] + 384*PolyLog[4, E^((-I/2)*x)] + 384*PolyLog[4, -E^((I/2)*x)])*Sin[x/
2])/Sqrt[a - a*Cos[x]]
```

Maple [F] time = 0.125, size = 0, normalized size = 0.

$$\int x^{3}\frac{1}{\sqrt{a-a\cos\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a-a*cos(x))^(1/2),x)
```

```
[Out] int(x^3/(a-a*cos(x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\sqrt{-a\cos\left(x\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(-a*cos(x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-a\cos\left(x\right)+a}x^{3}}{a\cos\left(x\right)-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*cos(x) + a)*x^3/(a*cos(x) - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\sqrt{-a\left(\cos\left(x\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a-a*cos(x))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(-a*(cos(x) - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{-a\cos(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a-a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(-a*cos(x) + a), x)
```

3.176 $\int \frac{x^2}{\sqrt{a-a\cos(x)}}\,dx$

Optimal. Leaf size=163

$$\frac{8ix\mathrm{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{8ix\mathrm{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{16\mathrm{Li}_3\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}+\frac{16\mathrm{Li}_3\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{4x^2\sin\left(\frac{x}{2}\right)\tanh^{-1}\left(e^{\frac{ix}{2}}\right)}{\sqrt{a-a\cos(x)}}$$

```
[Out] (-4*x^2*ArcTanh[E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] + ((8*I)*x*PolyLo
g[2, -E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] - ((8*I)*x*PolyLog[2, E^((I
/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] - (16*PolyLog[3, -E^((I/2)*x)]*Sin[x/2
])/Sqrt[a - a*Cos[x]] + (16*PolyLog[3, E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Co
s[x]]
```

Rubi [A] time = 0.141194, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3319, 4183, 2531, 2282, 6589}

$$\frac{8ix\mathrm{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{8ix\mathrm{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{16\mathrm{Li}_3\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}+\frac{16\mathrm{Li}_3\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{4x^2\sin\left(\frac{x}{2}\right)\tanh^{-1}\left(e^{\frac{ix}{2}}\right)}{\sqrt{a-a\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a - a*Cos[x]],x]
```

```
[Out] (-4*x^2*ArcTanh[E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] + ((8*I)*x*PolyLo
g[2, -E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] - ((8*I)*x*PolyLog[2, E^((I
/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] - (16*PolyLog[3, -E^((I/2)*x)]*Sin[x/2
])/Sqrt[a - a*Cos[x]] + (16*PolyLog[3, E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Co
s[x]]
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a-a\cos(x)}}\,dx = \frac{\sin\left(\frac{x}{2}\right)\int x^2\csc\left(\frac{x}{2}\right)dx}{\sqrt{a-a\cos(x)}}$$
$$= -\frac{4x^2\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{\left(4\sin\left(\frac{x}{2}\right)\right)\int x\log\left(1-e^{\frac{ix}{2}}\right)dx}{\sqrt{a-a\cos(x)}} + \frac{\left(4\sin\left(\frac{x}{2}\right)\right)\int x\log\left(1+e^{\frac{ix}{2}}\right)dx}{\sqrt{a-a\cos(x)}}$$
$$= -\frac{4x^2\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{8ix\mathrm{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{8ix\mathrm{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{\left(8i\sin\left(\frac{x}{2}\right)\right)\int \mathrm{Li}_2\left(\right.}{\sqrt{a-a\cos(}}$$
$$= -\frac{4x^2\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{8ix\mathrm{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{8ix\mathrm{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{\left(16\sin\left(\frac{x}{2}\right)\right)\mathrm{Subst}}{\sqrt{a-}}$$
$$= -\frac{4x^2\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{8ix\mathrm{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{8ix\mathrm{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{16\mathrm{Li}_3\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}$$

Mathematica [A] time = 0.0485859, size = 117, normalized size = 0.72

$$\frac{2\sin\left(\frac{x}{2}\right)\left(4ix\mathrm{Li}_2\left(-e^{\frac{ix}{2}}\right)-4ix\mathrm{Li}_2\left(e^{\frac{ix}{2}}\right)-8\mathrm{Li}_3\left(-e^{\frac{ix}{2}}\right)+8\mathrm{Li}_3\left(e^{\frac{ix}{2}}\right)+x^2\log\left(1-e^{\frac{ix}{2}}\right)-x^2\log\left(1+e^{\frac{ix}{2}}\right)\right)}{\sqrt{a-a\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a - a*Cos[x]],x]
```

```
[Out] (2*(x^2*Log[1 - E^((I/2)*x)] - x^2*Log[1 + E^((I/2)*x)] + (4*I)*x*PolyLog[2
, -E^((I/2)*x)] - (4*I)*x*PolyLog[2, E^((I/2)*x)] - 8*PolyLog[3, -E^((I/2)*
x)] + 8*PolyLog[3, E^((I/2)*x)])*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int x^2\frac{1}{\sqrt{a-a\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a-a*cos(x))^(1/2),x)
```

```
[Out] int(x^2/(a-a*cos(x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-a\cos(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(-a*cos(x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-a\cos(x)+a}\,x^2}{a\cos(x)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*cos(x) + a)*x^2/(a*cos(x) - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-a(\cos(x)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a-a*cos(x))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(-a*(cos(x) - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-a\cos(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a-a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(-a*cos(x) + a), x)
```

3.177 $\int \frac{x}{\sqrt{a-a\cos(x)}}\,dx$

Optimal. Leaf size=97

$$\frac{4i\text{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{4i\text{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{4x\sin\left(\frac{x}{2}\right)\tanh^{-1}\left(e^{\frac{ix}{2}}\right)}{\sqrt{a-a\cos(x)}}$$

```
[Out] (-4*x*ArcTanh[E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] + ((4*I)*PolyLog[2,
 -E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] - ((4*I)*PolyLog[2, E^((I/2)*x)
]*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

Rubi [A] time = 0.0784426, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3319, 4183, 2279, 2391}

$$\frac{4i\text{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{4i\text{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{4x\sin\left(\frac{x}{2}\right)\tanh^{-1}\left(e^{\frac{ix}{2}}\right)}{\sqrt{a-a\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a - a*Cos[x]],x]
```


```
[Out] (-4*x*ArcTanh[E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] + ((4*I)*PolyLog[2,
 -E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] - ((4*I)*PolyLog[2, E^((I/2)*x)
]*Sin[x/2])/Sqrt[a - a*Cos[x]]
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x}{\sqrt{a-a\cos(x)}}\,dx = \frac{\sin\left(\frac{x}{2}\right)\int x\csc\left(\frac{x}{2}\right)dx}{\sqrt{a-a\cos(x)}}$$
$$= -\frac{4x\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{\left(2\sin\left(\frac{x}{2}\right)\right)\int\log\left(1-e^{\frac{ix}{2}}\right)dx}{\sqrt{a-a\cos(x)}} + \frac{\left(2\sin\left(\frac{x}{2}\right)\right)\int\log\left(1+e^{\frac{ix}{2}}\right)dx}{\sqrt{a-a\cos(x)}}$$
$$= -\frac{4x\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{\left(4i\sin\left(\frac{x}{2}\right)\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}dx, x, e^{\frac{ix}{2}}\right)}{\sqrt{a-a\cos(x)}} - \frac{\left(4i\sin\left(\frac{x}{2}\right)\right)\text{Subst}\left(\int\frac{\log(1+}{x}\right.}{\sqrt{a-a\cos(x)}}$$
$$= -\frac{4x\tanh^{-1}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{4i\text{Li}_2\left(-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{4i\text{Li}_2\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}$$

Mathematica [A] time = 0.0330293, size = 83, normalized size = 0.86

$$\frac{2\sin\left(\frac{x}{2}\right)\left(2i\text{Li}_2\left(-e^{\frac{ix}{2}}\right)-2i\text{Li}_2\left(e^{\frac{ix}{2}}\right)+x\left(\log\left(1-e^{\frac{ix}{2}}\right)-\log\left(1+e^{\frac{ix}{2}}\right)\right)\right)}{\sqrt{a-a\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a - a*Cos[x]],x]
```

```
[Out] (2*(x*(Log[1 - E^((I/2)*x)] - Log[1 + E^((I/2)*x)]) + (2*I)*PolyLog[2, -E^(
(I/2)*x)] - (2*I)*PolyLog[2, E^((I/2)*x)])*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt{a-a\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a-a*cos(x))^(1/2),x)
```

```
[Out] int(x/(a-a*cos(x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-a\cos(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(-a*cos(x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a\cos(x)+a}x}{a\cos(x)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*cos(x) + a)*x/(a*cos(x) - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-a(\cos(x)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a-a*cos(x))**(1/2),x)
```

```
[Out] Integral(x/sqrt(-a*(cos(x) - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-a\cos(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a-a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(-a*cos(x) + a), x)
```

3.178 $\int \frac{1}{\sqrt{a-a\cos(x)}}\,dx$

Optimal. Leaf size=37

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a-a\cos(x)}}\right)}{\sqrt{a}}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[a - a*Cos[x]])])/Sqrt[a])
```

Rubi [A] time = 0.0204732, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2649, 206}

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a-a\cos(x)}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - a*Cos[x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[a - a*Cos[x]])])/Sqrt[a])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a-a\cos(x)}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\sin(x)}{\sqrt{a-a\cos(x)}}\right)\right)\\ &= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a-a\cos(x)}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.0150268, size = 36, normalized size = 0.97

$$\frac{2\sin\left(\frac{x}{2}\right)\left(\log\left(\sin\left(\frac{x}{4}\right)\right)-\log\left(\cos\left(\frac{x}{4}\right)\right)\right)}{\sqrt{a-a\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - a*Cos[x]],x]
```

```
[Out] (2*(-Log[Cos[x/4]] + Log[Sin[x/4]])*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

Maple [A] time = 0.889, size = 25, normalized size = 0.7

$$-\sqrt{2}\sin\left(\frac{x}{2}\right)\operatorname{Artanh}\left(\cos\left(\frac{x}{2}\right)\right)\frac{1}{\sqrt{a\left(\sin\left(\frac{x}{2}\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*cos(x))^(1/2),x)
```

```
[Out] -sin(1/2*x)*arctanh(cos(1/2*x))*2^(1/2)/(a*sin(1/2*x)^2)^(1/2)
```

Maxima [B] time = 2.02647, size = 109, normalized size = 2.95

$$-\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)\right)\right)^{2}+\sin\left(\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)\right)\right)^{2}+2\cos\left(\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)\right)\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(2)*log(cos(1/2*arctan2(sin(x), cos(x)))^2 + sin(1/2*arctan2(sin(
x), cos(x)))^2 + 2*cos(1/2*arctan2(sin(x), cos(x))) + 1) - sqrt(2)*log(cos(
1/2*arctan2(sin(x), cos(x)))^2 + sin(1/2*arctan2(sin(x), cos(x)))^2 - 2*cos
(1/2*arctan2(sin(x), cos(x))) + 1))/sqrt(a)
```

Fricas [A] time = 1.63184, size = 273, normalized size = 7.38

$$\left[\frac{\sqrt{2}\log\left(-\frac{(\cos(x)+3)\sin(x)-\frac{2\sqrt{2}\sqrt{-a\cos(x)+a}(\cos(x)+1)}{\sqrt{a}}}{(\cos(x)-1)\sin(x)}\right)}{2\sqrt{a}}, \sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{-a\cos\left(x\right)+a}\sqrt{-\frac{1}{a}}}{\sin\left(x\right)}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log(-((cos(x) + 3)*sin(x) - 2*sqrt(2)*sqrt(-a*cos(x) + a)*(cos
(x) + 1)/sqrt(a))/((cos(x) - 1)*sin(x)))/sqrt(a), sqrt(2)*sqrt(-1/a)*arctan
(sqrt(2)*sqrt(-a*cos(x) + a)*sqrt(-1/a)/sin(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-a\cos\left(x\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cos(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-a*cos(x) + a), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a\cos(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-a*cos(x) + a), x)
```

3.179 $\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx$

Optimal. Leaf size=17

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{a-a\cos(x)}},x\right)$$

```
[Out] Unintegrable[1/(x*Sqrt[a - a*Cos[x]]), x]
```

Rubi [A] time = 0.0772828, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[a - a*Cos[x]]),x]
```

```
[Out] Defer[Int][1/(x*Sqrt[a - a*Cos[x]]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx=\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx$$

Mathematica [A] time = 3.12421, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[a - a*Cos[x]]),x]
```

```
[Out] Integrate[1/(x*Sqrt[a - a*Cos[x]]), x]
```

Maple [A] time = 0.087, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt{a-a\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a-a*cos(x))^(1/2),x)
```

```
[Out] int(1/x/(a-a*cos(x))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a\cos(x) + a}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a*cos(x) + a)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a\cos(x) + a}}{ax\cos(x) - ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*cos(x) + a)/(a*x*cos(x) - a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-a(\cos(x) - 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a-a*cos(x))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-a*(cos(x) - 1))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a\cos(x) + a}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a-a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a*cos(x) + a)*x), x)
```

3.180 $\int \frac{x^3}{(a+a\cos(x))^{3/2}}\,dx$

Optimal. Leaf size=423

$$\frac{3ix^2\text{Li}_2\left(-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}-\frac{3ix^2\text{Li}_2\left(ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}-\frac{12x\text{Li}_3\left(-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}+\frac{12x\text{Li}_3\left(ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}+\frac{24i\text{Li}_2\left(-ie^{\frac{ix}{2}}\right)\text{co}}{a\sqrt{a\cos(x)+}}$$

```
[Out] (-3*x^2)/(a*Sqrt[a + a*Cos[x]]) - ((24*I)*x*ArcTan[E^((I/2)*x)]*Cos[x/2])/(
a*Sqrt[a + a*Cos[x]]) - (I*x^3*ArcTan[E^((I/2)*x)]*Cos[x/2])/(a*Sqrt[a + a*
Cos[x]]) + ((24*I)*Cos[x/2]*PolyLog[2, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos
[x]]) + ((3*I)*x^2*Cos[x/2]*PolyLog[2, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos
[x]]) - ((24*I)*Cos[x/2]*PolyLog[2, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]])
- ((3*I)*x^2*Cos[x/2]*PolyLog[2, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) - (
12*x*Cos[x/2]*PolyLog[3, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + (12*x*
Cos[x/2]*PolyLog[3, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) - ((24*I)*Cos[x/
2]*PolyLog[4, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + ((24*I)*Cos[x/2]*
PolyLog[4, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + (x^3*Tan[x/2])/(2*a*Sqr
t[a + a*Cos[x]])
```

Rubi [A] time = 0.259535, antiderivative size = 423, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {3319, 4186, 4181, 2279, 2391, 2531, 6609, 2282, 6589}

$$\frac{3ix^2\text{Li}_2\left(-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}-\frac{3ix^2\text{Li}_2\left(ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}-\frac{12x\text{Li}_3\left(-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}+\frac{12x\text{Li}_3\left(ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}+\frac{24i\text{Li}_2\left(-ie^{\frac{ix}{2}}\right)\text{co}}{a\sqrt{a\cos(x)+}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + a*Cos[x])^(3/2),x]
```

```
[Out] (-3*x^2)/(a*Sqrt[a + a*Cos[x]]) - ((24*I)*x*ArcTan[E^((I/2)*x)]*Cos[x/2])/(
a*Sqrt[a + a*Cos[x]]) - (I*x^3*ArcTan[E^((I/2)*x)]*Cos[x/2])/(a*Sqrt[a + a*
Cos[x]]) + ((24*I)*Cos[x/2]*PolyLog[2, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos
[x]]) + ((3*I)*x^2*Cos[x/2]*PolyLog[2, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos
[x]]) - ((24*I)*Cos[x/2]*PolyLog[2, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]])
- ((3*I)*x^2*Cos[x/2]*PolyLog[2, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) - (
12*x*Cos[x/2]*PolyLog[3, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + (12*x*
Cos[x/2]*PolyLog[3, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) - ((24*I)*Cos[x/
2]*PolyLog[4, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + ((24*I)*Cos[x/2]*
PolyLog[4, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + (x^3*Tan[x/2])/(2*a*Sqr
t[a + a*Cos[x]])
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
```

```
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3}{(a+a\cos(x))^{3/2}}\,dx = \frac{\cos\left(\frac{x}{2}\right)\int x^3\sec^3\left(\frac{x}{2}\right)dx}{2a\sqrt{a+a\cos(x)}}$$

$$= -\frac{3x^2}{a\sqrt{a+a\cos(x)}} + \frac{x^3\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}} + \frac{\cos\left(\frac{x}{2}\right)\int x^3\sec\left(\frac{x}{2}\right)dx}{4a\sqrt{a+a\cos(x)}} + \frac{\left(6\cos\left(\frac{x}{2}\right)\right)\int x\sec\left(\frac{x}{2}\right)dx}{a\sqrt{a+a\cos(x)}}$$

$$= -\frac{3x^2}{a\sqrt{a+a\cos(x)}} - \frac{24ix\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} - \frac{ix^3\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{x^3\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}} -$$

$$= -\frac{3x^2}{a\sqrt{a+a\cos(x)}} - \frac{24ix\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} - \frac{ix^3\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{3ix^2\cos\left(\frac{x}{2}\right)\text{Li}_2\left(-i\right.}{a\sqrt{a+a\cos(x)}}$$

$$= -\frac{3x^2}{a\sqrt{a+a\cos(x)}} - \frac{24ix\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} - \frac{ix^3\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{24i\cos\left(\frac{x}{2}\right)\text{Li}_2\left(-ie\right.}{a\sqrt{a+a\cos(x)}}$$

$$= -\frac{3x^2}{a\sqrt{a+a\cos(x)}} - \frac{24ix\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} - \frac{ix^3\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{24i\cos\left(\frac{x}{2}\right)\text{Li}_2\left(-ie\right.}{a\sqrt{a+a\cos(x)}}$$

$$= -\frac{3x^2}{a\sqrt{a+a\cos(x)}} - \frac{24ix\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} - \frac{ix^3\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{24i\cos\left(\frac{x}{2}\right)\text{Li}_2\left(-ie\right.}{a\sqrt{a+a\cos(x)}}$$

Mathematica [A] time = 0.459208, size = 257, normalized size = 0.61

$$-\frac{i\cos\left(\frac{x}{2}\right)\left(-6\left(x^2+8\right)\text{Li}_2\left(-ie^{\frac{ix}{2}}\right)\cos^2\left(\frac{x}{2}\right)+6\left(x^2+8\right)\text{Li}_2\left(ie^{\frac{ix}{2}}\right)\cos^2\left(\frac{x}{2}\right)-24ix\text{Li}_3\left(-ie^{\frac{ix}{2}}\right)\cos^2\left(\frac{x}{2}\right)+24ix\text{Li}_3\left(ie^{\frac{ix}{2}}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + a*Cos[x])^(3/2),x]
```

```
[Out] ((-I)*Cos[x/2]*((-6*I)*x^2*Cos[x/2] + 48*x*ArcTan[E^((I/2)*x)]*Cos[x/2]^2 +
 2*x^3*ArcTan[E^((I/2)*x)]*Cos[x/2]^2 - 6*(8 + x^2)*Cos[x/2]^2*PolyLog[2, (
-I)*E^((I/2)*x)] + 6*(8 + x^2)*Cos[x/2]^2*PolyLog[2, I*E^((I/2)*x)] - (24*I
)*x*Cos[x/2]^2*PolyLog[3, (-I)*E^((I/2)*x)] + (24*I)*x*Cos[x/2]^2*PolyLog[3
, I*E^((I/2)*x)] + 48*Cos[x/2]^2*PolyLog[4, (-I)*E^((I/2)*x)] - 48*Cos[x/2]
^2*PolyLog[4, I*E^((I/2)*x)] + I*x^3*Sin[x/2]))/(a*(1 + Cos[x]))^(3/2)
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int x^3\left(a+a\cos(x)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+a*cos(x))^(3/2),x)
```

```
[Out] int(x^3/(a+a*cos(x))^(3/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\cos\left(x\right)+a}x^{3}}{a^{2}\cos\left(x\right)^{2}+2\,a^{2}\cos\left(x\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cos(x) + a)*x^3/(a^2*cos(x)^2 + 2*a^2*cos(x) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{3}}{\left(a\left(\cos\left(x\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+a*cos(x))**(3/2),x)
```

```
[Out] Integral(x**3/(a*(cos(x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{3}}{\left(a\cos\left(x\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cos(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(a*cos(x) + a)^(3/2), x)
```

3.181 $\int \frac{x^2}{(a+a\cos(x))^{3/2}}\,dx$

Optimal. Leaf size=257

$$\frac{2ix\text{Li}_2\left(-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}-\frac{2ix\text{Li}_2\left(ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}-\frac{4\text{Li}_3\left(-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}+\frac{4\text{Li}_3\left(ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}-\frac{ix^2\cos\left(\frac{x}{2}\right)\tan^{-1}\left(e^{\frac{ix}{2}}\right)}{a\sqrt{a\cos(x)+a}}-$$

```
[Out] (-2*x)/(a*Sqrt[a + a*Cos[x]]) - (I*x^2*ArcTan[E^((I/2)*x)]*Cos[x/2])/(a*Sqr
t[a + a*Cos[x]]) + (4*ArcTanh[Sin[x/2]]*Cos[x/2])/(a*Sqrt[a + a*Cos[x]]) +
((2*I)*x*Cos[x/2]*PolyLog[2, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) - ((
2*I)*x*Cos[x/2]*PolyLog[2, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) - (4*Cos[
x/2]*PolyLog[3, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + (4*Cos[x/2]*Pol
yLog[3, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + (x^2*Tan[x/2])/(2*a*Sqrt[a
 + a*Cos[x]])
```

Rubi [A] time = 0.188985, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3319, 4186, 3770, 4181, 2531, 2282, 6589}

$$\frac{2ix\text{Li}_2\left(-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}-\frac{2ix\text{Li}_2\left(ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}-\frac{4\text{Li}_3\left(-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}+\frac{4\text{Li}_3\left(ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}-\frac{ix^2\cos\left(\frac{x}{2}\right)\tan^{-1}\left(e^{\frac{ix}{2}}\right)}{a\sqrt{a\cos(x)+a}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + a*Cos[x])^(3/2),x]
```



```
[Out] (-2*x)/(a*Sqrt[a + a*Cos[x]]) - (I*x^2*ArcTan[E^((I/2)*x)]*Cos[x/2])/(a*Sqr
t[a + a*Cos[x]]) + (4*ArcTanh[Sin[x/2]]*Cos[x/2])/(a*Sqrt[a + a*Cos[x]]) +
((2*I)*x*Cos[x/2]*PolyLog[2, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) - ((
2*I)*x*Cos[x/2]*PolyLog[2, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) - (4*Cos[
x/2]*PolyLog[3, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + (4*Cos[x/2]*Pol
yLog[3, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + (x^2*Tan[x/2])/(2*a*Sqrt[a
 + a*Cos[x]])
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{(a+a\cos(x))^{3/2}}\,dx = \frac{\cos\left(\frac{x}{2}\right)\int x^2\sec^3\left(\frac{x}{2}\right)dx}{2a\sqrt{a+a\cos(x)}}$$

$$= -\frac{2x}{a\sqrt{a+a\cos(x)}} + \frac{x^2\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}} + \frac{\cos\left(\frac{x}{2}\right)\int x^2\sec\left(\frac{x}{2}\right)dx}{4a\sqrt{a+a\cos(x)}} + \frac{\left(2\cos\left(\frac{x}{2}\right)\right)\int\sec\left(\frac{x}{2}\right)dx}{a\sqrt{a+a\cos(x)}}$$

$$= -\frac{2x}{a\sqrt{a+a\cos(x)}} - \frac{ix^2\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{4\tanh^{-1}\left(\sin\left(\frac{x}{2}\right)\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{x^2\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}} - \cdots$$

$$= -\frac{2x}{a\sqrt{a+a\cos(x)}} - \frac{ix^2\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{4\tanh^{-1}\left(\sin\left(\frac{x}{2}\right)\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{2ix\cos\left(\frac{x}{2}\right)\text{Li}_2\left(-ie^{\cdots}\right.}{a\sqrt{a+a\cos(x)}}$$

$$= -\frac{2x}{a\sqrt{a+a\cos(x)}} - \frac{ix^2\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{4\tanh^{-1}\left(\sin\left(\frac{x}{2}\right)\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{2ix\cos\left(\frac{x}{2}\right)\text{Li}_2\left(-ie^{\cdots}\right.}{a\sqrt{a+a\cos(x)}}$$

$$= -\frac{2x}{a\sqrt{a+a\cos(x)}} - \frac{ix^2\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{4\tanh^{-1}\left(\sin\left(\frac{x}{2}\right)\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{2ix\cos\left(\frac{x}{2}\right)\text{Li}_2\left(-ie^{\cdots}\right.}{a\sqrt{a+a\cos(x)}}$$

Mathematica [A] time = 0.117919, size = 185, normalized size = 0.72

$$\frac{\cos\left(\frac{x}{2}\right)\left(4ix\text{Li}_2\left(-ie^{\frac{ix}{2}}\right)\cos^2\left(\frac{x}{2}\right) - 4ix\text{Li}_2\left(ie^{\frac{ix}{2}}\right)\cos^2\left(\frac{x}{2}\right) - 8\text{Li}_3\left(-ie^{\frac{ix}{2}}\right)\cos^2\left(\frac{x}{2}\right) + 8\text{Li}_3\left(ie^{\frac{ix}{2}}\right)\cos^2\left(\frac{x}{2}\right) + x^2\sin\left(\frac{x}{2}\right) - 2ix^2 \cdots\right.}{(a(\cos(x)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + a*Cos[x])^(3/2),x]
```

```
[Out] (Cos[x/2]*(-4*x*Cos[x/2] - (2*I)*x^2*ArcTan[E^((I/2)*x)]*Cos[x/2]^2 + 8*Arc
Tanh[Sin[x/2]]*Cos[x/2]^2 + (4*I)*x*Cos[x/2]^2*PolyLog[2, (-I)*E^((I/2)*x)]
 - (4*I)*x*Cos[x/2]^2*PolyLog[2, I*E^((I/2)*x)] - 8*Cos[x/2]^2*PolyLog[3, (
-I)*E^((I/2)*x)] + 8*Cos[x/2]^2*PolyLog[3, I*E^((I/2)*x)] + x^2*Sin[x/2]))/
(a*(1 + Cos[x]))^(3/2)
```

Maple [F] time = 0.116, size = 0, normalized size = 0.

$$\int x^{2} \left(a + a\cos\left(x\right)\right)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+a*cos(x))^(3/2),x)
```

```
[Out] int(x^2/(a+a*cos(x))^(3/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\cos\left(x\right) + a}x^{2}}{a^{2}\cos\left(x\right)^{2} + 2\,a^{2}\cos\left(x\right) + a^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cos(x) + a)*x^2/(a^2*cos(x)^2 + 2*a^2*cos(x) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\left(a\left(\cos\left(x\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+a*cos(x))**(3/2),x)
```

```
[Out] Integral(x**2/(a*(cos(x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a\cos(x) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cos(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(a*cos(x) + a)^(3/2), x)
```

3.182 $\int \frac{x}{(a+a\cos(x))^{3/2}}\,dx$

Optimal. Leaf size=150

$$\frac{i\text{Li}_2\left(-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}-\frac{i\text{Li}_2\left(ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}-\frac{1}{a\sqrt{a\cos(x)+a}}-\frac{ix\cos\left(\frac{x}{2}\right)\tan^{-1}\left(e^{\frac{ix}{2}}\right)}{a\sqrt{a\cos(x)+a}}+\frac{x\tan\left(\frac{x}{2}\right)}{2a\sqrt{a\cos(x)+a}}$$

```
[Out] -(1/(a*Sqrt[a + a*Cos[x]])) - (I*x*ArcTan[E^((I/2)*x)]*Cos[x/2])/(a*Sqrt[a
+ a*Cos[x]]) + (I*Cos[x/2]*PolyLog[2, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[
x]]) - (I*Cos[x/2]*PolyLog[2, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + (x*T
an[x/2])/(2*a*Sqrt[a + a*Cos[x]])
```

Rubi [A] time = 0.11658, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3319, 4185, 4181, 2279, 2391}

$$\frac{i\text{Li}_2\left(-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}-\frac{i\text{Li}_2\left(ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}}-\frac{1}{a\sqrt{a\cos(x)+a}}-\frac{ix\cos\left(\frac{x}{2}\right)\tan^{-1}\left(e^{\frac{ix}{2}}\right)}{a\sqrt{a\cos(x)+a}}+\frac{x\tan\left(\frac{x}{2}\right)}{2a\sqrt{a\cos(x)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + a*Cos[x])^(3/2),x]
```

```
[Out] -(1/(a*Sqrt[a + a*Cos[x]])) - (I*x*ArcTan[E^((I/2)*x)]*Cos[x/2])/(a*Sqrt[a
+ a*Cos[x]]) + (I*Cos[x/2]*PolyLog[2, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[
x]]) - (I*Cos[x/2]*PolyLog[2, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + (x*T
an[x/2])/(2*a*Sqrt[a + a*Cos[x]])
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x}{(a+a\cos(x))^{3/2}}\,dx = \frac{\cos\left(\frac{x}{2}\right)\int x\sec^3\left(\frac{x}{2}\right)dx}{2a\sqrt{a+a\cos(x)}}$$
$$= -\frac{1}{a\sqrt{a+a\cos(x)}} + \frac{x\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}} + \frac{\cos\left(\frac{x}{2}\right)\int x\sec\left(\frac{x}{2}\right)dx}{4a\sqrt{a+a\cos(x)}}$$
$$= -\frac{1}{a\sqrt{a+a\cos(x)}} - \frac{ix\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{x\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}} - \frac{\cos\left(\frac{x}{2}\right)\int\log\left(1-ie^{\frac{ix}{2}}\right)dx}{2a\sqrt{a+a\cos(x)}} + \frac{\text{co}}{}$$
$$= -\frac{1}{a\sqrt{a+a\cos(x)}} - \frac{ix\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{x\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}} + \frac{\left(i\cos\left(\frac{x}{2}\right)\right)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx\right.}{a\sqrt{a+a\cos(x)}}$$
$$= -\frac{1}{a\sqrt{a+a\cos(x)}} - \frac{ix\tan^{-1}\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{i\cos\left(\frac{x}{2}\right)\text{Li}_2\left(-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} - \frac{i\cos\left(\frac{x}{2}\right)\text{Li}_2\left(ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} + \frac{x\,\text{t}}{2a\sqrt{a}}$$

Mathematica [A] time = 0.179946, size = 165, normalized size = 1.1

$$\frac{\sec\left(\frac{x}{2}\right)\left(2i\text{Li}_2\left(-ie^{\frac{ix}{2}}\right)(\cos(x)+1)-2i\text{Li}_2\left(ie^{\frac{ix}{2}}\right)(\cos(x)+1)+x\log\left(1-ie^{\frac{ix}{2}}\right)-x\log\left(1+ie^{\frac{ix}{2}}\right)+2x\sin\left(\frac{x}{2}\right)-4\cos\left(\frac{x}{2}\right)\right.}{4a\sqrt{a(\cos(x)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + a*Cos[x])^(3/2),x]
```

```
[Out] (Sec[x/2]*(-4*Cos[x/2] + x*Log[1 - I*E^((I/2)*x)] + x*Cos[x]*Log[1 - I*E^((
I/2)*x)] - x*Log[1 + I*E^((I/2)*x)] - x*Cos[x]*Log[1 + I*E^((I/2)*x)] + (2*
I)*(1 + Cos[x])*PolyLog[2, (-I)*E^((I/2)*x)] - (2*I)*(1 + Cos[x])*PolyLog[2
, I*E^((I/2)*x)] + 2*x*Sin[x/2]))/(4*a*Sqrt[a*(1 + Cos[x])])
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int x\left(a+a\cos(x)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+a*cos(x))^(3/2),x)
```

```
[Out] int(x/(a+a*cos(x))^(3/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\cos\left(x\right)+a}x}{a^2\cos\left(x\right)^2+2\,a^2\cos\left(x\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cos(x) + a)*x/(a^2*cos(x)^2 + 2*a^2*cos(x) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\left(a\left(\cos\left(x\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cos(x))**(3/2),x)
```

```
[Out] Integral(x/(a*(cos(x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\left(a\cos\left(x\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cos(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(a*cos(x) + a)^(3/2), x)
```

3.183 $\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{x(a\cos(x)+a)^{3/2}},x\right)$$

```
[Out] Unintegrable[1/(x*(a + a*Cos[x])^(3/2)), x]
```

Rubi [A] time = 0.0786225, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + a*Cos[x])^(3/2)),x]
```

```
[Out] Defer[Int][1/(x*(a + a*Cos[x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx = \int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx$$

Mathematica [A] time = 11.3057, size = 0, normalized size = 0.

$$\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + a*Cos[x])^(3/2)),x]
```

```
[Out] Integrate[1/(x*(a + a*Cos[x])^(3/2)), x]
```

Maple [A] time = 0.112, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+a\cos(x)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+a*cos(x))^(3/2),x)
```

```
[Out] int(1/x/(a+a*cos(x))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(x) + a)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(x) + a)^(3/2)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a\cos(x) + a}}{a^2 x\cos(x)^2 + 2\, a^2 x\cos(x) + a^2 x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cos(x) + a)/(a^2*x*cos(x)^2 + 2*a^2*x*cos(x) + a^2*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,(a\,(\cos(x) + 1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cos(x))**(3/2),x)
```

```
[Out] Integral(1/(x*(a*(cos(x) + 1))**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(x) + a)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cos(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(x) + a)^(3/2)*x), x)
```

3.184 $\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{\sqrt[3]{a\cos(c+dx)+a}}{x},x\right)$$

```
[Out] Unintegrable[(a + a*Cos[c + d*x])^(1/3)/x, x]
```

Rubi [A] time = 0.0648875, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + a*Cos[c + d*x])^(1/3)/x,x]
```

```
[Out] Defer[Int][(a + a*Cos[c + d*x])^(1/3)/x, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx=\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx$$

Mathematica [A] time = 1.0715, size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Cos[c + d*x])^(1/3)/x,x]
```

```
[Out] Integrate[(a + a*Cos[c + d*x])^(1/3)/x, x]
```

Maple [A] time = 0.164, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt[3]{a+\cos(dx+c)\,a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/3)/x,x)
```

```
[Out] int((a+cos(d*x+c)*a)^(1/3)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/3)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(1/3)/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/3)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a(\cos(c+dx)+1)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/3)/x,x)
```

```
[Out] Integral((a*(cos(c + d*x) + 1))**(1/3)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/3)/x,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(1/3)/x, x)
```

3.185 $\int \frac{x^3}{a+b\cos(x)}\,dx$

Optimal. Leaf size=383

$$-\frac{3x^2\text{Li}_2\left(-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{3x^2\text{Li}_2\left(-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{6ix\text{Li}_3\left(-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{6ix\text{Li}_3\left(-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{6\text{Li}_4\left(-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{6\text{Li}_4\left(-\cdots\right.}{\sqrt{a^2\cdots}}$$

```
[Out] ((-I)*x^3*Log[1 + (b*E^(I*x))/(a - Sqrt[a^2 - b^2])])/Sqrt[a^2 - b^2] + (I*
x^3*Log[1 + (b*E^(I*x))/(a + Sqrt[a^2 - b^2])])/Sqrt[a^2 - b^2] - (3*x^2*Po
lyLog[2, -((b*E^(I*x))/(a - Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2] + (3*x^2*Po
lyLog[2, -((b*E^(I*x))/(a + Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2] - ((6*I)*x*
PolyLog[3, -((b*E^(I*x))/(a - Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2] + ((6*I)*
x*PolyLog[3, -((b*E^(I*x))/(a + Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2] + (6*Po
lyLog[4, -((b*E^(I*x))/(a - Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2] - (6*PolyLo
g[4, -((b*E^(I*x))/(a + Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2]
```

Rubi [A] time = 0.558972, antiderivative size = 383, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3321, 2264, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3x^2\text{Li}_2\left(-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{3x^2\text{Li}_2\left(-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{6ix\text{Li}_3\left(-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{6ix\text{Li}_3\left(-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{6\text{Li}_4\left(-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{6\text{Li}_4\left(-\cdots\right.}{\sqrt{a^2\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Cos[x]),x]
```

```
[Out] ((-I)*x^3*Log[1 + (b*E^(I*x))/(a - Sqrt[a^2 - b^2])])/Sqrt[a^2 - b^2] + (I*
x^3*Log[1 + (b*E^(I*x))/(a + Sqrt[a^2 - b^2])])/Sqrt[a^2 - b^2] - (3*x^2*Po
lyLog[2, -((b*E^(I*x))/(a - Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2] + (3*x^2*Po
lyLog[2, -((b*E^(I*x))/(a + Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2] - ((6*I)*x*
PolyLog[3, -((b*E^(I*x))/(a - Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2] + ((6*I)*
x*PolyLog[3, -((b*E^(I*x))/(a + Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2] + (6*Po
lyLog[4, -((b*E^(I*x))/(a - Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2] - (6*PolyLo
g[4, -((b*E^(I*x))/(a + Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2]
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3}{a+b\cos(x)}\,dx = 2\int \frac{e^{ix}x^3}{b+2ae^{ix}+be^{2ix}}\,dx$$
$$= \frac{(2b)\int \frac{e^{ix}x^3}{2a-2\sqrt{a^2-b^2}+2be^{ix}}\,dx}{\sqrt{a^2-b^2}} - \frac{(2b)\int \frac{e^{ix}x^3}{2a+2\sqrt{a^2-b^2}+2be^{ix}}\,dx}{\sqrt{a^2-b^2}}$$
$$= -\frac{ix^3\log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{ix^3\log\left(1+\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{(3i)\int x^2\log\left(1+\frac{2be^{ix}}{2a-2\sqrt{a^2-b^2}}\right)dx}{\sqrt{a^2-b^2}} - \frac{(3i)\int x^2\log}{}$$
$$= -\frac{ix^3\log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{ix^3\log\left(1+\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{3x^2\mathrm{Li}_2\left(-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{3x^2\mathrm{Li}_2\left(-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{6\int}{}$$
$$= -\frac{ix^3\log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{ix^3\log\left(1+\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{3x^2\mathrm{Li}_2\left(-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{3x^2\mathrm{Li}_2\left(-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{6ix}{}$$
$$= -\frac{ix^3\log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{ix^3\log\left(1+\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{3x^2\mathrm{Li}_2\left(-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{3x^2\mathrm{Li}_2\left(-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{6ix}{}$$
$$= -\frac{ix^3\log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{ix^3\log\left(1+\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{3x^2\mathrm{Li}_2\left(-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{3x^2\mathrm{Li}_2\left(-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{6ix}{}$$

Mathematica [A] time = 0.898692, size = 290, normalized size = 0.76

$$\frac{-3x^2\mathrm{Li}_2\left(\frac{be^{ix}}{\sqrt{a^2-b^2}-a}\right) + 3x^2\mathrm{Li}_2\left(-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right) - 6ix\mathrm{Li}_3\left(\frac{be^{ix}}{\sqrt{a^2-b^2}-a}\right) + 6ix\mathrm{Li}_3\left(-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right) + 6\mathrm{Li}_4\left(\frac{be^{ix}}{\sqrt{a^2-b^2}-a}\right) - 6\mathrm{Li}_4\left(-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Cos[x]),x]
```

```
[Out] ((-I)*x^3*Log[1 + (b*E^(I*x))/(a - Sqrt[a^2 - b^2])] + I*x^3*Log[1 + (b*E^(
I*x))/(a + Sqrt[a^2 - b^2])] - 3*x^2*PolyLog[2, (b*E^(I*x))/(-a + Sqrt[a^2
- b^2])] + 3*x^2*PolyLog[2, -((b*E^(I*x))/(a + Sqrt[a^2 - b^2]))] - (6*I)*x
*PolyLog[3, (b*E^(I*x))/(-a + Sqrt[a^2 - b^2])] + (6*I)*x*PolyLog[3, -((b*E
^(I*x))/(a + Sqrt[a^2 - b^2]))] + 6*PolyLog[4, (b*E^(I*x))/(-a + Sqrt[a^2 -
 b^2])] - 6*PolyLog[4, -((b*E^(I*x))/(a + Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^
2]
```

Maple [F] time = 0.231, size = 0, normalized size = 0.

$$\int \frac{x^3}{a+b\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*cos(x)),x)
```

```
[Out] int(x^3/(a+b*cos(x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.53652, size = 2719, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*I*b*x^3*sqrt((a^2 - b^2)/b^2)*log(1/2*(2*a*cos(x) + 2*I*a*sin(x) +
2*(b*cos(x) + I*b*sin(x))*sqrt((a^2 - b^2)/b^2) + 2*b)/b) - 2*I*b*x^3*sqrt(
(a^2 - b^2)/b^2)*log(1/2*(2*a*cos(x) + 2*I*a*sin(x) - 2*(b*cos(x) + I*b*sin
(x))*sqrt((a^2 - b^2)/b^2) + 2*b)/b) - 2*I*b*x^3*sqrt((a^2 - b^2)/b^2)*log(
1/2*(2*a*cos(x) - 2*I*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt((a^2 - b^2)
/b^2) + 2*b)/b) + 2*I*b*x^3*sqrt((a^2 - b^2)/b^2)*log(1/2*(2*a*cos(x) - 2*I
*a*sin(x) - 2*(b*cos(x) - I*b*sin(x))*sqrt((a^2 - b^2)/b^2) + 2*b)/b) + 6*b
*x^2*sqrt((a^2 - b^2)/b^2)*dilog(-1/2*(2*a*cos(x) + 2*I*a*sin(x) + 2*(b*cos
(x) + I*b*sin(x))*sqrt((a^2 - b^2)/b^2) + 2*b)/b + 1) - 6*b*x^2*sqrt((a^2 -
 b^2)/b^2)*dilog(-1/2*(2*a*cos(x) + 2*I*a*sin(x) - 2*(b*cos(x) + I*b*sin(x)
)*sqrt((a^2 - b^2)/b^2) + 2*b)/b + 1) + 6*b*x^2*sqrt((a^2 - b^2)/b^2)*dilog
(-1/2*(2*a*cos(x) - 2*I*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt((a^2 - b^
2)/b^2) + 2*b)/b + 1) - 6*b*x^2*sqrt((a^2 - b^2)/b^2)*dilog(-1/2*(2*a*cos(x
) - 2*I*a*sin(x) - 2*(b*cos(x) - I*b*sin(x))*sqrt((a^2 - b^2)/b^2) + 2*b)/b
 + 1) + 12*I*b*x*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cos(x) + I*a*sin(x) +
 (b*cos(x) + I*b*sin(x))*sqrt((a^2 - b^2)/b^2))/b) - 12*I*b*x*sqrt((a^2 - b
^2)/b^2)*polylog(3, -(a*cos(x) + I*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(
(a^2 - b^2)/b^2))/b) - 12*I*b*x*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cos(x)
 - I*a*sin(x) + (b*cos(x) - I*b*sin(x))*sqrt((a^2 - b^2)/b^2))/b) + 12*I*b*
x*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cos(x) - I*a*sin(x) - (b*cos(x) - I*
b*sin(x))*sqrt((a^2 - b^2)/b^2))/b) - 12*b*sqrt((a^2 - b^2)/b^2)*polylog(4,
 -(a*cos(x) + I*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt((a^2 - b^2)/b^2))/b
) + 12*b*sqrt((a^2 - b^2)/b^2)*polylog(4, -(a*cos(x) + I*a*sin(x) - (b*cos(
x) + I*b*sin(x))*sqrt((a^2 - b^2)/b^2))/b) - 12*b*sqrt((a^2 - b^2)/b^2)*pol
ylog(4, -(a*cos(x) - I*a*sin(x) + (b*cos(x) - I*b*sin(x))*sqrt((a^2 - b^2)/
b^2))/b) + 12*b*sqrt((a^2 - b^2)/b^2)*polylog(4, -(a*cos(x) - I*a*sin(x) -
(b*cos(x) - I*b*sin(x))*sqrt((a^2 - b^2)/b^2))/b))/(a^2 - b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{a + b\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*cos(x)),x)
```

```
[Out] Integral(x**3/(a + b*cos(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{b\cos(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*cos(x) + a), x)
```

3.186 $\int \frac{x^2}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=329

$$-\frac{2x\text{Li}_2\left(-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\sqrt{a^2-b^2}}+\frac{2x\text{Li}_2\left(-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2\sqrt{a^2-b^2}}-\frac{2i\text{Li}_3\left(-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^3\sqrt{a^2-b^2}}+\frac{2i\text{Li}_3\left(-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^3\sqrt{a^2-b^2}}-\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}+\frac{ix^2\log}{}$$

```
[Out] ((-I)*x^2*Log[1 + (b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b
^2]*d) + (I*x^2*Log[1 + (b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a
^2 - b^2]*d) - (2*x*PolyLog[2, -((b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]))
])/(Sqrt[a^2 - b^2]*d^2) + (2*x*PolyLog[2, -((b*E^(I*(c + d*x)))/(a + Sqrt[
a^2 - b^2]))])/(Sqrt[a^2 - b^2]*d^2) - ((2*I)*PolyLog[3, -((b*E^(I*(c + d*x
)))/(a - Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*d^3) + ((2*I)*PolyLog[3, -((b
*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*d^3)
```

Rubi [A] time = 0.662918, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3321, 2264, 2190, 2531, 2282, 6589}

$$-\frac{2x\text{Li}_2\left(-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\sqrt{a^2-b^2}}+\frac{2x\text{Li}_2\left(-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2\sqrt{a^2-b^2}}-\frac{2i\text{Li}_3\left(-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^3\sqrt{a^2-b^2}}+\frac{2i\text{Li}_3\left(-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^3\sqrt{a^2-b^2}}-\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}+\frac{ix^2\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((-I)*x^2*Log[1 + (b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b
^2]*d) + (I*x^2*Log[1 + (b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a
^2 - b^2]*d) - (2*x*PolyLog[2, -((b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]))
])/(Sqrt[a^2 - b^2]*d^2) + (2*x*PolyLog[2, -((b*E^(I*(c + d*x)))/(a + Sqrt[
a^2 - b^2]))])/(Sqrt[a^2 - b^2]*d^2) - ((2*I)*PolyLog[3, -((b*E^(I*(c + d*x
)))/(a - Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*d^3) + ((2*I)*PolyLog[3, -((b
*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*d^3)
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{a+b\cos(c+dx)}\,dx = 2\int \frac{e^{i(c+dx)}x^2}{b+2ae^{i(c+dx)}+be^{2i(c+dx)}}\,dx$$

$$= \frac{(2b)\int \frac{e^{i(c+dx)}x^2}{2a-2\sqrt{a^2-b^2}+2be^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}} - \frac{(2b)\int \frac{e^{i(c+dx)}x^2}{2a+2\sqrt{a^2-b^2}+2be^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}}$$

$$= -\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} + \frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} + \frac{(2i)\int x\log\left(1+\frac{2be^{i(c+dx)}}{2a-2\sqrt{a^2-b^2}}\right)dx}{\sqrt{a^2-b^2}d} - \frac{(2i)\int x\,\mathrm{l}}{}$$

$$= -\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} + \frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} - \frac{2x\mathrm{Li}_2\left(-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} + \frac{2x\mathrm{Li}_2\left(-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} +$$

$$= -\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} + \frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} - \frac{2x\mathrm{Li}_2\left(-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} + \frac{2x\mathrm{Li}_2\left(-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} -$$

$$= -\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} + \frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} - \frac{2x\mathrm{Li}_2\left(-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} + \frac{2x\mathrm{Li}_2\left(-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} -$$

Mathematica [A] time = 0.701405, size = 379, normalized size = 1.15

$$\frac{e^{ic}\left(-i\left(d^2x^2\log\left(1+\frac{be^{i(2c+dx)}}{ae^{ic}-\sqrt{e^{2ic}\left(a^2-b^2\right)}}\right)-d^2x^2\log\left(1+\frac{be^{i(2c+dx)}}{\sqrt{e^{2ic}\left(a^2-b^2\right)}+ae^{ic}}\right)+2idx\mathrm{Li}_2\left(-\frac{be^{i(2c+dx)}}{e^{ic}a+\sqrt{\left(a^2-b^2\right)e^{2ic}}}\right)+2\mathrm{Li}_3\left(-\frac{be^{i(2c+dx)}}{ae^{ic}-\sqrt{\left(a^2-b^2\right)e^{2}}}\right.\right.\right.}{d^3\sqrt{e^{2ic}\left(a^2-b^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Cos[c + d*x]),x]
```

```
[Out] (E^(I*c)*(-2*d*x*PolyLog[2, -((b*E^(I*(2*c + d*x)))/(a*E^(I*c) - Sqrt[(a^2
- b^2)*E^((2*I)*c)]))] - I*(d^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))/(a*E^(I*c
) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - d^2*x^2*Log[1 + (b*E^(I*(2*c + d*x)))
/(a*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + (2*I)*d*x*PolyLog[2, -((b*E
^(I*(2*c + d*x)))/(a*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] + 2*PolyLog
[3, -((b*E^(I*(2*c + d*x)))/(a*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] -
 2*PolyLog[3, -((b*E^(I*(2*c + d*x)))/(a*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I
)*c)]))])))/(d^3*Sqrt[(a^2 - b^2)*E^((2*I)*c)])
```

Maple [F] time = 0.44, size = 0, normalized size = 0.

$$\int \frac{x^2}{a + b\cos(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*cos(d*x+c)),x)
```

```
[Out] int(x^2/(a+b*cos(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.51322, size = 3218, normalized size = 9.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*b*d*x*sqrt((a^2 - b^2)/b^2)*dilog(-1/2*(2*a*cos(d*x + c) + 2*I*a*si
n(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) +
2*b)/b + 1) - 4*b*d*x*sqrt((a^2 - b^2)/b^2)*dilog(-1/2*(2*a*cos(d*x + c) +
2*I*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)
/b^2) + 2*b)/b + 1) + 4*b*d*x*sqrt((a^2 - b^2)/b^2)*dilog(-1/2*(2*a*cos(d*x
 + c) - 2*I*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^
2 - b^2)/b^2) + 2*b)/b + 1) - 4*b*d*x*sqrt((a^2 - b^2)/b^2)*dilog(-1/2*(2*a
*cos(d*x + c) - 2*I*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*
sqrt((a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*I*b*c^2*sqrt((a^2 - b^2)/b^2)*log(2
*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 2
*I*b*c^2*sqrt((a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) +
```

```
2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 2*I*b*c^2*sqrt((a^2 - b^2)/b^2)*log(-2*b
*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) - 2*a) + 2*I
*b*c^2*sqrt((a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2
*b*sqrt((a^2 - b^2)/b^2) - 2*a) + 2*(I*b*d^2*x^2 - I*b*c^2)*sqrt((a^2 - b^2
)/b^2)*log(1/2*(2*a*cos(d*x + c) + 2*I*a*sin(d*x + c) + 2*(b*cos(d*x + c) +
 I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + 2*b)/b) + 2*(-I*b*d^2*x^2 + I*b*
c^2)*sqrt((a^2 - b^2)/b^2)*log(1/2*(2*a*cos(d*x + c) + 2*I*a*sin(d*x + c) -
 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + 2*b)/b) + 2*
(-I*b*d^2*x^2 + I*b*c^2)*sqrt((a^2 - b^2)/b^2)*log(1/2*(2*a*cos(d*x + c) -
2*I*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^2)
/b^2) + 2*b)/b) + 2*(I*b*d^2*x^2 - I*b*c^2)*sqrt((a^2 - b^2)/b^2)*log(1/2*(
2*a*cos(d*x + c) - 2*I*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c
))*sqrt((a^2 - b^2)/b^2) + 2*b)/b) + 4*I*b*sqrt((a^2 - b^2)/b^2)*polylog(3,
 -1/2*(2*a*cos(d*x + c) + 2*I*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(
d*x + c))*sqrt((a^2 - b^2)/b^2))/b) - 4*I*b*sqrt((a^2 - b^2)/b^2)*polylog(3
, -1/2*(2*a*cos(d*x + c) + 2*I*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin
(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) - 4*I*b*sqrt((a^2 - b^2)/b^2)*polylog(
3, -1/2*(2*a*cos(d*x + c) - 2*I*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*si
n(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) + 4*I*b*sqrt((a^2 - b^2)/b^2)*polylog
(3, -1/2*(2*a*cos(d*x + c) - 2*I*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*s
in(d*x + c))*sqrt((a^2 - b^2)/b^2))/b))/((a^2 - b^2)*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{a + b\cos(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral(x**2/(a + b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{b\cos(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*cos(d*x + c) + a), x)
```

3.187 $\int \frac{x}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=214

$$-\frac{\text{Li}_2\left(-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\sqrt{a^2-b^2}}+\frac{\text{Li}_2\left(-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2\sqrt{a^2-b^2}}-\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}+\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d\sqrt{a^2-b^2}}$$

```
[Out] ((-I)*x*Log[1 + (b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2
]*d) + (I*x*Log[1 + (b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 -
 b^2]*d) - PolyLog[2, -((b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]))]/(Sqrt[a
^2 - b^2]*d^2) + PolyLog[2, -((b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2]))]/(
Sqrt[a^2 - b^2]*d^2)
```

Rubi [A] time = 0.400171, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3321, 2264, 2190, 2279, 2391}

$$-\frac{\text{Li}_2\left(-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\sqrt{a^2-b^2}}+\frac{\text{Li}_2\left(-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2\sqrt{a^2-b^2}}-\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}+\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Cos[c + d*x]),x]
```



```
[Out] ((-I)*x*Log[1 + (b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2
]*d) + (I*x*Log[1 + (b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 -
 b^2]*d) - PolyLog[2, -((b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]))]/(Sqrt[a
^2 - b^2]*d^2) + PolyLog[2, -((b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2]))]/(
Sqrt[a^2 - b^2]*d^2)
```


Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{a+b\cos(c+dx)}\,dx &= 2\int \frac{e^{i(c+dx)}x}{b+2ae^{i(c+dx)}+be^{2i(c+dx)}}\,dx\\
&= \frac{(2b)\int \frac{e^{i(c+dx)}x}{2a-2\sqrt{a^2-b^2}+2be^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}} - \frac{(2b)\int \frac{e^{i(c+dx)}x}{2a+2\sqrt{a^2-b^2}+2be^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}}\\
&= -\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} + \frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} + \frac{i\int\log\left(1+\frac{2be^{i(c+dx)}}{2a-2\sqrt{a^2-b^2}}\right)dx}{\sqrt{a^2-b^2}d} - \frac{i\int\log\left(1+\frac{2b}{2a+}\right.}{\sqrt{a^2-}}\\
&= -\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} + \frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} + \frac{\text{Subst}\left(\int\frac{\log\left(1+\frac{2bx}{2a-2\sqrt{a^2-b^2}}\right)}{x}dx, x, e^{i(c+dx)}\right)}{\sqrt{a^2-b^2}d^2} - \frac{\text{Su}}{}\\
&= -\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} + \frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} - \frac{\text{Li}_2\left(-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} + \frac{\text{Li}_2\left(-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2}
\end{aligned}$$

Mathematica [B] time = 0.810934, size = 756, normalized size = 3.53

$$\frac{i\left(\text{Li}_2\left(\frac{\left(a-i\sqrt{b^2-a^2}\right)\left(a+b-\sqrt{b^2-a^2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b\left(a+b+\sqrt{b^2-a^2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}\right)-\text{Li}_2\left(\frac{\left(a+i\sqrt{b^2-a^2}\right)\left(a+b-\sqrt{b^2-a^2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b\left(a+b+\sqrt{b^2-a^2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}\right)\right)+2(c+dx)\tanh^{-1}\left(\frac{(a+b)\cot\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*(c + d*x)*ArcTanh[((a + b)*Cot[(c + d*x)/2])/Sqrt[-a^2 + b^2]] - 2*(c +
ArcCos[-(a/b)])*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]] + (Ar
cCos[-(a/b)] - (2*I)*ArcTanh[((a + b)*Cot[(c + d*x)/2])/Sqrt[-a^2 + b^2]] +
 (2*I)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])*Log[Sqrt[-a^2
 + b^2]/(Sqrt[2]*Sqrt[b]*E^((I/2)*(c + d*x))*Sqrt[a + b*Cos[c + d*x]])] + (
ArcCos[-(a/b)] + (2*I)*(ArcTanh[((a + b)*Cot[(c + d*x)/2])/Sqrt[-a^2 + b^2]
] - ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]]))*Log[(Sqrt[-a^2
+ b^2]*E^((I/2)*(c + d*x)))/(Sqrt[2]*Sqrt[b]*Sqrt[a + b*Cos[c + d*x]])] - (
ArcCos[-(a/b)] - (2*I)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]
])*Log[((a + b)*(-a + b - I*Sqrt[-a^2 + b^2])*(1 + I*Tan[(c + d*x)/2]))/(b*
(a + b + Sqrt[-a^2 + b^2]*Tan[(c + d*x)/2]))] - (ArcCos[-(a/b)] + (2*I)*Arc
Tanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])*Log[((a + b)*(I*a - I*b
 + Sqrt[-a^2 + b^2])*(I + Tan[(c + d*x)/2]))/(b*(a + b + Sqrt[-a^2 + b^2]*T
an[(c + d*x)/2]))] + I*(PolyLog[2, ((a - I*Sqrt[-a^2 + b^2])*(a + b - Sqrt[
-a^2 + b^2]*Tan[(c + d*x)/2]))/(b*(a + b + Sqrt[-a^2 + b^2]*Tan[(c + d*x)/2
]))] - PolyLog[2, ((a + I*Sqrt[-a^2 + b^2])*(a + b - Sqrt[-a^2 + b^2]*Tan[(
c + d*x)/2]))/(b*(a + b + Sqrt[-a^2 + b^2]*Tan[(c + d*x)/2]))]))/(Sqrt[-a^2
 + b^2]*d^2)
```

Maple [B] time = 0.418, size = 414, normalized size = 1.9

$$\frac{ix}{d}\ln\left(\left(b\mathrm{e}^{i(dx+c)}+\sqrt{a^2-b^2}+a\right)\left(a+\sqrt{a^2-b^2}\right)^{-1}\right)\frac{1}{\sqrt{a^2-b^2}}-\frac{ix}{d}\ln\left(\left(-b\mathrm{e}^{i(dx+c)}+\sqrt{a^2-b^2}-a\right)\left(-a+\sqrt{a^2-b^2}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*cos(d*x+c)),x)
```

```
[Out] I/d/(a^2-b^2)^(1/2)*ln((b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1
/2)))*x-I/d/(a^2-b^2)^(1/2)*ln((-b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)-a)/(-a+(a
^2-b^2)^(1/2)))*x+I/d^2/(a^2-b^2)^(1/2)*ln((b*exp(I*(d*x+c))+(a^2-b^2)^(1/2
)+a)/(a+(a^2-b^2)^(1/2)))*c-I/d^2/(a^2-b^2)^(1/2)*ln((-b*exp(I*(d*x+c))+(a^
2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*c-1/d^2/(a^2-b^2)^(1/2)*dilog((-b*exp
(I*(d*x+c))+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))+1/d^2/(a^2-b^2)^(1/2)*
dilog((b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))+2*I/d^2*c/(
-a^2+b^2)^(1/2)*arctan(1/2*(2*b*exp(I*(d*x+c))+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.37501, size = 2338, normalized size = 10.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*I*b*c*sqrt((a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x +
c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 2*I*b*c*sqrt((a^2 - b^2)/b^2)*log(2
*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 2
*I*b*c*sqrt((a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2
*b*sqrt((a^2 - b^2)/b^2) - 2*a) - 2*I*b*c*sqrt((a^2 - b^2)/b^2)*log(-2*b*co
s(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) - 2*a) + 2*b*sq
rt((a^2 - b^2)/b^2)*dilog(-1/2*(2*a*cos(d*x + c) + 2*I*a*sin(d*x + c) + 2*(
b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*
b*sqrt((a^2 - b^2)/b^2)*dilog(-1/2*(2*a*cos(d*x + c) + 2*I*a*sin(d*x + c) -
 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + 2*b)/b + 1)
+ 2*b*sqrt((a^2 - b^2)/b^2)*dilog(-1/2*(2*a*cos(d*x + c) - 2*I*a*sin(d*x +
c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + 2*b)/b +
 1) - 2*b*sqrt((a^2 - b^2)/b^2)*dilog(-1/2*(2*a*cos(d*x + c) - 2*I*a*sin(d*
x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + 2*b)
/b + 1) + 2*(I*b*d*x + I*b*c)*sqrt((a^2 - b^2)/b^2)*log(1/2*(2*a*cos(d*x +
c) + 2*I*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 -
```

```
 b^2)/b^2) + 2*b)/b) + 2*(-I*b*d*x - I*b*c)*sqrt((a^2 - b^2)/b^2)*log(1/2*(
2*a*cos(d*x + c) + 2*I*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I*b*sin(d*x + c
))*sqrt((a^2 - b^2)/b^2) + 2*b)/b) + 2*(-I*b*d*x - I*b*c)*sqrt((a^2 - b^2)/
b^2)*log(1/2*(2*a*cos(d*x + c) - 2*I*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I
*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + 2*b)/b) + 2*(I*b*d*x + I*b*c)*sqrt
((a^2 - b^2)/b^2)*log(1/2*(2*a*cos(d*x + c) - 2*I*a*sin(d*x + c) - 2*(b*cos
(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + 2*b)/b))/((a^2 - b^2)
*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{a + b\cos(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral(x/(a + b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b\cos(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x/(b*cos(d*x + c) + a), x)
```

3.188 $\int \frac{1}{x(a+b\cos(x))} \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{1}{x(a+b\cos(x))},x\right)$$

```
[Out] Unintegrable[1/(x*(a + b*Cos[x])), x]
```

Rubi [A] time = 0.0485615, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x(a+b\cos(x))} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + b*Cos[x])),x]
```

```
[Out] Defer[Int][1/(x*(a + b*Cos[x])), x]
```

Rubi steps

$$\int \frac{1}{x(a+b\cos(x))} \, dx = \int \frac{1}{x(a+b\cos(x))} \, dx$$

Mathematica [A] time = 0.818925, size = 0, normalized size = 0.

$$\int \frac{1}{x(a+b\cos(x))} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + b*Cos[x])),x]
```

```
[Out] Integrate[1/(x*(a + b*Cos[x])), x]
```

Maple [A] time = 0.191, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\cos\left(x\right)\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*cos(x)),x)
```

```
[Out] int(1/x/(a+b*cos(x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(x)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(x) + a)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bx\cos(x)+ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x*cos(x) + a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x(a+b\cos(x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cos(x)),x)
```

```
[Out] Integral(1/(x*(a + b*cos(x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(x)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(x) + a)*x), x)
```

3.189 $\int \frac{e+fx}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=296

$$-\frac{af\mathrm{Li}_2\left(-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\left(a^2-b^2\right)^{3/2}}+\frac{af\mathrm{Li}_2\left(-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2\left(a^2-b^2\right)^{3/2}}-\frac{f\log(a+b\cos(c+dx))}{d^2\left(a^2-b^2\right)}-\frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{ia(e+fx)\log}{d\left(a^2\right.}$$

```
[Out] ((-I)*a*(e + f*x)*Log[1 + (b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2
 - b^2)^(3/2)*d) + (I*a*(e + f*x)*Log[1 + (b*E^(I*(c + d*x)))/(a + Sqrt[a^2
 - b^2])])/((a^2 - b^2)^(3/2)*d) - (f*Log[a + b*Cos[c + d*x]])/((a^2 - b^2)
*d^2) - (a*f*PolyLog[2, -((b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]))])/((a^
2 - b^2)^(3/2)*d^2) + (a*f*PolyLog[2, -((b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
 b^2]))])/((a^2 - b^2)^(3/2)*d^2) - (b*(e + f*x)*Sin[c + d*x])/((a^2 - b^2)
*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.522643, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3324, 3321, 2264, 2190, 2279, 2391, 2668, 31}

$$-\frac{af\mathrm{Li}_2\left(-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\left(a^2-b^2\right)^{3/2}}+\frac{af\mathrm{Li}_2\left(-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2\left(a^2-b^2\right)^{3/2}}-\frac{f\log(a+b\cos(c+dx))}{d^2\left(a^2-b^2\right)}-\frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{ia(e+fx)\log}{d\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] ((-I)*a*(e + f*x)*Log[1 + (b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2
 - b^2)^(3/2)*d) + (I*a*(e + f*x)*Log[1 + (b*E^(I*(c + d*x)))/(a + Sqrt[a^2
 - b^2])])/((a^2 - b^2)^(3/2)*d) - (f*Log[a + b*Cos[c + d*x]])/((a^2 - b^2)
*d^2) - (a*f*PolyLog[2, -((b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]))])/((a^
2 - b^2)^(3/2)*d^2) + (a*f*PolyLog[2, -((b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
 b^2]))])/((a^2 - b^2)^(3/2)*d^2) - (b*(e + f*x)*Sin[c + d*x])/((a^2 - b^2)
*d*(a + b*Cos[c + d*x]))
```


Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{e+fx}{(a+b\cos(c+dx))^2}\,dx = -\frac{b(e+fx)\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{a\int \frac{e+fx}{a+b\cos(c+dx)}\,dx}{a^2-b^2} + \frac{(bf)\int \frac{\sin(c+dx)}{a+b\cos(c+dx)}\,dx}{\left(a^2-b^2\right)d}$$

$$= -\frac{b(e+fx)\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{(2a)\int \frac{e^{i(c+dx)}(e+fx)}{b+2ae^{i(c+dx)}+be^{2i(c+dx)}}\,dx}{a^2-b^2} - \frac{f\,\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\,c\right.}{\left(a^2-b^2\right)d^2}$$

$$= -\frac{f\log(a+b\cos(c+dx))}{\left(a^2-b^2\right)d^2} - \frac{b(e+fx)\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{(2ab)\int \frac{e^{i(c+dx)}(e+fx)}{2a-2\sqrt{a^2-b^2}+2be^{i(c+dx)}}\,d}{\left(a^2-b^2\right)^{3/2}}$$

$$= -\frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} + \frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{f\log(a+b\cos(c+dx))}{\left(a^2-b^2\right)d^2}$$

$$= -\frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} + \frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{f\log(a+b\cos(c+dx))}{\left(a^2-b^2\right)d^2}$$

$$= -\frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} + \frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{f\log(a+b\cos(c+dx))}{\left(a^2-b^2\right)d^2}$$

Mathematica [B] time = 9.65293, size = 933, normalized size = 3.15

$$\left(\frac{2a(de-cf)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}} + f\log\left(\sec^2\left(\frac{1}{2}(c+dx)\right)\right) - f\log\left((a+b\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\right) - \frac{iaf\left(\log\left(1-i\tan\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-(b*d*e*Sin[c + d*x]) + b*c*f*Sin[c + d*x] - b*f*(c + d*x)*Sin[c + d*x])/(
(a - b)*(a + b)*d^2*(a + b*Cos[c + d*x])) + (Cos[(c + d*x)/2]^2*((2*a*(d*e
- c*f)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqr
t[a + b]) + f*Log[Sec[(c + d*x)/2]^2] - f*Log[(a + b*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2] - (I*a*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(Sqrt[a + b] - Sqrt[-
a + b]*Tan[(c + d*x)/2])/(I*Sqrt[-a + b] + Sqrt[a + b])] + PolyLog[2, (Sqrt
[-a + b]*(1 - I*Tan[(c + d*x)/2]))/(Sqrt[-a + b] - I*Sqrt[a + b])]))/(Sqrt[
-a + b]*Sqrt[a + b]) + (I*a*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(I*(Sqrt[a +
 b] + Sqrt[-a + b]*Tan[(c + d*x)/2]))/(Sqrt[-a + b] + I*Sqrt[a + b])] + Pol
yLog[2, (Sqrt[-a + b]*(1 - I*Tan[(c + d*x)/2]))/(Sqrt[-a + b] + I*Sqrt[a +
b])]))/(Sqrt[-a + b]*Sqrt[a + b]) - (I*a*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log
[(Sqrt[a + b] + Sqrt[-a + b]*Tan[(c + d*x)/2])/(I*Sqrt[-a + b] + Sqrt[a + b
])] + PolyLog[2, (Sqrt[-a + b]*(1 + I*Tan[(c + d*x)/2]))/(Sqrt[-a + b] - I*
Sqrt[a + b])]))/(Sqrt[-a + b]*Sqrt[a + b]) + (I*a*f*(Log[1 + I*Tan[(c + d*x
)/2]]*Log[(I*(Sqrt[a + b] - Sqrt[-a + b]*Tan[(c + d*x)/2]))/(Sqrt[-a + b] +
 I*Sqrt[a + b])] + PolyLog[2, (Sqrt[-a + b]*(1 + I*Tan[(c + d*x)/2]))/(Sqrt
[-a + b] + I*Sqrt[a + b])]))/(Sqrt[-a + b]*Sqrt[a + b]))*(a*d*e + a*d*f*x +
 b*f*Sin[c + d*x])*(Sqrt[a + b] - Sqrt[-a + b]*Tan[(c + d*x)/2])*(Sqrt[a +
b] + Sqrt[-a + b]*Tan[(c + d*x)/2]))/((a^2 - b^2)*d^2*(a + b*Cos[c + d*x])*
(a*(d*e - c*f + I*f*Log[1 - I*Tan[(c + d*x)/2]] - I*f*Log[1 + I*Tan[(c + d*
x)/2]]) + b*f*Sin[c + d*x]))
```

Maple [B] time = 0.795, size = 674, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2*I*(f*x+e)*(a*exp(I*(d*x+c))+b)/d/(-a^2+b^2)/(b*exp(2*I*(d*x+c))+2*a*exp(I
*(d*x+c))+b)-2/d^2/(-a^2+b^2)*f*ln(exp(I*(d*x+c)))+1/d^2/(-a^2+b^2)*f*ln(b*
exp(2*I*(d*x+c))+2*a*exp(I*(d*x+c))+b)+2*I/d/(-a^2+b^2)^(3/2)*a*e*arctan(1/
2*(2*b*exp(I*(d*x+c))+2*a)/(-a^2+b^2)^(1/2))+I/d/(-a^2+b^2)*a*f/(a^2-b^2)^(
1/2)*ln((-b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*x+I/d^2
/(-a^2+b^2)*a*f/(a^2-b^2)^(1/2)*ln((-b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)-a)/(-
a+(a^2-b^2)^(1/2)))*c-I/d/(-a^2+b^2)*a*f/(a^2-b^2)^(1/2)*ln((b*exp(I*(d*x+c
))+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*x-I/d^2/(-a^2+b^2)*a*f/(a^2-b^2)
^(1/2)*ln((b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*c+1/d^2
/(-a^2+b^2)*a*f/(a^2-b^2)^(1/2)*dilog((-b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)-a)
/(-a+(a^2-b^2)^(1/2)))-1/d^2/(-a^2+b^2)*a*f/(a^2-b^2)^(1/2)*dilog((b*exp(I*
(d*x+c))+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))-2*I/d^2/(-a^2+b^2)^(3/2)*a
*f*c*arctan(1/2*(2*b*exp(I*(d*x+c))+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.88441, size = 3575, normalized size = 12.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a*b^2*f*cos(d*x + c) + a^2*b*f)*sqrt((a^2 - b^2)/b^2)*dilog(-1/2*(2*
a*cos(d*x + c) + 2*I*a*sin(d*x + c) + 2*(b*cos(d*x + c) + I*b*sin(d*x + c))
*sqrt((a^2 - b^2)/b^2) + 2*b)/b + 1) - (a*b^2*f*cos(d*x + c) + a^2*b*f)*sqr
t((a^2 - b^2)/b^2)*dilog(-1/2*(2*a*cos(d*x + c) + 2*I*a*sin(d*x + c) - 2*(b
*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + 2*b)/b + 1) + (a*
b^2*f*cos(d*x + c) + a^2*b*f)*sqrt((a^2 - b^2)/b^2)*dilog(-1/2*(2*a*cos(d*x
 + c) - 2*I*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^
2 - b^2)/b^2) + 2*b)/b + 1) - (a*b^2*f*cos(d*x + c) + a^2*b*f)*sqrt((a^2 -
b^2)/b^2)*dilog(-1/2*(2*a*cos(d*x + c) - 2*I*a*sin(d*x + c) - 2*(b*cos(d*x
+ c) - I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + 2*b)/b + 1) - (-I*a^2*b*d*
f*x - I*a^2*b*c*f + (-I*a*b^2*d*f*x - I*a*b^2*c*f)*cos(d*x + c))*sqrt((a^2
- b^2)/b^2)*log(1/2*(2*a*cos(d*x + c) + 2*I*a*sin(d*x + c) + 2*(b*cos(d*x +
```

```
 c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + 2*b)/b) - (I*a^2*b*d*f*x +
I*a^2*b*c*f + (I*a*b^2*d*f*x + I*a*b^2*c*f)*cos(d*x + c))*sqrt((a^2 - b^2)/
b^2)*log(1/2*(2*a*cos(d*x + c) + 2*I*a*sin(d*x + c) - 2*(b*cos(d*x + c) + I
*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + 2*b)/b) - (I*a^2*b*d*f*x + I*a^2*b
*c*f + (I*a*b^2*d*f*x + I*a*b^2*c*f)*cos(d*x + c))*sqrt((a^2 - b^2)/b^2)*lo
g(1/2*(2*a*cos(d*x + c) - 2*I*a*sin(d*x + c) + 2*(b*cos(d*x + c) - I*b*sin(
d*x + c))*sqrt((a^2 - b^2)/b^2) + 2*b)/b) - (-I*a^2*b*d*f*x - I*a^2*b*c*f +
 (-I*a*b^2*d*f*x - I*a*b^2*c*f)*cos(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(1/2
*(2*a*cos(d*x + c) - 2*I*a*sin(d*x + c) - 2*(b*cos(d*x + c) - I*b*sin(d*x +
 c))*sqrt((a^2 - b^2)/b^2) + 2*b)/b) + ((a^2*b - b^3)*f*cos(d*x + c) + (a^3
 - a*b^2)*f - (I*a^2*b*d*e - I*a^2*b*c*f + (I*a*b^2*d*e - I*a*b^2*c*f)*cos(
d*x + c))*sqrt((a^2 - b^2)/b^2))*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c)
+ 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + ((a^2*b - b^3)*f*cos(d*x + c) + (a^3 -
 a*b^2)*f - (-I*a^2*b*d*e + I*a^2*b*c*f + (-I*a*b^2*d*e + I*a*b^2*c*f)*cos(
d*x + c))*sqrt((a^2 - b^2)/b^2))*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c)
+ 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + ((a^2*b - b^3)*f*cos(d*x + c) + (a^3 -
 a*b^2)*f - (I*a^2*b*d*e - I*a^2*b*c*f + (I*a*b^2*d*e - I*a*b^2*c*f)*cos(d*
x + c))*sqrt((a^2 - b^2)/b^2))*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) +
 2*b*sqrt((a^2 - b^2)/b^2) - 2*a) + ((a^2*b - b^3)*f*cos(d*x + c) + (a^3 -
a*b^2)*f - (-I*a^2*b*d*e + I*a^2*b*c*f + (-I*a*b^2*d*e + I*a*b^2*c*f)*cos(d
*x + c))*sqrt((a^2 - b^2)/b^2))*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c)
+ 2*b*sqrt((a^2 - b^2)/b^2) - 2*a) + 2*((a^2*b - b^3)*d*f*x + (a^2*b - b^3)
*d*e)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d^2*cos(d*x + c) + (a^5 - 2*
a^3*b^2 + a*b^4)*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(b*cos(d*x + c) + a)^2, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal, debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
                print("both result and optimal complex");
             fi;
             #both result and optimal complex
             if leaf_count_result<=2*leaf_count_optimal then
                return "A";
             else
                return "B";
             end if
          else #result contains complex but optimal is not
             if debug then
                print("result contains complex but optimal is not");
             fi;
             return "C";
          end if
       else # result do not contain complex
            # this assumes optimal do not as well
            if debug then
                print("result do not contain complex, this assumes optimal do
   not as well");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
               if debug then
                  print("leaf_count_result<=2*leaf_count_optimal");
               fi;
               return "A";
            else
               if debug then
                  print("leaf_count_result>2*leaf_count_optimal");
               fi;
               return "B";
            end if
       end if
  else #ExpnType(result) > ExpnType(optimal)
       if debug then
          print("ExpnType(result) > ExpnType(optimal)");
       fi;
       return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
   (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```